On a credit oscillatory system with the inclusion of stick-slip

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Abstract. The work was a mathematical model that describes the effect of the sliding attachment (stick-slip), taking into account hereditarity. This model can be regarded as a mechanical model of earthquake preparation. For such a model was proposed explicit finite- difference scheme, on which were built the waveform and phase trajectories hereditarity effect of stick-slip.

1 Introduction

The effect of stick-slip occurs into tribology problems. For example, we study movement of cargo on a spring along the surface. Due to the adhesion of the goods adheres to the surface, and due to the spring tension breaks off and slides along it at the same place his hesitation [1–3]. Also stick-slip effect may be laid in the foundation of the mechanical model of the earthquake in the subduction zone of lithospheric plates [4].

In this paper we will explore hereditarity nonlinear oscillator which characterizes the stick-slip effect [5, 6]. Recall that in [5, 6], we considered the model equation stick-slip effect, c considering hereditarity with derivatives of fractional order permanent.

In this paper we consider the oscillator hereditarity stick-slip effect with fractional derivatives variables governmental orders, which are bounded functions of. And the orders of fractional derivatives may depend not only on the transfer variable t, and shift function $x(t)$. Introduction thus fractional control parameters in the model equation generalizes considered into the third chapter hereditarity nonlinear oscillator based on stick-slip effects that can lead to new properties or effects as a result of mathematical modeling.

Next, we construct an explicit finite-difference scheme for the numerical calculation of the solution of the generalized model of stick-slip effect with variable fractional order. Next, look at some examples of the application of this scheme with different values of the control parameters, we plot and investigate phase trajectories.

2 Statement of the problem and methods of solution

A problem. Find the solution $x(t)$, where $t \in [0, T]$ the following Cauchy problem:

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$$\partial_{0t}^{\beta(x(t),t)} x(\tau) + \lambda \partial_{0t}^{\gamma(x(t),t)} x(\tau) + \omega^{\beta(x(t),t)} x(t) = f(x(t), t), \tag{1}$$

$$x(0) = x_0,\ \dot{x}(0) = y_0, \tag{2}$$

Where $\partial_{0t}^{\beta(x(t),t)} x(\tau)$ and $\partial_{0t}^{\gamma(x(t),t)} x(\tau)$ – the derivatives operators Gerasimova-Caputo different orders $1 < \beta(x(t), t) < 2, 0 < \gamma(x(t), t) < 1, f(x(t), t) = bt + c \sum_{n=1}^{\infty} a_n \sin(nx(t))$, b – the speed of movement of the spring, c – the energy of adhesion, λ – coefficient of friction, ω – the frequency of free oscillations, $a_n = 2n \int_0^1 \frac{\cos(\pi n \tau)}{\cosh^2(\pi\tau)} d\tau$ – expansion coefficients Fourier series.

Note. Gerasimov-Caputo Operators into the model equation (1) can be written as:

$$\partial_{0t}^{\beta(x(t),t)} x(\tau) = \int_0^t \frac{\ddot{x}(\tau)\, d\tau}{\Gamma(2 - \beta(x(\tau), \tau))(t - \tau)^{\beta(x(\tau),\tau) - 1}}, \tag{3}$$

$$\partial_{0t}^{\gamma(x(t),t)} x(\tau) = \int_0^t \frac{\dot{x}(\tau)\, d\tau}{\Gamma(1 - \gamma(x(\tau), \tau))(t - \tau)^{\gamma(x(\tau),\tau)}}.$$

Therefore, considering the representations (3) are placed two Cauchy problems (1) and (2).

Note. The Cauchy problem (1) and (2) when the fractional parameters β and γ are constants, becomes the Cauchy problem in [5], if these constants are $\beta = 2$ and $\gamma = 1$, we arrive at the Cauchy problem for the classical stick-slip effect [3].

The Cauchy problem (1) and (2) generally has no clear decision, so we will solve this problem numerically using the theory of finite difference schemes [7, 8].

We construct an explicit finite-difference scheme, because it is easiest to implement on a computer. We divide the time interval $[0, T]$ for N equal parts increments $\tau = T/N$. Then the function solutions $x(t)$ of the differential of the Cauchy problem (1) and (2) pass grid function in $x\left(t_j\right)$, where $t_j = j\tau,\ j = 0, 1, \cdots, N - 1$, which will be solution of the corresponding Cauchy difference problem.

Let us more detail on the approximation of the operators of fractional differentiation of (3). Following definitions [9], we obtain the following for their discrete counterparts operators (3):

$$\partial_{0t}^{\beta(x(t),t)} x(\tau) \approx \sum_{k=0}^{j-1} \frac{p_k^j \tau^{-\beta(x_k,t_k)}}{\Gamma(3 - \beta(x_k, t_k))} \left(x_{j-k-1} - 2x_{j-k} + x_{j-k+1}\right), \tag{4}$$

$$\partial_{0t}^{\gamma(x(t),t)} x(\tau) \approx \sum_{k=0}^{j-1} \frac{q_k^j \tau^{-\gamma(x_k,t_k)}}{\Gamma(2 - \gamma(x_k, t_k))} j \left(x_{j-k-1} - 2x_{j-k} + x_{j-k+1}\right), \tag{5}$$

$$p_j^k = (k+1)^{2-\beta\left(x_j,t_j\right)} - k^{2-\beta\left(x_j,t_j\right)},\ p_j^k = (k+1)^{1-\gamma\left(x_j,t_j\right)} - k^{1-\gamma\left(x_j,t_j\right)}.$$

Note. In (4)-(5) was used: $x(t_j) = x_j$.

Subject to the initial conditions (2), put relations (4)-(5) into equation (1) we get explicit finite-difference scheme:

$$x_1 = \tau y_0 + x_0, j = 0,$$

$$x_{j+1} = Ax_j - Bx_{j-1} - D \sum_{k=1}^{j-1} \frac{\tau^{-\beta(x(t_k),t_k)}}{\Gamma(3 - \beta(x(t_k), t_k))} p_k^j \left(x_{j-k+1} - 2x_{j-k} + x_{j-k-1}\right) - \tag{6}$$

$$-D\sum_{k=1}^{j-1}\frac{\lambda\tau^{-\gamma(x(t_k),t_k)}q_k^j\left(x_{j-k+1}-x_{j-k}\right)}{\Gamma\left(2-\gamma\left(x\left(t_k\right),t_k\right)\right)}+Df_j,\ j=1,...,N-1,$$

$$A=\frac{2A_0+B_0-\omega^{\beta(x(t_j)),t_j}}{A_0+B_0},B=\frac{A_0}{A_0+B_0},\eta=\frac{\omega^{\beta(x(t_j),t_j)}}{A_0+B_0},$$

$$D=\frac{1}{A_0+B_0},A_0=\frac{\tau^{-\beta(x_0,t_0)}}{\Gamma\left(3-\beta\left(x_0,t_0\right)\right)},B_0=\frac{\lambda\tau^{-\gamma(x_0,t_0)}}{\Gamma\left(2-\gamma\left(x_0,t_0\right)\right)},$$

$$p_k^j=(k+1)^{2-\beta(x(t_k),t_k)}-k^{2-\beta(x(t_k),t_k)},q_k^j=(k+1)^{1-\gamma(x(t_k),t_k)}-k^{1-\gamma(x(t_k),t_k)}.$$

Note. Explicit finite difference scheme as are usually conditional stable, ie, there is a limitation on the step τ of difference grid. The scheme (6) are no exceptions. The step of the grid can be controlled by the method of double recalculation or Runge rule. In this paper we carry out a numerical experiment and see how the maximum absolute value error between the exact and numerical solutions, depending on the increase in the number of N nodes of the computational grid.

Note. The explicit finite difference scheme (6) in the case where the fractional parameters are constants and have values of $\beta = 2$ and $\gamma = 1$, become to the local settlement explicit finite-difference scheme for the classical optical effect stick-slip [3].

3 Results of numerical simulation

In [3] it was said that it is enough to take the first seven coefficients a_k in the expansion of the function $f(x(t), t)$ to obtain reliable solutions (1). The values of these coefficients take from [3] $a_1 = 0.436, a_2 = 0.344, a_3 = 0.164, a_4 = 0.058, a_4 = 0.021, a_6 = 0.004, a_7 = 0.003$.

Numerical modeling hereditarity stick-slip effect on the formulas (6) and obtained in [5] depending on the control parameters. First, we consider the dependence of the fractional control parameters only on the time t, ie, $\beta = \beta(t)$ and $\gamma = \gamma(t)$. Then we look at the more general case, when the fractional parameters also depend on $x(t)$ of displacement.

Example 1. The control parameters: $\beta(t) = 1.8 - 0.03\sin(\pi t), \gamma(t) = 0.6 - 0.04\cos(\pi t), N = 3000, \delta = 50, \tau = 0.05, \lambda = 0.3, b = 1, \omega = 1, y_0 = 0.3, x_0 = 0$.

In Fig.1 shows the calculated curves of displacement, velocity displacement and the phase trajectory. In Fig.1a shows an oscillogramma for example 1. It can be seen that in the separation of the load fluctuates, the rate of these oscillations in the potential well decays slowly enough (Fig.1b). This effect is ereditarnostyu process. In the phase trajectory (Fig.1c) shows that potential wells are stable focuses.

Example 2. The control parameters: $\beta(t) = 1.6 - 0.1\sin(\pi t), \gamma(t) = 0.6 - 0.2\cos(\pi t), N = 3000, \tau = 0.05, \delta = 50, \lambda = 0.3, b = 1, \omega = 1, y_0 = 0.3, x_0 = 0$.

In Fig.2 shows the calculated curves obtained by numerical formulas (6) and [5]. Unlike the examples discussed above, in this example the fractional parameters depend not only on the time t, but on the $x(t)$ displacement.

In Fig.2a shows the oscillogrammas obtained by the formulas (6) and [5], which characterize the effect of stick-slip given hereditarity. You may notice that the goods slide along the surface, testing damped oscillations of small amplitude. It can be seen that the amplitude of these oscillations increases slightly for the subsequent potential wells (Fig.2b). Phase trajectories to this case are given in Fig.2c.

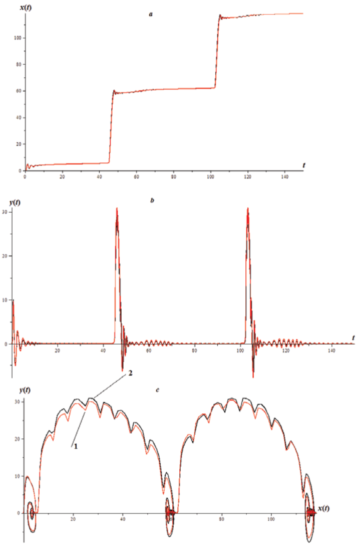

Figure 1. The calculated curves obtained from the formulas (6) (curve 1) and [5] (curve 2): a) - oscillogramma, b) - oscillator speed, c) - the phase trajectory

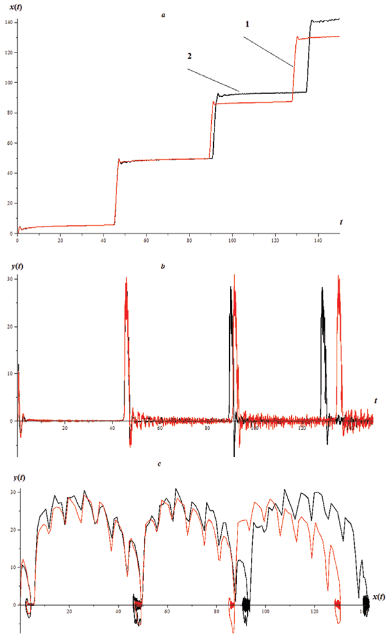

Figure 2. The calculated curves obtained from the formulas (6) and [5] a) - oscillogramma b) - oscillator speed, c) - the phase trajectory

4 Conclusion

It can be concluded that both finite-difference scheme (6) give good agreement with the results of [5]. In the future we plan a more detailed study of the proposed finite-difference.

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