Assessing flood forecast uncertainty with fuzzy arithmetic

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Abstract. Providing forecasts for flow rates and water levels during floods have to be associated with uncertainty estimates. The forecast sources of uncertainty are plural. For hydrological forecasts (rainfall-runoff) performed using a deterministic hydrological model with basic physics, two main sources can be identified. The first obvious source is the forcing data: rainfall forecast data are supplied in real time by meteorological forecasting services to the Flood Forecasting Service within a range between a lowest and a highest predicted discharge. These two values define an uncertainty interval for the rainfall variable provided on a given watershed. The second source of uncertainty is related to the complexity of the modeled system (the catchment impacted by the hydro-meteorological phenomenon), the number of variables that may describe the problem and their spatial and time variability. The model simplifies the system by reducing the number of variables to a few parameters. Thus it contains an intrinsic uncertainty. This model uncertainty is assessed by comparing simulated and observed rates for a large number of hydro-meteorological events. We propose a method based on fuzzy arithmetic to estimate the possible range of flow rates (and levels) of water making a forecast based on possible rainfalls provided by forcing and uncertainty model. The model uncertainty is here expressed as a range of possible values. Both rainfall and model uncertainties are combined with fuzzy arithmetic. This method allows to evaluate the prediction uncertainty range. The Flood Forecasting Service of Oise and Aisne rivers, in particular, monitors the upstream watershed of the Oise at Hirson. This watershed's area is 310 km². Its response time is about 10 hours. Several hydrological models are calibrated for flood forecasting in this watershed and use the rainfall forecast. This method presents the advantage to be easily implemented. Moreover, it permits to be carried out in real time (for making information available to the user as quickly as possible). It provides end users Flood Forecasting (crisis managers, public) valuable information that allows everyone to better prepare its possible actions (warning, crisis preparation, safekeeping of the furniture, etc.).

1 Context

Many rivers in France are monitored by Flood Forecasting Offices (SPC). These offices are supervised by the hydro-meteorological office for flood forecasting support (SCHAPI). They provide twice a day at least the flood risk forecast level in the rivers monitored by the French State through a color: green, yellow, orange or red (increasing level going from no serious flood to heavy flood with damages). They now increasingly produce additional information such as the levels or flows in the closed future (between 2 and 72 hours) when flood risk is detected.

In smaller watersheds (some hundreds of km² area), barely upstream of the first monitoring water level measurement location (hydrometric station) some SPC have developed rainfall-runoff models.

These models simulate flows from meteorological data (including rain). They help to anticipate flood risks. Models can be fed by scenarios that combine the

observed rainfalls and forecast rainfalls to carry out flood forecast.

Hydrological models (rainfall-runoff) produce uncertain results. Uncertainties arise from the simplified representation of phenomena that models try to simulate, but also from the great diversity and spatial and temporal variability of the variables that explain the uncertainty of forcing data. Data assimilation methods, based on the use of real time observed data (for example, water levels in a river), provide such a way to balance these imperfections [9].

Forcing data are also uncertain. Rainfall measurements on the watershed are now better known thanks to the deployment of a significant RADAR coverage. Rainfall forecasts are provided to SPC with some indications about their possible range at a future date (eg. Rainfall expected to be between 10 and 20 mm for the next 24H). On the one hand, recently, flood forecasting services wished they provided users flood numerical forecast as soon flood risk is detected and

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exists. Because of sources' uncertainties, a range of possible values is often offered. This range is usually determined by experts in flood forecasting. In real time, they choose the likely scenarios and, based on their experiences, they provide a minimum forecast value (floor) and a maximum forecast value (ceiling). This is the method currently used in the Flood Forecast Service for the Oise and Aisne rivers.

On the other hand, fuzzy arithmetic is used to represent numbers as overall ability [11]. Here we propose a calculation code of a modified hydrological model to allow to carry out scenarios using fuzzy numbers. This method permits, from state or possible forcing values represented with fuzzy numbers, to determine the shape of fuzzy numbers possible outcomes. In this way the hydrological model integrates in a single calculation the uncertainty necessary to initialize a numerical model state (eg. the saturation level of the soil) and forecast rainfalls provided by weather services as a range of possible values between two extrema.

2 Rainfall-Runoff model

The flood forecasting office of Oise and Aisne rivers monitors the main rivers of the Oise watershed. It uses rainfall-runoff hydrological models to forecast water levels in the upstream of the watershed it oversees. These models are used to predict the flows expected at the most upstream stations (water levels measuring stations) from rainfalls at the basin heads. The main rainfall-runoff models are based on the GRP software [2][10] and HydraBV [7].

2.1 Program overview

The numerical program used to perform the simulations in the context of this article is a code similar to that used by the Forecasting Flood Service of Oise and Aisne rivers in usual operational conditions for flood forecast. It is a reservoir model. The topsoil is represented by a surface tank: Highly Usable Reserve (RFU) whose capacity is RFU_0 . The deeper soil layers are considered by a reservoir capacity of J_0 .

The surface reservoir is fed by rain and emptied by evapotranspiration. Once the tank is full, the surplus feeds the reservoir depth. Part of this tank trickles, the other is drained. The distribution is computed using the SCS (Soil Conservative Service) method (Runoff curve number). The rapid runoff flow is routed to the outlet according to a unit hydrograph.

2.2 Rainfall-runoff model of the river Oise at Hirson

Several model parameters were evaluated for the watershed of the Oise at Hirson.

The city of Hirson is a small town in northern France. It is located at the confluence of the Oise in the north (100 km²) and Gland in the east (210 km²); the watershed total area is around 310 km².

The watershed of the river Oise at Hirson is shown on figure 1.

Figure 1. Watershed of the river Oise at Hirson.

This watershed was described and studied in [5]. It consists in an impermeable ground (marl, clay, phyllites and quartzites). It is mainly covered by forests (45%) and grassland and crops (50%) largely dominated by grazing. The basin concentration time is evaluated within 24 hours. The runoff coefficient C_r was estimated at about 0.2.

2.3 Calibration of model parameters

Model variables were adjusted by drawing some comparisons of the results obtained with several well-known flood situations (especially for emblematic floods such as January 1993 flood, December 1995 or December 2003). The calibration step gives the following values:

- Maximum capacity of readily usable reserve: $RFU_0 = 0.07$ mm.
- Maximum interception altitude of the soil intermediate layer: $J_0 = 0.075$ mm.
- Duration for the drying up of the soil layer : $T_{res} = 48$ h;
- loss through seepage into groundwater $Q_f = 0.000147$ mm / h.
- Reaction time of the watershed: $Truiss K = Truis = 10$ h.

2.4 Operational forecasting operating

Usually, the flood forecasting office performs flood forecast for the next 24 hours. It simulates the flow at a station by using rainfall-runoff models previously calibrated and building a rainfall scenario from observed rainfall amounts and expected (predicted) rainfall amounts.

The initialization of the computation is a difficulty. Including the level of saturation of the easily usable tank (RFU) is a sensitive data for starting the calculation. To overcome this difficulty, only one rain-flow is computed since the beginning of the hydrological year (September 1). This initialization method raises the problem of computing time and the availability of rainfall chronicles which would be necessary to start computation. This method is also not completely effective.

3 Fuzzy arithmetic

Fuzzy logic has emerged in the late sixties to meet the needs of auto and computer sciences [11]. In the nineties, it proved to be a tool increasingly used in the thematic discipline of Environmental Sciences [1] [6] [4].

The fuzzy set theory expands the traditional concept of set. Conventional mathematical sets offer only two choices: an element belongs or doesn't belong to a given set. The theory of fuzzy sets allows only that an element partially belongs to a set and also belongs to its complementary at the same time (e.g., a person whose size is 1.75 m belongs to the set of tall people at 70% and small at 30%).

This theory differs from the theory of probability as the former person is not seven over ten chances to be tall and three over ten chances to be small. It is large and small at the same time, it is a matter of judgment.

The fuzzy arithmetic results in fuzzy logic. It uses fuzzy numbers represented by a membership function which reflects the possibility of values that these numbers may take [8]. Current operations (sum, multiplication, etc.) can be performed through the fuzzy arithmetic which allows to integrate the imprecision of parameters into the calculation itself.

Some imprecise calculation parameters are converted into fuzzy numbers and possible rainfall values beyond precision time also. In both cases, the ignorance of uncertainty leads to preferentially use simple membership functions. The choice of the membership functions was put on triangular membership functions. The summit is the most likely value and the basis length reflects the range of inaccuracy.

4 Results

4.1 test case: the river Oise flood of January 2011 at Hirson

The flood that occurred on the 6 and 7 January 2011 at Hirson was an unprecedented flood. The flood frequency was estimated to be about 100 years. From December 2010 to early January 2011 a significant amount of snow accumulated on the watershed of the river Oise at Hirson. Snow accumulations and its spatial distribution on the basin were poorly estimated and still are. The volume of snow which fell in that part of the basin is approximated to be around 10 cm at Hirson and 50 to 60 cm upstream from Hirson.

The convergence of two masses of moist air at the Belgian frontier upstream of the watershed of the Oise river generated strong total rainfalls on a localized band: about 80 mm during 24h between the 6th and the 7th of January 2011. Rainfalls were accompanied by a significant warming of the atmosphere: at Signy-le-Petit near Hirson temperature rose from -5 °C on the 6th January to 7.7 °C on the 7th January at 21:00.

Combined, these three phenomena have led to the formation at Hirson of a flood of exceptional magnitude.

The flood then spread by reducing from Hirson to the confluence between the Oise river and the Aisne river in about 8 days (flood peak).

The model has not been calibrated using this flood.

4.2 Difficulties encountered in operational forecasting (a few hours before the event)

The anticipation of this flood has been difficult for two main reasons. The first deals with the forecasting of rainfall. The rain event was uncertain. The range of possible values was important. Moreover, the rains have been very localized.

The second reason is explained just now. When the rain event occurred, the watershed was covered by snow. This snow fell about 1 month before. Since that time, the RFU (easily usable tank water depth) kept draining (depending on the model used). Computations indeed shows that the latter is almost empty. However, the presence of snow changed the flow conditions of the watershed. Indeed, when snow is present in the watershed, it behaves actually similarly to a catchment area with a readily usable reserve which would be closed to saturation. The initialization mode of this variable as it has been realised when planning the event could not lead to a correct prediction.

Figure 2 shows the observed chronicle flow during 2011 flood at Hirson and the simulations made based on observed rainfalls (calculated retrospectively) with a readily usable tank (RFU) initialized to 0 mm (empty).

Figure 2. Comparison between a simulation of the river Oise flow at Hirson and the observed river flow during the flood. The calculation is very low in comparison with the real discharge observed during the event.

4.3- Proposed tool that takes into account the possible levels of easily usable tank and rain forecast

To overcome this problem we propose to use fuzzy arithmetic to realize a model that uses both possible initialization levels of readily usable tank and possible rainfalls.

Thus we introduce the ability to calculate forecast rainfalls in the model using an initial fuzzy height in the tank and, from this time, also fuzzy. The result of the calculation is provided as a fuzzy number.

The forecast bulletin of the afternoon is achieved by flood forecasters at 14 pm. In the case of the flood of January 2011 at Hirson, the 2011/01/06 at 14 pm, rainfalls

expected until the 2011/01/7 6 am could have been between 20 to 40 mm. The floor of the interval is generally not used for predicting the risk of flood. The interval is thus reduced between 30 and 40 mm remaining to fall for 16 h to come either from 1.9 to 2.5 mm / h uniformly distributed. The rainfall forecasts used for the next two days were:

- from the 2010/01/07 6:00 am to the 2010/01/08 6 am : 10 to 15 mm uniformly distributed;
- from 2010/01/08 6:00 am to the 2010/01/09 6 am : 10 to 20 mm uniformly distributed.

The fuzzy number of the initial level in the easily usable tank (RFU) and an example of a fuzzy number representing the rainfalls forecast are presented in the following figure 3.

Figure 3. Example of fuzzy input data.

The result is a chronicle consisting of fuzzy numbers. The representation of the chronicle was obtained by cutting implementation for the following possibilities: 0, 0.1, 0.2, 0.3, 0.4, etc.

The result is shown in the following figure 4. Two examples of fuzzy flows obtained at the output of the model are provided in Figure 5.

Figure 4. Fuzzy simulation of the flow of the Oise river at Hirson, with RFU initial and rainfall forecast as fuzzy number.

Figure 4. Fuzzy simulation of the Oise river flow at Hirson, with RFU initial and rainfall forecast as fuzzy number.

These results give a wide spectrum of possibility and covers a range starting on without overflowing flood (less than 50 m³/s) and ending at the catastrophic flood (over

150 m³/s). However this simulation provides other information that might allow the forecaster to make a good prediction.

At this moment of the forecast (red line of Figure 4) the rate began to rise. However all simulated curves have not started to react. These latter correspond to the lowest values of the initial RFU. Thus the forecaster may eliminate the curves for which the flow rate has not varied. All chronics predict a flow level at least as high as the flow actually observed represented by a green dot in Figure 4 (the observed chronicle represented is the one that was observed by the defective measuring instruments during the flood).

The highest chronicles are included in a narrow beam possibility. The method does not eliminate the uncertainties in the calculation (real rainfalls, rainfall distribution, etc.), but, in this case, the possible values seem grouped and this leads to think that this result was a great opportunity to be in the highest values.

The use of this method might make us able to detect from 14h to 1/6 the magnitude of the flood which was going to occur at the date of the forecast and might have helped to provide a reasonable level of risk.

5 conclusion and perspectives

The sensitive variable initialization of the model and the expected rainfalls were made fuzzy, the forecaster he can then use a wide range of possible floods. During January 2011 flood of the river Oise at Hirson, the Event detection was not realised properly especially because the saturation state of the watershed had been poorly understood but also because rainfalls' possibilities could not be analyzed. Our proposal of variable fuzzification for the initialization of the model and for forecasted rainfalls allows the forecaster during the operational step forecast to obtain the range of possible and probable floods. Moreover in the case of Hirson at the moment of the forecast, comparison between possible simulations and reality happening can permit to detect the tendencies. This attempt to implement a model with some fuzzy variables opens possibility for flood forecast that should be studied in more various cases. It could further be tested in real time on real floods (besides the usual assessments) to ensure it is able to improve the forecast of flood risk.

From a technical perspective this experiment opens up opportunities related to the use of fuzzy logic (the current tool only uses fuzzy arithmetic). This possibility could thus allow to cross the results with others.

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