Reliability assessment of composite column according to Monte Carlo Simulation and Latin Hypercube Sampling

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Abstract. When a structural element, like a column, is evaluated for resistance using the finite element method or analytical calculation, all parameters related to the resistance side and action effect are treated as deterministic. Nevertheless, in probabilistic analyses, these parameters are regarded as random variables, which are defined using various distributions like the normal distribution, log-normal distribution, beta, or gamma distribution. Monte Carlo Simulation (MCS) and Latin Hypercube Sampling (LHS) are two sampling methods widely used for generating random samples from statistical distribution. The number of samples generated influences the accuracy of the sampling methods especially the Monte Carlo Simulation; generally speaking, the more samples generated, the more accurate the results and this also implies a significant amount of work and computation time. Consequently, the purpose of this paper is to compare the two sampling methods for the reliability assessment of the resistance of the composite column to flexural buckling under compression force.

1 Introduction

In the standard finite element method (FEM) the resistance as well as the action effect for the considered structural element such as composite column in steel and concrete are considered deterministic. On the other hand, the probabilistic analysis or so-called stochastic finite element method (SFEM) involve the consideration of the randomness related to resistance and action effect. In structural engineering, the normal distribution or log-normal distribution are used respectively, to express the randomness of the resistance part, such as the cross-sectional geometry or the material parameters according to **Chyba! Nenašiel sa žiaden zdroj o dkazov.**

As per Eurocode 0 1, a full probabilistic analysis is considered when the action effect and the resistance of the structural element are both regarded as random variables. A semi-probabilistic analysis is on the other hand when the randomness is limited to the resistance part of the structural element.
Monte Carlo Simulation (MCS) and Latin Hypercube Sampling (LHS) are two sampling methods widely used for generating random samples from statistical distribution of main

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geometric and material parameters. Generally speaking, the accuracy of sampling methods is directly correlated with the number of samples produced; the higher the number, the more accurate the results will be.

Therefore, this study compares the Monte Carlo Simulation (MCS) and Latin Hypercube Sampling (LHS) methods for the reliability assessment of the resistance of the composite column to flexural buckling under compression force using non-linear stochastic finite element method (SFEM).

Software such as OpenSees 4 and QuoFEM 5 are used to conduct non-linear stochastic finite element analysis (SFEA).

2 Reliability assessment – SFEM

Stochastic finite element method (SFEM) combines the standard finite element method with the probability theory 9, by considering the resistance parameters as random variables. To perform reliability assessment using SFEM, the following steps are required:

- First step is to create a deterministic model to perform non-linear finite element analysis (NLFEA)
- Second step is to transform the deterministic model into stochastic model by treating the main geometric and material parameters as random variables described by the normal and log-normal distribution to perform non-linear stochastic finite element analysis (SFEA)
- Third step is to generate random samples using one of the sampling methods such as MCS or LHS
- Fourth step is to analyze and evaluate the results of probabilistic analysis by checking the statistical distribution of the generated samples
- Fifth step is to repeat and refine the model until the desired results is obtained.

Fig. **1** shows schematically the procedure to perform reliability assessment for composite columns using SFEM.

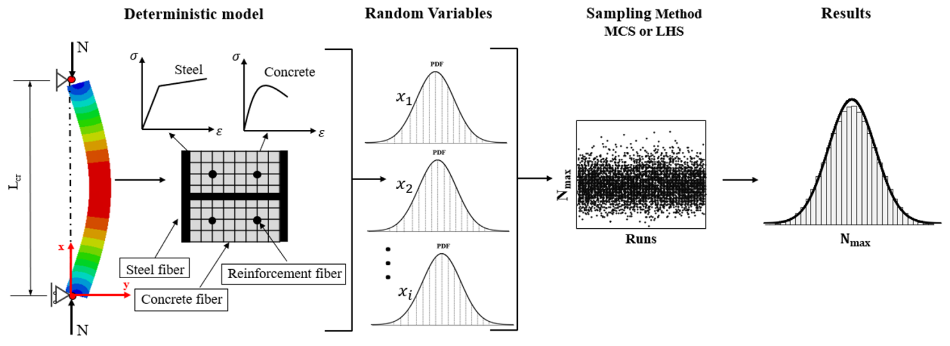

Fig. 1. Stochastic Finite Element Method for composite columns (adopted from 7).

2.1 Monte Carlo Sampling (MCS)

The name Monte Carlo Sampling (MCS) comes from casino games of Monte Carlo, Monaco 9 and is a statistical approach based on random sampling of a set of random variables. The objective is to use a stochastic process to approximate the probability of a given event.

The steps below must be followed in order to perform MCS.

1) Determine the parameters to consider as a random variable.

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2) Select the appropriate probability density function to describe the random variable e.g., normal distribution, log-normal distribution, gamma distribution, beta distribution etc.
3) Generate random samples.
4) Evaluate the outcomes.
5) Modify the number of samples and repeat until the desired results is obtained.

As previously mentioned, the number of samples directly affects the accuracy and reliability of the MCS; the more samples, the more accurate the result, and this implies more effort and computational time.

To overcome the issue with the number of samplings, a new sampling method called Latin Hypercube Sampling (LHS) first introduced by McKay, et. al 10. will be presented and explained in detail.

2.2 Latin Hypercube Sampling (LHS)

Latin Hypercube Sampling (LHS) also known as "Stratified Sampling Technique" 9 is an advanced sampling method which uses fewer samples to achieve the same level of accuracy in the results as MCS. To do this, the probability density function (PDF) of the random variables is divided into *n* equal non-overlapping intervals 7, 9 and then selecting random variables from each interval as it is presented in Fig. **2**.

Assuming that there is just a single random variable (such as the one shown in Fig. **2**) that is described by a normal distribution, in case of MCS, the selection of the variables is completely random and according to the empirical rule of the normal distribution, 68% of the values fall between the mean and one standard deviation, and 95% and 99.7% of the values fall between two and three standard deviations respectively from the mean value, and because most points are chosen around the mean value, the tails of the normal distribution may not be well-represented in the case of a small number of samples 7.

However, when LHS is used, at least one point is chosen from each of the *n* intervals, better representing the normal distribution of the random variable even with a smaller number of samples.

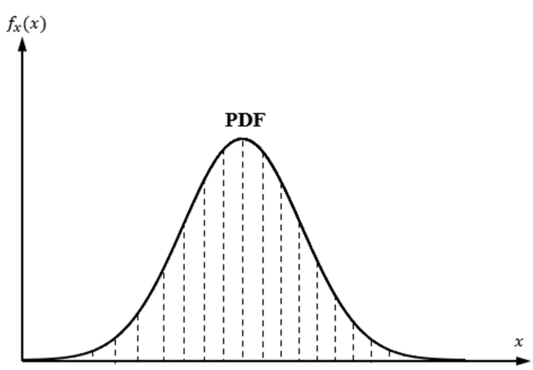

Fig. 2. Latin hypercube Sampling (Adopted from 11).

3 Considered column

The considered partial concrete encased composite column's geometry, material parameters, and boundary conditions presented in this study are adopted from 7 presented also in 8.

As shown in Fig. *3*, a simple supported HEB 280 steel profile is partially encased in a reinforced concrete with a class of C50/60 and four reinforcement bars with $\varnothing 20\ mm$ with yield strength $f_{sk} = 500\ N/mm^2$.

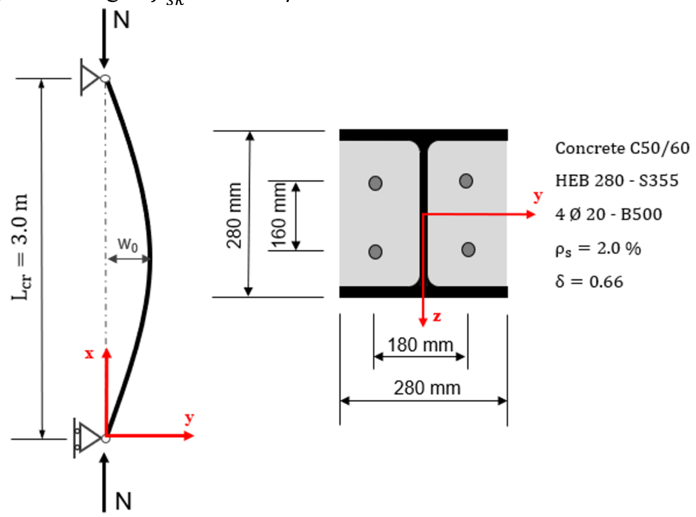

Fig. 3 Considered partial encased composite column.

To perform reliability assessment using MCS and LHS, the main parameters related to the resistance part of the column are considered as random variable as presented in Table 1, defined by normal and log-normal distribution in accordance with JCSS Model Code 6 prEN 1992-1-1 2 and prEN 1993-1-1 3.

Table 1. Summary of random variables considered for reliability assessment.

Symbol	Parameter	Mean	COV	Distribution
f_c	Concrete compressive strength	$f_{ck} + 8$	0.15	Log-normal
f_y	Steel yield stress	$1.2 \cdot f_{yk}$	0.07	Log-normal
f_s	Reinforcement yield stress	$1.078 \cdot f_{sk}$	0.04	Log-normal
E_c	Concrete elastic modulus	$9500 \cdot f_{cm}^{1/3}$	0.15	Log-normal
E_y	Steel elastic modulus	$E_{y,nom}$	0.03	Log-normal
w_0	Geometrical imperfection	$L/1000$	0.25	Normal
h_p	Height of steel profile (concrete)	$h_{p,nom}$	0.009	Normal
b_p	Width of steel profile (concrete)	$b_{p,nom}$	0.009	Normal

The sample sizes are adjusted to range from 100 to 10,000 in order to compare the outcomes of the two sampling techniques.

Based on standard deterministic non-linear finite element analysis with the material parameters for concrete, steel and reinforcement based on mean values, the resistance of the column shown in Fig. *3* is equal to $R_{max} = 8560$ kN, which is considered as reference value (true mean value μ).

4 Results

In this section the results from Monte Carlo Sampling and Latin Hypercube Sampling will be presented. As previously mentioned, the sampling size range from 100 to 10,000. First, the histogram from MCS and LHS with sample size 100, 1000 and 10,000 will be presented, then the convergence of the mean value from the true values in MCS and LHS.

The maximum resistance determined by the non-linear stochastic finite element method (SFEM) using Monte Carlo Sampling (MCS) and Latin Hypercube Sampling (LHS) is represented by the shape of the histograms with sample sizes of 100, 1000, and 10,000 in Figure 4 and Fig. **5.**

From the sensitivity analysis presented in 8, it was shown that, the main parameters which influence the resistance of the partial encased composite (PCEC) column are concrete compressive strength (f_c), and steel yield stress f_y. Since the log-normal distribution function is used to describe both parameters f_c and f_y, it is also expected that the resistance generated from the SFEM analysis will likewise follow a log-normal distribution.

The first observation from Figure 4 and Fig. ***5*** is that, as the sample size increases, the shape of the histogram become more symmetric and closer to the expected log-normal distribution. However, in comparison with MCS, the LHS have a more symmetric and closer to the expected log-normal distribution even for a sample size of 1000.

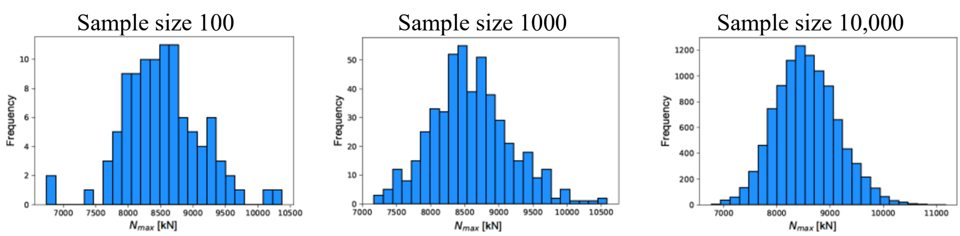

Figure 4. Histograms with sample size 100, 1000, and 10,000 using LHS

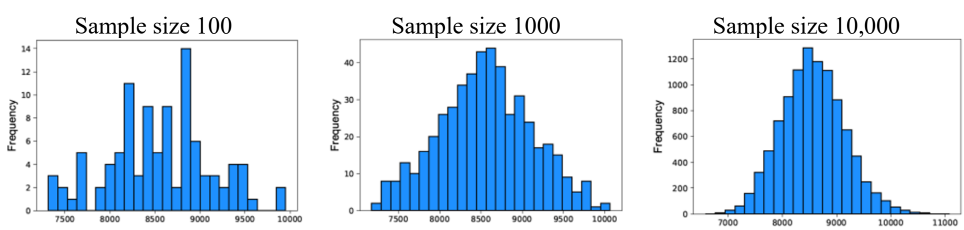

Fig. 5. Histograms with sample size 100, 1000, and 10,000 using MCS.

Moreover, after it has been identified that the LHS approach is more representative than the MCS method, the convergence analysis of these two sampling techniques will be discussed.

Fig. ***6*** shows the convergence of the LHS and MCS to the true mean value (μ) calculated using deterministic non-linear finite element analysis. The dashed black line denotes the reference value or true mean value, while the blue line represents the Monte Carlo sampling, and the red line corresponds to the Latin Hypercube sampling.

The results show that, LHS converges faster to the true mean value compared to MCS. In case of LHS, even with the sample size of 1000, the sample mean is very close to the true mean value, while on the other hand, the MCS fluctuates around the actual mean value and truly begins to approach to the true mean value after a sample size of 5000. This supports the previous claim that LHS requires fewer samples than MCS in order to produce the intended results.

This particularly important when dealing with lot of data and computational time such as the reliability assessment of composite columns using non-linear stochastic finite element analysis, as the computation time is directly correlated to the number of iterations.

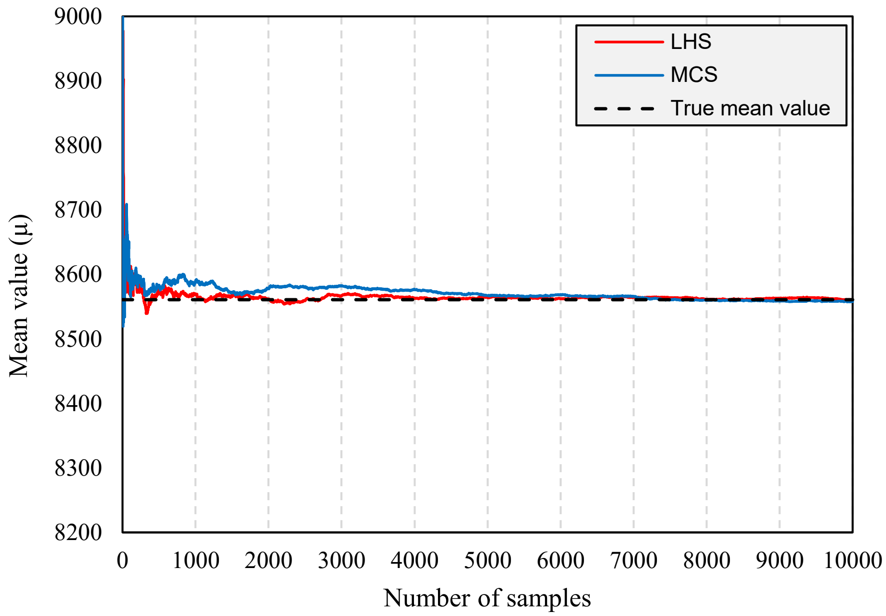

Fig. 6. Convergence of MCS and LHS.

5 Conclusion

This paper compares the Monte Carlo Simulation (MCS) and Latin Hypercube Sampling (LHS) methods for the reliability assessment of the resistance of the composite column to flexural buckling under compression force using non-linear stochastic finite element method (SFEM).

The results show that, for both methods the sample size is important and as the size increases, the shape of the histograms get closer to the desired distribution. However, in case of Latin Hypercube Sampling we need a smaller number of samples compared to Monte Carlo Sampling to reach desired result.

This is further supported by the convergence analysis between these two sampling techniques, which shows that the LHS converges to the true mean value more quickly than the MCS, as with LHS this is reached with a sample size of 1000, while in the case of MCS the sample mean approaches the true mean value after a sample size of 5000.

References

1. EN 1990 (2002): Eurocode 0 – Basic of structural design, CEN – European committee for Standardization, Brussels, Belgium, (2002)
2. prEN 1992-1-1, Eurocode 2: Design of Concrete Structures – Part 1-1, General rules, and rules for Buildings, bridges, and civil engineering structures, (2021)
3. prEN 1993-1-1, Eurocode 3: Design of Steel Structures – Part 1-1: General rules and rules for building, (2019)
4. S. Mazzoni, F. McKenna, M.H. Scott, and G. L. Fenves: 'Open system for earthquake engineering simulation user command language manual (OpenSees)', University of California, Berkeley, California, (2006)
5. F. McKenna, A. Zsarnóczay, S. Yi, A. Satish, N. Padhye: 'Quantified Uncertainty with Optimization for the Finite Element Method (QuoFEM)', University of California, Berkeley, California (2018)
6. Joint Committee on Structural Safety (JCSS) – Probabilistic Model Code – Part 3: Material properties, October (2000)
7. P. Zogu: 'Safety and Reliability Assessment of Composite Columns in Steel and Concrete Using Non-linear Finite Element Analysis', University of Luxembourg, Luxembourg (2024)
8. P. Zogu, M. Schäfer: 'Global sensitivity analysis of composite columns in steel and concrete', 24th DASt-Colloquium, Technical University of Munich (TUM), Munich, Germany, (2024)
9. S.-K. Choi, R. V. Grandhi, and R. A. Canfield, Reliability based structural design, (2006)
10. M. D. McKay, R. J. Beckman, and W. J. Conover, 'A comparison of three methods for selecting values of input variables in the analysis of output from a computer code', Technometrics, vol. **42**, no. 1, pp. 55–61, (2000), doi: 10.1080/00401706.2000.10485979
11. S. Shayan, 'System Reliability-Based Design of 2D Steel Frames by Advanced Analysis', PhD Thesis, The University of Sydney, Sydney, Australia (2013)