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Study of thermoelastic waves at the interface between layers during 3D printing of building structures

Yu. A. Chirkunov[1*], *M. Yu.* Chirkunov[1], *V. V.* Molodin[1], *A. A.* Lapidus[2], and *D. V.* Topchiy[2]

[1]Novosibirsk State University of Architecture and Civil Engineering (Sibstrin), 113 Leningradskaya st., 630008 Novosibirsk, Russia
[2] National Research Moscow State University of Civil Engineering" (NRUMGSU), 26 Yaroslavskoe shosse, 129337 Moscow, Russia

Abstract. In our paper, we study a traveling wave in a transversely isotropic medium caused by the resulting temperature gradient between layers during 3D printing of building structures. To describe this process, we use a three-dimensional model defined by a system of first-order differential equations, which, in addition to the components of the displacement vector and temperature, contains additional functions. Formulas are obtained that express the components of the displacement vector and temperature through the wave parameter. This made it possible to obtain the dependence of the displacement vector on the temperature in the traveling wave. As an example, for specific values of the quantities on which the traveling wave depends, we plotted the dependence of the components of the displacement vector on temperature.

1 Introduction

The impressive successes of 3D printing could not allow the construction industry to miss the opportunity to robotize building construction processes. Already the first proposals for the use of additive technologies for construction using monolithic concrete have shown their clear advantages [1].

At 3D printing, very important is the adhesion of the laid layers to each other, which ensures the solidity of the structure.

However, on real construction sites, a 3D printer goes around the perimeter of the building being constructed over a long period - from 20 to 60 minutes. During this time, the laid layer transfers its heat to the substrate and to the environment. In the conditions of a construction site, in the open air, concrete cools quickly even when a heated layer of concrete is laid on it. The resulting temperature gradient causes the traveling wave. There is an intermittent separation of the newly laid concrete layer from the previously laid one. This compromises the continuity of the material and the strength of the structure as a whole [2, 3].

A similar phenomenon is observed in electric arc welding of metals and explosion welding [4, 5].

* Corresponding author: chr101@mail.ru

With the advent of additive technologies in relation to concrete work, the issue of the quality of adhesion of layers during heat treatment of laid concrete has become especially acute.

To describe thermoelastic waves arising at the boundary of layers during 3D printing of building structures, we use a traveling wave of a general form in a transversally isotropic thermoelastic medium that satisfies the Gassmann condition widely used in geophysics.

The system of equations for dynamic deformation of a transversally isotropic thermoelastic medium under the Gassmann condition is written in the form [6–11]

$$\boldsymbol{u}_{tt} - \left(\lambda + 2\mu\right)\nabla \operatorname{div}\boldsymbol{u} + \operatorname{rot}\left(M \operatorname{rot}\boldsymbol{u}\right) + \nabla\theta = \boldsymbol{0}, \quad \theta_t + div\boldsymbol{u}_t = \Delta\theta, \tag{1}$$

where

$$t = \frac{c_0}{k\rho}\tilde{t}, \quad \boldsymbol{x} = \frac{c_0}{k\sqrt{\rho}}\tilde{\boldsymbol{x}}, \quad \boldsymbol{x} = \left(x,y,z\right), \quad \theta = \frac{k\sqrt{\rho}}{c_0}\tilde{\theta},$$
$$M = \operatorname{diag}\left(G, G, \mu\right), \quad \nabla = \partial_{\boldsymbol{x}}, \quad \Delta = \nabla^2; \tag{2}$$

$\boldsymbol{u} = \left(u_1\left(x, y, z, t\right), u_2\left(x, y, z, t\right), u_3\left(x, y, z, t\right)\right)$ is a displacement vector; $\tilde{\boldsymbol{x}} = \left(\tilde{x}, \tilde{y}, \tilde{z}\right)$ is a coordinate of a point in space R^3; $\tilde{t}$ is a time; λ, μ, G are the coefficients of a transversely isotropic medium; $\tilde{\theta} = \tilde{\theta}\left(x, y, z, t\right)$ is the temperature; k is a thermal conductivity coefficients; c_0 is the heat capacity coefficient and ρ is density.

Let $\boldsymbol{\omega}$, $\boldsymbol{\varphi}$, q are additional functions of variables x, y, z, t. From the system

$$\boldsymbol{u}_t = \left(\lambda + 2\mu\right)\nabla q - \operatorname{rot}\left(M\boldsymbol{\omega}\right) - \boldsymbol{\varphi}, \ \operatorname{div}\left(\boldsymbol{\varphi} - \boldsymbol{u}\right) = \theta, \ \boldsymbol{\omega}_t = \operatorname{rot}\boldsymbol{u}, \ \boldsymbol{\varphi}_t = \nabla\theta,$$
$$q_t = \operatorname{div}\boldsymbol{u}, \ \operatorname{rot}\boldsymbol{\varphi} = \boldsymbol{0}, \ \operatorname{div}\boldsymbol{\omega} = 0, \tag{3}$$

it follows that for its any solution $\left(\boldsymbol{u}, \boldsymbol{\omega}, \boldsymbol{\varphi}, q\right)$ a pair of the functions $\boldsymbol{u}$ and $\theta = \operatorname{div}\left(\boldsymbol{\varphi} - \boldsymbol{u}\right)$ is the solution of system (1).

2 Wave traveling

We consider a traveling wave of a general kind for system (2). This solution has a form

$$\boldsymbol{u} = \boldsymbol{u}\left(\xi\right) = \left(u_1, u_2, u_3\right), \ \theta = \theta\left(\xi\right), \ \boldsymbol{\omega} = \boldsymbol{\omega}\left(\xi\right) = \left(\omega_1, \omega_2, \omega_3\right),$$
$$\boldsymbol{\varphi} = \boldsymbol{\varphi}\left(\xi\right) = \left(\varphi_1, \varphi_2, \varphi_3\right), \ q = q\left(\xi\right), \ \xi = \alpha t + \beta x + \gamma y - z, \ \left(\alpha\beta\gamma \neq 0\right). \tag{4}$$

The corresponding traveling wave for system (1) is determined by the formulas

$$\boldsymbol{u} = \boldsymbol{u}\left(\xi\right), \ \theta = \operatorname{div}\left(\boldsymbol{\varphi}\left(\xi\right) - \boldsymbol{u}\left(\xi\right)\right). \tag{5}$$

Substituting (4) into (3) gives the following system of the equations

$$au_1' + fu_2' + \beta g u_3' = \beta\theta + \alpha c_1 - \beta c_3,\quad fu_1' + bu_2' + \gamma g u_3' = \gamma\theta + \alpha c_2 - \gamma c_3,$$

$$\beta g u_1' + \gamma g u_2' + cu_3' = \gamma\theta + \alpha c_2 - \gamma c_3,$$

$$\frac{\beta^2 + \gamma^2 + 1}{\alpha}\theta' - \theta = \beta\left(u_1' - c_1\right) + \gamma\left(u_2' - c_2\right) - u_3', \tag{6}$$

$$\alpha\omega_1' = \gamma u_3' + u_2',\quad \alpha\omega_2' = -u_1' - \beta u_3',\quad \alpha\omega_3' = \beta u_2' - \gamma u_1',\quad \alpha\varphi_3 = c_3 - \theta,$$

$$\alpha q' = \beta u_1' + \gamma u_2' - u_3',\quad \varphi_1 = -\frac{\beta}{\alpha}\theta + c_1 - \frac{\beta}{\alpha}c_3,\quad \varphi_2 = -\frac{\gamma}{\alpha}\theta + c_2 - \frac{\gamma}{\alpha}c_3.$$

Here c_1, c_2, c_3 are arbitrary real constants; the prime denotes derivatives with respect to a variable ξ; $a = -\alpha^2 + \beta^2(\lambda+2\mu) + \gamma^2\mu + G$, $b = -\alpha^2 + \gamma^2(\lambda+2\mu) + \beta^2\mu + G$, $c = -\alpha^2 + (\lambda+2\mu) + \left(\beta^2 + \gamma^2\right)G$, $f = \beta\gamma(\lambda+\mu)$, $g = G - (\lambda+2\mu)$.

Let's take additional functions φ_1 and φ_2 so that $c_1 = c_2 = 0$. In this case, the solution to the first three equations of system (6) has the form

$$u_1' = (\theta - c_3)\frac{\Delta_1}{\Delta},\quad u_2' = (\theta - c_3)\frac{\Delta_2}{\Delta},\quad u_3' = (\theta - c_3)\frac{\Delta_3}{\Delta}, \tag{7}$$

where

$$\Delta = \det\begin{vmatrix} a & f & \beta g \\ f & b & \gamma g \\ \beta g & \gamma g & c \end{vmatrix} \neq 0,\quad \Delta_1 = \det\begin{vmatrix} \beta & f & \beta g \\ \gamma & b & \gamma g \\ -1 & \gamma g & c \end{vmatrix},$$

$$\Delta_2 = \det\begin{vmatrix} a & \beta & \beta g \\ f & \gamma & \gamma g \\ \beta g & -1 & c \end{vmatrix},\quad \Delta_3 = \det\begin{vmatrix} a & f & \beta \\ f & b & \gamma \\ \beta g & \gamma g & -1 \end{vmatrix}.$$

Substituting (7) into the fourth equation of system (6) and integrating the resulting equation gives the following expression for the function $\theta(\xi)$ which, by virtue of formulas (2), determines the temperature

$$\theta(\xi) = c_4 \exp(h\xi) + c_3\left(\frac{\beta^2 + \gamma^2 + 1}{\alpha} - \frac{\beta}{h}\right). \tag{8}$$

Here c_4 is an arbitrary real constant; $h = \dfrac{\alpha\left(\Delta + \beta\Delta_1 + \gamma\Delta_2 - \Delta_3\right)}{\Delta\left(\beta^2 + \gamma^2 + 1\right)}$.

Substituting (8) into (7) and integrating the resulting equation gives the following expression for the displacement vector components

$$
\begin{aligned}
u_1 &= c_4 \frac{\Delta_1}{h\Delta} \exp(h\xi) + c_3 \frac{\Delta_1}{\Delta}\left(\frac{\beta^2+\gamma^2+1}{\alpha} - \frac{\beta}{h} - 1\right)\xi + c_5, \\
u_2 &= c_4 \frac{\Delta_2}{h\Delta} \exp(h\xi) + c_3 \frac{\Delta_2}{\Delta}\left(\frac{\beta^2+\gamma^2+1}{\alpha} - \frac{\beta}{h} - 1\right)\xi + c_6, \\
u_3 &= c_4 \frac{\Delta_3}{h\Delta} \exp(h\xi) + c_3 \frac{\Delta_3}{\Delta}\left(\frac{\beta^2+\gamma^2+1}{\alpha} - \frac{\beta}{h} - 1\right)\xi + c_7,
\end{aligned} \tag{9}
$$

where c_5, c_6, c_7 are arbitrary real constants.

To find the dependence of the displacement vector on temperature, we express (00) by (88) from (8). We have

$$
\xi = \frac{1}{h}\ln\left(\frac{\alpha h\theta + c_3\left(\alpha\beta - h\left(\beta^2+\gamma^2+1\right)\right)}{\alpha h c_4}\right). \tag{10}
$$

Substituting (10) into (9) gives the following expression for the displacement vector components on temperature

$$
\begin{aligned}
u_1 &= \frac{\Delta_1}{\alpha h^2\Delta}\Bigg(\alpha h\theta + c_3\left(\alpha\beta - h\left(\beta^2+\gamma^2+1\right)\right) - \\
&\quad - c_3\left(\alpha(\beta+h) - h\left(\beta^2+\gamma^2+1\right)\right)\ln\left(\frac{\alpha h\theta + c_3\left(\alpha\beta - h\left(\beta^2+\gamma^2+1\right)\right)}{\alpha h c_4}\right)\Bigg) + c_5, \\
u_2 &= \frac{\Delta_2}{\alpha h^2\Delta}\Bigg(\alpha h\theta + c_3\left(\alpha\beta - h\left(\beta^2+\gamma^2+1\right)\right) - \\
&\quad - c_3\left(\alpha(\beta+h) - h\left(\beta^2+\gamma^2+1\right)\right)\ln\left(\frac{\alpha h\theta + c_3\left(\alpha\beta - h\left(\beta^2+\gamma^2+1\right)\right)}{\alpha h c_4}\right)\Bigg) + c_6, \\
u_3 &= \frac{\Delta_3}{\alpha h^2\Delta}\Bigg(\alpha h\theta + c_3\left(\alpha\beta - h\left(\beta^2+\gamma^2+1\right)\right) - \\
&\quad - c_3\left(\alpha(\beta+h) - h\left(\beta^2+\gamma^2+1\right)\right)\ln\left(\frac{\alpha h\theta + c_3\left(\alpha\beta - h\left(\beta^2+\gamma^2+1\right)\right)}{\alpha h c_4}\right)\Bigg) + c_7,
\end{aligned} \tag{11}
$$

For example, at $\alpha = 3$, $\beta = 1$, $\gamma = 2$, $\lambda = \mu = 1$, $G = 2$, $c_3 = 5$, $c_4 = 1$, $c_5 = c_6 = c_7 = 0$ the dependence of displacement vector components on temperature is shown at Figs. 1 – 3.

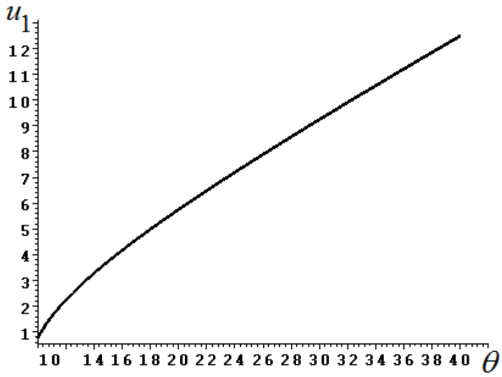

Fig. 1. Dependence of u_1 on temperature.

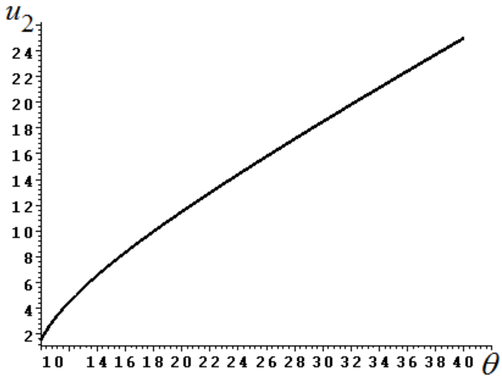

Fig. 2. Dependence of u_2 on temperature.

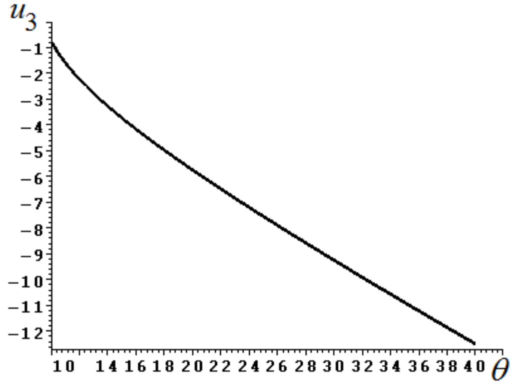

Fig. 3. Dependence of u_3 on temperature.

Figures 1 - 3 show that in this traveling wave, stretching occurs along the axes Ox and Oy . And along the axis Oz , compression occurs. As it cools down, the absolute value of each component of the displacement vector decreases. The fading of the traveling wave occurs.

Substituting (2) into (11) gives an expression of the displacement vector components through temperature and physical variables.

3 Conclusion

Dynamic deformation of a transversally isotropic thermoelastic medium under the Gassmann condition is described by a three-dimensional system of second-order differential equations (1).

Using dimensionless variables (2), we moved to a system of first-order differential equations (3), which, in addition to the components of displacement vector and temperature, contains additional functions. Formulas (8) and (9) are obtained, expressing the components of displacement vector components and temperature through the wave parameter $\xi = \alpha t + \beta x + \gamma y - z$. This made it possible to obtain formulas (11), expressing the dependence of the displacement vector on the temperature in the traveling wave. As an example, for specific values of the quantities on which the traveling wave depends, we plotted the dependence of the components of displacement vector on temperature. These graphs are shown in Figs. 1 - 3. From these graphs it follows that in this traveling wave, stretching occurs along the axes Ox and Oy . And along the axis Oz , compression occurs. As it cool down, the absolute value of each component of the displacement vector decreases. The fading of the traveling wave occurs.

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