Solution of the Heat and Mass Transfer Problem for Soil Radiant Heating Conditions Using the Method of Finite Integral Fourier Transform

Khristina Maksudovna Vafaeva[1,2], *Denis Fedorovich Karpov*[3], *Mikhail Vasilyevich Pavlov*[4], *Aleksei Marlenovich Ismailov*[5], *Sivaraman Baskar*[6], *Tarun Kapoor*[7], *Devendra Singh*[8], *Nishant Bhardwaj*[9], *Perumalla Varaprasada Rao*[10], *Jitendra Gudainiyan*[11]

[1]Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russia
[2]Division of research and development, Lovely Professional University, Phagwara, Punjab, India
[3]Department of Heat, Gas and Water Supply, Vologda State University, Vologda, Russian Federation
[4]Department of Heat, Gas and Water Supply, Vologda State University, Vologda, Russian Federation
[5]Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russia
[6]Department of H&S, KG Reddy College of Engineering and Technology, Chilkur(Vil), Moinabad(M), Ranga Reddy(Dist), Hyderabad, 500075,Telangana, India.
[7]Centre of Research Impact and Outcome, Chitkara University, Rajpura- 140417, Punjab, India
[8]Uttaranchal University, Dehradun - 248007, India
[9]Chitkara Centre for Research and Development, Chitkara University, Himachal Pradesh-174103 India
[10]Department of CSE, GRIET, Bachupally, Hyderabad, Telangana, India.
[11] Department of Civil Engineering, GLA University, Mathura-281406 (U.P.), India
Corresponding Email: vafaeva.khm@gmail.com

Abstract. To achieve high agricultural yields, it is essential to predict the soil temperature and moisture regime, considering the heating technology employed. The research object is soil heated by a ceiling-mounted infrared emitter. The research subject encompasses one-dimensional unsteady fields of soil moisture content and temperature. The research goal is to forecast the soil temperature and moisture regime under radiant heating conditions. The research methods involve the analytical solution of heat and mass transfer differential equations using the method of finite integral Fourier transforms. Research results indicate that the top layer of milled peat, with an initial moisture content of 3.7 kg/kg, will reach a final moisture content of 1.0 kg/kg in approximately 6 hours during infrared drying. As a result of radiant heating, the soil temperature will rise from an initial 5 °C to a final 22.6 °C in approximately 3 hours. The analytical solution of the mass transfer differential equation can be utilized for theoretical studies of drying capillary-porous materials, such as determining the drying period or the thickness of the material layer that will dry to a specified final moisture content. The analytical solution of the heat transfer differential equation, accounting for both thermal conductivity and the Dufour effect, can be employed to manage the operation of the infrared radiation source, such as determining its operational and shutdown periods when the soil surface temperature reaches its maximum (critical) value. The mathematical solutions discussed in the article do not consider thermodiffusion processes in the soil layer (Soret effect), which presents a promising direction for further scientific research.

Keywords: Moisture Content, Temperature, Heat and Mass Transfer Differential Equations; One-Dimensional Unsteady Field, Soil, Radiant Heating, Dufour Effect, Soret Effect.

1 Introduction

Research on thermal and mass transfer processes occurring in cultivation structures, such as greenhouses designed for year-round plant cultivation, can be conducted at various hierarchical levels [1–5]. On one hand, in the development of software and engineering methods for designing and calculating heating systems, it is essential to consider the thermal and material balances of the entire facility where agricultural crops (e.g., tomatoes, cucumbers) are grown, including its enclosure and soil surface (integral approach) [6–10]. This approach necessitates accounting for the interconnection of heat and mass flows and their impact on the microclimate parameters within the facility: the temperature and relative humidity of the indoor air, and the soil surface temperature. On the other hand, there is scientific interest in a differential approach, where heat and mass transfer are studied locally within an individual element of the greenhouse, rather than in the entire greenhouse as a whole [11, 12]. From the perspective of favorable plant growth, development, and fruiting, it is evident that this element is the soil, where the desired variables may be moisture content and temperature of the layer, varying with coordinates and time.

As is well known, research on various heat and mass transfer processes occurring in physical bodies [13–15] is associated with solving problems of molecular energy and substance transport, which typically follow linear laws (e.g., heat conduction is described by Fourier's linear law). These linear laws form the basis for deriving the corresponding differential equations. Solving these equations under specific initial and boundary conditions, which characterize the initial state of the body and its surface interactions with the surrounding environment, presents well-known mathematical challenges [16–21]. Since the late 18th century, mathematicians from different countries have developed various methods for solving heat and mass transfer differential equations. Through the development of experimental methods, a direct relationship between energy and substance transport processes has been established (e.g., thermal energy transport can occur not only due to thermal conductivity but also due to concentration differences in a mixture, concentration gradient – Dufour effect) [22–30]. This is also characteristic of soil, which is a colloidal capillary-porous body, particularly under conditions of radiant heating [31–33].

Research object: soil heated by a ceiling-mounted infrared emitter.

Research subject: one-dimensional unsteady fields of soil moisture content and temperature.

Research objective: to predict the soil temperature and moisture regime under radiant heating conditions.

Research tasks:

1. Formulation of the problem. Establishment of boundary conditions corresponding to the radiant heating of the soil surface.

2. Obtaining solutions to the heat and mass transfer differential equations using the method of finite integral Fourier transforms.
3. Examination of the analytical solution of the heat and mass transfer differential equations in a specific example.
4. Description of the obtained solution results.

2 Methodology

Formulation of the mass transfer problem. Consider a soil layer of infinite dimensions along the coordinate axes $0x$ and $0y$ (Figure 1), with a characteristic linear dimension along the $0z$ – thickness L, m. The initial moisture content of the soil layer (at time $t = 0$) along the coordinate interval $z \in [0; L]$ is known and equals W_0, kg/kg. Due to radiant heating, moisture evaporates from the soil surface into the surrounding environment at a constant rate j, $\mathrm{kg/(m^2 \cdot s)}$, resulting in a change in soil moisture content along the $0z$ coordinate over time t, s. Determine the moisture content distribution $W(z,t)$.

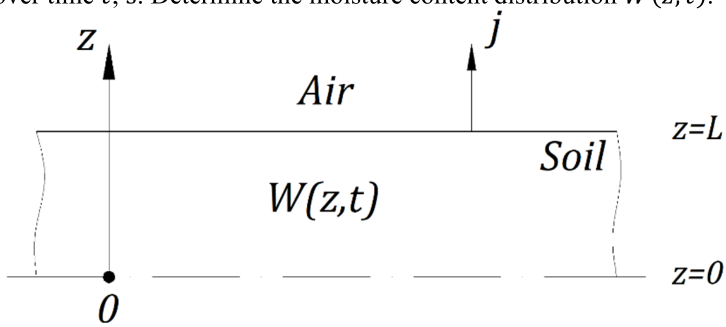

Fig. 1. Formulation of the mass transfer problem.

In radiant heating, the evaporation rate at the soil surface can be determined using Dalton's formula:

$$j = \alpha_m (p_{surf} - p_\infty),\ \mathrm{kg/(m^2 \cdot s)}, \quad (1)$$

where α_m is the external moisture exchange coefficient (mass transfer coefficient), $\mathrm{kg/(m^2 \cdot s \cdot Pa)}$, which is calculated using criterion similarity equations for mass transfer processes or approximated by Lewis's formula; p_{surf} and p_∞ are the partial vapor pressures at the soil surface and in the ambient environment, respectively, Pa.

The differential mass transfer equation and the boundary conditions for its solution are as follows (Figure 1):

$$\frac{\partial W(z,t)}{\partial t} = D \frac{\partial W^2(z,t)}{\partial z^2}\ (0 \le z \le L, t > 0), \quad (2)$$

$$W(z,0) = W_0, \quad (3)$$

$$D\rho \frac{\partial W(L,t)}{\partial z} + j = 0, \quad (4)$$

$$W(0,t) = W_0, \frac{\partial W(0,t)}{\partial z} = 0, \quad (5)$$

where W is the moisture content, kg/kg; z is the coordinate, m; t is time, s; D is the soil diffusion coefficient, $\mathrm{m^2/s}$; ρ is the soil density, $\mathrm{kg/m^3}$; j is the evaporation rate at the soil surface, $\mathrm{kg/(m^2 \cdot s)}$.

The solution to the mass transfer differential equation (2) with initial (3) and boundary (4), (5) conditions is known and is given by:

$$W(z,t) = W_0 - \frac{jL}{D\rho}\left[\mathrm{Fo}_W - \frac{1}{6}\left(1 - 3\frac{z^2}{L^2}\right) + \right.$$
$$+\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{n^2\pi^2}\cos\left(n\pi\frac{z}{L}\right)e^{-n^2\pi^2\mathrm{Fo}_W} + \tag{6}$$
$$\left. + \frac{2W_0}{L^2}\sum_{n=1}^{\infty}\cos\left(n\pi\frac{z}{L}\right)e^{-n^2\pi^2\mathrm{Fo}_W}\cdot\int_0^h\cos\left(n\pi\frac{z}{L}\right)dz\right],$$

where $\mathrm{Fo}_W = \frac{Dt}{L^2}$ this is the Fourier number for mass transfer.

Formulation of the heat transfer problem. Consider a soil layer of infinite dimensions along the coordinate axes $0x$ and $0y$ (Figure 2), with a characteristic linear dimension along the $0z$ axis – thickness L, m. The initial temperature of the soil layer (at time $t = 0$) along the coordinate interval $z \in [0; L]$ is known and is T_0, °C. Under the influence of a radiant flux with constant intensity q, $\mathrm{W/m^2}$, the soil temperature varies along the $0z$ coordinate over time t, s. The temperature field is influenced not only by thermal conductivity but also by the transfer of thermal energy with water vapor as it moves through the soil capillaries, induced by concentration differences of the mixture components (Dufour effect). Determine the temperature distribution $T(z,t)$.

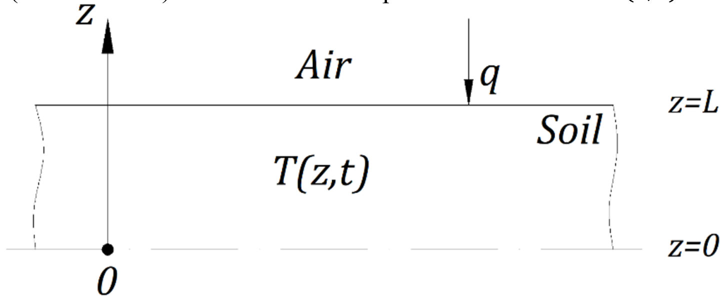

Fig. 2. Formulation of the heat transfer problem.

The portion of the thermal flux from the infrared radiation source affecting the temperature field of the soil layer is calculated using the heat balance equation for its surface:

$$q = \left(1 - \frac{R}{100}\right)q_{inf} - q_{vp} - q_{conv},\ \mathrm{W/m^2}, \tag{7}$$

where R is the surface thermal reflectance of the soil, %; q_{inf} is the thermal flux density from the infrared radiation source, $\mathrm{W/m^2}$; q_{vp} is the thermal flux density consumed for moisture evaporation from the soil surface (directly proportional to the evaporation intensity j), $\mathrm{W/m^2}$; q_{conv} is the thermal flux density due to convective heat exchange between the soil surface and the surrounding environment (air), $\mathrm{W/m^2}$, determined using Newton's law of cooling.

The differential heat transfer equation, accounting for changes in the temperature field of the soil layer due to both external thermal influences and internal vapor diffusion processes (Dufour effect), along with the boundary conditions for its solution, is as follows (Figure 2):

$$\frac{\partial T(z,t)}{\partial t} = \alpha\frac{\partial T^2(z,t)}{\partial z^2} + \frac{\varepsilon h_{fg}}{c}\frac{\partial W(z,t)}{\partial t}\ (0 \le z \le L, t > 0), \tag{8}$$

$$T(z,0) = T_0, \tag{9}$$

$$k\frac{\partial T(L,t)}{\partial z} + q = 0, \tag{10}$$

$$T(0,t) = T_0, \frac{\partial T(0,t)}{\partial z} = 0, \tag{11}$$

where T is the temperature, °C; z is the coordinate, m; t is time, s; α is the soil thermal diffusivity, $\mathrm{m^2/s}$; ε is the phase transition criterion, which represents the fraction of moisture flux in the form of water vapor within the total moisture flux; h_{fg} is the latent heat of vaporization, J/kg; c is the specific heat capacity of the soil, J/(kg · K); k is the thermal conductivity of the soil, W/(m · K); q is the thermal flux density at the soil surface, $\mathrm{W/m^2}$.

The solution to the heat transfer differential equation (8) with initial (9) and boundary (10), (11) conditions is known and is given by:

$$\begin{aligned} T(z,t) = T_0 + \frac{qL}{k}\Bigg[\mathrm{Fo}_T - \frac{1}{6}\left(1 - 3\frac{z^2}{L^2}\right) + \\ + \sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{n^2\pi^2}\cos\left(n\pi\frac{z}{L}\right)e^{-n^2\pi^2\mathrm{Fo}_T} + \\ + \frac{2T_0 k}{qL^2}\sum_{n=1}^{\infty}\cos\left(n\pi\frac{z}{L}\right)e^{-n^2\pi^2\mathrm{Fo}_T}\cdot\int_0^L\cos\left(n\pi\frac{z}{L}\right)dz\Bigg] + \\ + \frac{\varepsilon h_{fg}}{c}\frac{1}{1-\mathrm{Le}}\frac{jL}{D\rho}\Bigg\{\mathrm{Fo}_T - \mathrm{Fo}_W + \sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{n^2\pi^2}\cos\left(n\pi\frac{z}{L}\right)\times \\ \times\left[e^{-n^2\pi^2\mathrm{Fo}_T} - e^{-n^2\pi^2\mathrm{Fo}_W}\right] - \frac{2W_0 D\rho}{jL^2}\sum_{n=1}^{\infty}\cos\left(n\pi\frac{z}{L}\right)\times \\ \times\left[e^{-n^2\pi^2\mathrm{Fo}_T} - e^{-n^2\pi^2\mathrm{Fo}_W}\right]\cdot\int_0^L\cos\left(n\pi\frac{z}{L}\right)dz\Bigg\}, \end{aligned} \quad (12)$$

where $\mathrm{Fo}_T = \frac{\alpha t}{L^2}$ is the Fourier number; $\mathrm{Le} = \frac{\alpha}{D}$ is the Lewis number.

3 Results and Discussion

Let's present the solution of the mass transfer differential equation (2) using the example of milled peat (Figure 3) with the following initial data: $L = 0.02$ m; $\rho = 74\,\mathrm{kg/m^3}$; $D = 2.0\cdot10^{-8}\,\mathrm{m^2/s}$; $W_0 = 3.7$ kg/kg; $j = 170\cdot10^{-6}\,\mathrm{kg/(m^2\cdot s)}$.

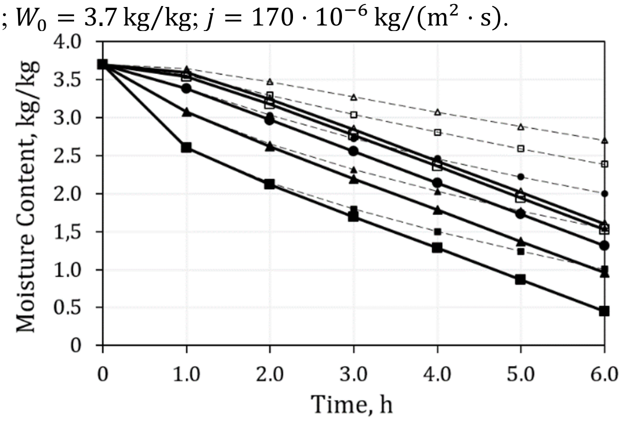

Fig. 3. Solution of the mass transfer differential equation (solid line - method of finite integral Fourier transform; dashed line - method using the error function) at z: ■ – 20 mm; ▲ – 15 mm; ● – 10 mm; □ – 5 mm; △ – 0 mm.

Over the drying period of milled peat, which is 6 hours, the average deviation in soil moisture content values (Figure 3), determined using the method of finite integral Fourier transform and the error function, does not exceed 20 %.

Let's present the solution of the heat transfer differential equation (8) using the example of milled peat (Figure 4) with the following initial data: $L = 0.02$ m; $k = 0.302\ \mathrm{W/(m \cdot K)}$; $\alpha = 14.8 \cdot 10^{-8}\ \mathrm{m^2/s}$; $\rho = 74\ \mathrm{kg/m^3}$; $D = 2.0 \cdot 10^{-8}\ \mathrm{m^2/s}$; Le $= 7.4$; $\varepsilon = 0.1$; $h_{fg} = 2.472 \cdot 10^6$ J/kg; $c = 2.75 \cdot 10^4$ J/(kg · K); $T_0 = 5$ °C; $W_0 = 3.7$ kg/kg; $q = 100\ \mathrm{W/m^2}$; $j = 170 \cdot 10^{-6}\ \mathrm{kg/(m^2 \cdot s)}$.

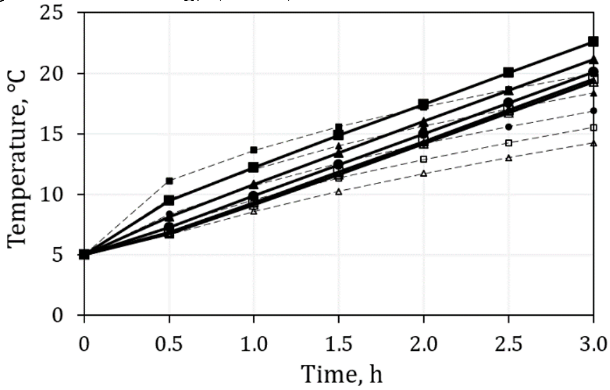

Fig. 4. Solution of the heat transfer differential equation (solid line - method of finite integral Fourier transform; dashed line - method using the error function, excluding the Dufour effect) at *z*: ■ – 20 mm; ▲ – 15 mm; ● – 10 mm; □ – 5 mm; △ – 0 mm.

Over the heating period of milled peat, equal to 3 hours, the average deviation of soil temperature values (Figure 4), found using the method of finite integral Fourier transform and the error function (excluding the Dufour effect), does not exceed 10 %.

Based on the mathematical solutions (6) and (12) of the heat and mass transfer differential equations (2) and (8), respectively, a program was developed in the mathematical editor Mathcad to determine the moisture content and temperature of an infinite slab.

4 Conclusions

According to the results of computational modeling, the top layer of milled peat with a thickness of 5 mm and an initial moisture content of 3.7 kg/kg will reach a final moisture content of 1.0 kg/kg in approximately 6 hours during infrared drying (Figure 3). As a result of radiant heating, the soil will reach a final temperature of 22.6 °C from an initial temperature of 5 °C in approximately 3 hours (Figure 4).

The analytical solutions of the heat and mass transfer differential equations (2) and (8), represented as algebraic functions (6) and (12), respectively, allow for the investigation of the influence of initial parameters (e.g., the intensity of moisture evaporation from the soil surface or the magnitude of heat flux density) on the soil temperature and moisture regime. This is a crucial factor in creating a favorable microclimate in cultivation structures for growing seedlings or for the growth of greens, vegetables, and flowers in protected environments. Additionally, equation (6) can be used for theoretical studies of drying capillary-porous materials (e.g., lump, ground, or milled peat), while equation (12) can be employed to manage the operation of the infrared radiation source.

At this stage of scientific research, the mass transfer differential equation (2) does not include a term accounting for the cross-process of heat and mass transfer – thermodiffusion (Soret effect) [22, 24, 25], i.e., moisture conductivity induced by the temperature gradient of the substance in the mixture. Therefore, obtaining a solution to the mass transfer differential equation using the combined application of the Laplace integral transform method [26, 34, 35] and the Bubnov-Galerkin variational method [36–38] presents scientific interest. This approach takes into account both vapor diffusion processes (Dufour effect) and the phenomenon of thermo-moisture conductivity (Soret effect) [27–30].

5 Funding

This research was funded by the Ministry of Science and Higher Education of the Russian Federation within the framework of the state assignment No. 075-03-2022-010 dated 14 January 2022 and No. 075--01568-23-04 dated 28 March 2023(Additional agreement 075-03-2022-010/10 dated 09 November 2022, Additional agreement 075-03-2023-004/4 dated 22 May 2023), FSEG-2022-0010.

References

1. Huang, Shuanggen and Liu, Quanyao and Wu, Yan and Chen, Minmin and Yin, Hua and Zhao, Jinhui. *Edible Mushroom Greenhouse Environment Prediction Model Based on Attention CNN-LSTM*. Agronomy. **14**. 473. (2024). https://doi.org/10.3390/agronomy14030473
2. Behroozeh, Samira and Hayati, Dariush and Karami, Ezatollah. *Factors influencing energy consumption efficiency in greenhouse cropping systems*. Environment, Development and Sustainability. 1-36. (2024). https://doi.org/10.1007/s10668-024-04851-8
3. Ngwarati, Tendai and Kevin, Mtondo and Francis, Hove and Sibanda, Simelokuhle. Optimization of temperature, humidity and airflow in African Indigenous Vegetable (AIV) greenhouse. i-manager's Journal on Future Engineering and Technology. **18**. 6. (2023). https://doi.org/10.26634/jfet.18.4.19760
4. Mikhail Vasilyevich Pavlov, Khristina Maksudovna Vafaeva, Denis Fedorovich Karpov, Saurav Dixit, Prashanth Kumar, Abhishek Joshi, Rahul Mishra and Manvinder Brar. *Impact of Environmental Factors on Indoor Air Temperature in Gas-Fired Radiant Heated Cultivated Structures*. E3S Web of Conferences. **511**. 01036. (2024). https://doi.org/10.1051/e3sconf/202451101036
5. Denis Fedorovich Karpov, Khristina Maksudovna Vafaeva, Mikhail Vasilyevich Pavlov, Saurav Dixit, P. Ravikanth, Rishi Dev Nautiyal, Ankit Punia and Praney Madan. *Parametric Analysis of a Radiant Gas Heating System for Controlled-Environment Agriculture with Preheated Ventilation*. E3S Web of Conferences. **511**. 01010. (2024). https://doi.org/10.1051/e3sconf/202451101010
6. Singh, R., Hussain, A., Fezaa, L.H.A., Gupta, G., Singh, A.P., Dogra, A. *Greenhouse Automation System Using ESP32 and ThingSpeak for Temperature, Humidity, and Light Control*. In: Tiwari, S., Trivedi, M.C., Kolhe, M.L., Singh, B.K. (eds) Advances in Data and Information Sciences. ICDIS 2023. Lecture Notes in Networks and Systems. Vol. **796**. Springer. Singapore. (2024). https://doi.org/10.1007/978-981-99-6906-7_43
7. Hsu, Hsin and Dirmeyer, Paul. *Soil moisture-evaporation coupling shifts into new gears under increasing* CO_2. Nature Communications. **14**. (2023). https://doi.org/10.1038/s41467-023-36794-5
8. Sabziparvar, Ali-Akbar and Jahromi, Fatemeh. *Evaluating the most effective climatic parameters affecting the monthly mean soil temperature estimates using the PLS method*. Arabian Journal of Geosciences. **15**. (2022). https://doi.org/10.1007/s12517-022-10297-x
9. Si, Changqing and Qi, Fei and Ding, Xiaoming and He, Fen and Gao, Zhenjun and Feng, Qian and Zheng, Liang. *CFD Analysis of Solar Greenhouse Thermal and*

Humidity Environment Considering Soil–Crop–Back Wall Interactions. Energies. **16**. 2305. (2023). https://doi.org/10.3390/en16052305

10. Wu, Mingge. *Research on decoupling greenhouse temperature and humidity based on feedback linearization*. Journal of Electrical Systems. **20**. 394-399. (2024). https://doi.org/10.52783/jes.1192
11. Nagarajah, Arravinthen and Azizan, Fathin and Zalani, Fatnin. *Assessing Changes in Soil Moisture Distribution for Before and After Irrigation in a Harumanis Greenhouse*. Advanced and Sustainable Technologies (ASET). 3. 36-44. (2024). https://doi.org/10.58915/aset.v3i.580
12. Nam, Sang-Woon. *The Effect of Soil Warming on the Greenhouse Heating Load*. Journal of The Korean Society of Agricultural Engineers. **48**. (2006). https://doi.org/10.5389/KSAE.2006.48.5.051
13. Vafaeva, Khristina Maksudovna and Dhyani, Manoj and Acharya, Puja and Parik, Khushbu and Ledalla, Sukanya. *Glass-basalt-plastic materials for construction in temperate and Arctic climatic regions*. BIO Web of Conferences. **86**. (2024). https://doi.org/10.1051/bioconf/20248601111
14. Vafaeva, Khristina Maksudovna and Duklan, Nitin and Mohan, Chandra and Kumar, Yogesh and Ledalla, Sukanya. *Comparative Analysis of Glass-Basalt-Plastic Materials for Construction in Arctic Conditions*. BIO Web of Conferences. **86**. (2024). https://doi.org/10.1051/bioconf/20248601112
15. Vafaeva, Khristina Maksudovna and Zegait, Rachid. *Carbon nanotubes: revolutionizing construction materials for a sustainable future: A review*. Research on Engineering Structures and Materials. (2023). https://doi.org/10.17515/resm2023.42ma0818rv
16. Sha, Bibhu and Mohanty, Dr-Rajiv and Patel, Dishant and Das, Mihir. *Thermo-Hydraulic Evaluation of Isothermally Heated Bluff Bodies with Different Shapes*. 401-413. (2024). https://doi.org/10.1007/978-981-99-7827-4_32
17. Shadmanov, Istam and Shafiyev, Tursun. *Mathematical modeling of the processes of combined heat and moisture transfer during storage and drying of raw cotton*. E3S Web of Conferences. **431**. 01060. (2023). https://doi.org/10.1051/e3sconf/202343101060
18. Campo, Antonio. *Inclusion of unsteady heat conduction in regular bodies subject to uniform surface heat flux in a heat transfer course*. International Journal of Mechanical Engineering Education. (2023). https://doi.org/10.1177/03064190231188318
19. Tushavina, O. and Egorova, M. *Problems of heat and mass transfer in chemically reacting boundary layers on blunted bodies*. Uchenye Zapiski Kazanskogo Universiteta. Seriya Fiziko-Matematicheskie Nauki. **165**. 294-306. (2024). https://doi.org/10.26907/2541-7746.2023.3.294-306
20. Korotkov, Aleksey and Andrey, Kozelkov and Kurkin, Andrey and Giniyatullin, Robert and Lashkin, Sergey. *Numerical Simulation of the Conjugate Heat Transfer of a "Fluid–Solid Body" System on an Unmatched Grid Interface*. Fluids. **8**. 266. (2023). https://doi.org/10.3390/fluids8100266
21. Kondratyev, S and Golosov, S and Shmakova, Marina and Ershova, Alexandra and Zverev, Ilia and Ivanova, Ekaterina and Korobchenkova, Ksenia. *System of models for assessment and forecast of heat- and mass-transfer in the system "catchment-watercourse-water body"*. IOP Conference Series: Materials Science and Engineering. **1047**. 012156. (2021). https://doi.org/10.1088/1757-899X/1047/1/012156
22. Kavitha, S. and Sugapriya, V. and Ratchagar, Nirmala. *Porosity variations on the spread of spilled oil in the soil with Soret and Dufour effects*. AIP Conference Proceedings. **2516**. 170018. (2022). https://doi.org/10.1063/5.0109579
23. Wiese, Connor and Rutledge, James. *Mass-Heat Coupling in Film Cooling Experiments–The Influence of Temperature and Turbulence on the Dufour Effect*. Journal of Turbomachinery. **145**. (2023). https://doi.org/10.1115/1.4063411

24. Chaudhary, Santosh and Deshwal, Jyoti. *Thompson and Troian velocity slip flow of the Casson hybrid nanofluid past a Darcy–Forchheimer porous inclined surface with Ohmic heating, Dufour and Soret effects*. Pramana. **98**. (2024). https://doi.org/10.1007/s12043-024-02738-x
25. Chillingo, Kidney. *Fractal fractional model of radiative heat and mass transfer characteristics of time dependent flow with variable viscosity, induced magnetic field, Dufour and Soret effects: an entropy generation*. International Journal of Advanced Research in Computer Science. **15**. 1-14. (2024). https://doi.org/10.26483/ijarcs.v15i2.7069
26. Rath, Chinmoy and Nayak, Anita. *Hall current and Dufour effect on the flow of a viscous fluid in the presence of suction, chemical reaction, and heat source: Laplace transform procedure*. Heat Transfer. **53**. (2024). n/a-n/a. https://doi.org/10.1002/htj.23024
27. Mohamed Isa, Siti Suzilliana Putri and Azmi, Hazirah and Balakrishnan, Nanthini and Arifin, N. and Rosali, Haliza. *Soret-Dufour Effects on Heat and Mass Transfer of Newtonian Fluid Flow over the Inclined Sheet and Magnetic Field*. Journal of Advanced Research in Numerical Heat Transfer. **14**. 39-48. (2023). https://doi.org/10.37934/arnht.14.1.3948
28. Obulesu, Mopuri and Ganteda, Charankumar and Alsalhi, Sarah and Khan, Sami and Ganjikunta, Aruna and Govindan, Dr.V. and Alqurashi, Faris and Kchaou, Mohamed. *Inclined surface mixed convection flow of viscous fluid with porous medium and Soret effects*. Open Physics. **22**. (2024). https://doi.org/10.1515/phys-2023-0209
29. Rafique, Khuram and Alqahtani, Aisha and Ahmad, Shahzad and Alotaibi, Hammad and Khan, Ilyas and Singh, Abha. *Buongiorno Model of Micropolar Nanofluid with Surface Inclination and Soret Effect*. BioNanoScience. 1-11. (2024). https://doi.org/10.1007/s12668-024-01368-2
30. P.A., Dinesh and Swaminathan, Gayathri and Venkatapathiah, Kuncha and B V, Shilpa and Chandrashekhar, D.V. *Soret Effect on Mixed Convection of Casson Fluid Flow in Presence of Porous Media in a Porous Channel*. Journal of Mines, Metals and Fuels. 2837-2847. (2024). https://doi.org/10.18311/jmmf/2023/41943
31. Fu, Shenghua and Pan, Yang and Wan, Xingping and Wang, Jiyue and Jia, Xiaofeng and Luo, Xinmei and Ma, Hongqiang. *Numerical Simulation of Heat Transfer Characteristics of Capillary Radiant Heating Floor*. KSCE Journal of Civil Engineering. **28**. (2024). https://doi.org/10.1007/s12205-024-1518-1
32. Gendelis, Stanislavs and Telicko, Jevgenijs and Jakovics, Andris and Bukans, Indulis. *Radiant capillary heat exchangers – power calculation for optimal heating and cooling*. Journal of Physics: Conference Series. **2423**. 012011. (2023). https://doi.org/10.1088/1742-6596/2423/1/012011
33. Dong, Jiankai and Zheng, Wenke and Ran, Zhilin and Zhang, Bei. *Experimental investigation on heating performance of a novel radiant-convective heating terminal*. Renewable Energy. **164**. 804-814. (2021). https://doi.org/10.1016/j.renene.2020.09.100
34. Almalki, Yahya and Akel, Mohamed and Abul-Dahab, Mohamed. *Solutions of certain initial-boundary value problems via a new extended Laplace transform*. Nonlinear Engineering. **13**. (2024). https://doi.org/10.1515/nleng-2022-0353
35. Li, Xin and Ma, Weiyuan and Bao, Xionggai. *Generalized fractional calculus on time scales based on the generalized Laplace transform*. Chaos Solitons and Fractals. **180**. 114599. (2024). https://doi.org/10.1016/j.chaos.2024.114599
36. Sun, Hao and Jiao, Xiaodong and Tao, Jin and Sun, Qinglin and Chen, Zengqiang. *Dynamic modelling and Chaos control for thin plate oscillator Using Bubnov-Galerkin integral method*. (2022). https://doi.org/10.22541/au.166006452.29420893/v1

37. Hamadneh, Tareq and Merker, Jochen and Schimmel, Willi and Schuldt, Gregor. *Simplicial Bernstein form and positivity certificates for solutions obtained in a stationary digital twin by Bernstein Bubnov-Galerkin method*. 41-46. (2022). https://doi.org/10.1145/3545839.3545846
38. Ofondu, Iheanyichukwu and U, Ikwueze and Ike, Charles. *Bubnov-Galerkin method for the elastic buckling of euler columns*. Malaysian Journal of Civil Engineering. **30**. (2018). https://doi.org/10.11113/mjce.v30n2.483