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Solar-Terrestrial Relations and Physics of Earthquakes Precursors

Reversals in the six-jet Geodynamo model

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Abstract. We describe a large-scale geodynamo model based on hypothesis about 6-cells convection in the Earth's core. This hypothesis suggests indirect data of inhomogeneities in the density of the Earth?s core. The convection pattern is associated with a spherical harmonic Y_4^2 which defines the basic poloidal component of velocity. The model takes into account the feedback effect of the magnetic field on convection. It was ascertained that the model contains stable regimes of field generation with reversals. The velocity of convection and the dipole component of the magnetic field are similar to the observed ones.

1 Introduction

The process of formation of magnetic fields of planets and stars is successfully explained by the theory of hydromagnetic dynamo [1, 2]. The existing models reproduce MHD flows on a small-scale spatial grid at relatively small timescales (~ 50 kyr), or make it possible to calculate only the long-evolution large spatial structures. Clearly, that large-scale spatial structures of convection need to be specified for the models of the second type. This is a key question of the real structure of large-scale convection.

In this paper, we describe a model of geodynamo with six convective cells. The poloidal component of the velocity is determined by the spherical harmonic Y_4^2. In [3], a hypothesis on the six-cells structure of convection was proposed, based on analysis of the splitting of the Earth's free oscillations. It is clear that with such convection two cells are located in the equatorial region, and two cells are located still in each hemisphere. Note that the convective structure of six cells for the liquid core of the Earth has been obtained as a result of direct numerical simulation for some value of the parameters [4]. In this work, three convective cells were located in each hemisphere. The poloidal component of velocity of this convection is associated with the spherical harmonic Y_4^3.

The known properties of real cosmic dynamo-systems is the presence of reversals – change of the sign of the poloidal component of the field. The great interesting are reversals not related to the restructuring of convection.

In this paper, we discuss the question about of the field generation with six-jet Y_4^2-structure, taking into account turbulent α-effect, and reversals in this system.

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2 Model equations

The equations of geodynamo considering the α-effect in the Boussinesq approximation have the form:

$$\begin{aligned}
&\frac{\mathrm{E}}{\mathrm{Pm}}\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\mathbf{v} = \mathrm{E}\nabla^2\mathbf{v} - \nabla p - 2\mathbf{e}_z \times \mathbf{v} + \mathrm{RaPm}T\mathbf{r} + \mathbf{rotB}\times\mathbf{B},\\
&\frac{\partial T}{\partial t} + (\mathbf{v}\cdot\nabla)(T + T_s) = \frac{\mathrm{Pm}}{\mathrm{Pr}}\nabla^2 T,\\
&\frac{\partial \mathbf{B}}{\partial t} = \mathbf{rot}(\mathbf{v}\times\mathbf{B}) + \mathrm{R}_\alpha\mathbf{rot}(\alpha\mathbf{B}) + \nabla^2\mathbf{B},\\
&\nabla\cdot\mathbf{v} = 0, \quad \nabla\cdot\mathbf{B} = 0.
\end{aligned} \tag{1}$$

The governing dimensionless parameters of the model are the Ekman number E, the Rayleigh number Ra, the Prandtl number Pr, the magnetic Prandtl number Pm and the magnitude of α-effect R_α. The variable T is the temperature deviation from the equilibrium profile T_s.

It is assumed that turbulence is isotropic and the α-effect is used for scalar parametrization in the form $\alpha = a(r)\cos\theta$, where $\max|a(r)| = 1$. For the velocity, no-slip boundary conditions are applied. For the magnetic field there are specified potential boundary conditions at the CMB and finiteness at the center of the Earth. Finally, the temperature deviation is zero at the ICB and CMB.

For Eqs. (1) we derived of small-mode approximations. The hydrodynamical part of our model is the same as the Lorenz system for convection in a plane layer, but describes six-jet convection structure.

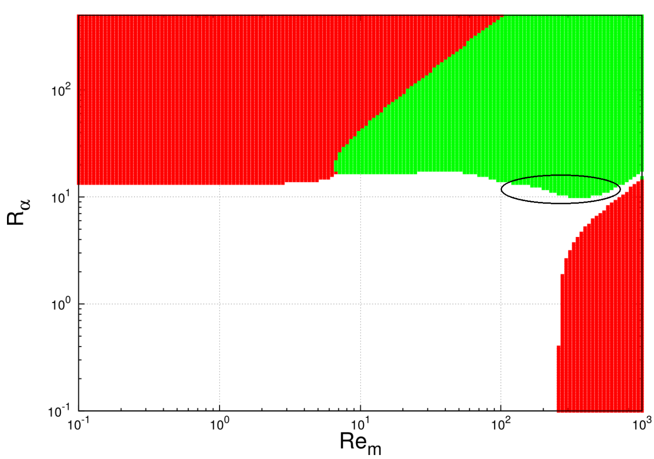

Figure 1. The areas of field generation. Red dots: non-oscillatory dynamo; green dots: oscillatory dynamo.

We use only one velocity mode $\mathbf{v}(\mathbf{r}, t) = u(t)\mathbf{v}_0(\mathbf{r})$, where $\mathbf{v}_0(\mathbf{r})$ is a simplest approximation of the Poincaré modes for the spherical shell. The details of construction of this mode are described in the authors paper [5]. For temperature introduce two-mode approximation $T(\mathbf{r}, t) = \theta_0(t)T_0(\mathbf{r}) + \theta_1(t)T_1(\mathbf{r})$, where $T_0(\mathbf{r})$ matched to the radial component of the velocity mode, and $T_1(\mathbf{r})$ is uniform on the sphere.

For representation of the magnetic field we use some free decay modes, which are selected following scheme borrowed from [6].

First, we order these modes as ascending eigenvalues (dissipation rate). The following sequence was obtained, where the box denotes doublets with identical eigenvalues: $\mathbf{P}_{01}^{-1..1}$, $\boxed{\mathbf{T}_{01}^{-1..1}, \mathbf{P}_{02}^{-2..2}}$, $\boxed{\mathbf{T}_{02}^{-2..2}, \mathbf{P}_{03}^{-3..3}}$, $\mathbf{P}_{11}^{-1..1}$, $\boxed{\mathbf{T}_{03}^{-3..3}, \mathbf{P}_{04}^{-4..4}}$, $\boxed{\mathbf{T}_{11}^{-1..1}, \mathbf{P}_{12}^{-2..2}}$, $\boxed{\mathbf{T}_{04}^{-4..4}, \mathbf{P}_{05}^{-5..5}}$, ...

Next, we selected several modes with the lowest eigenvalues, denoting them by $\mathbf{B}_k(\mathbf{r})$. The magnetic field is written as $\mathbf{B}(\mathbf{r}, t) = \sum_k g_k(t)\mathbf{B}_k(\mathbf{r})$ and substituted into the induction equation (the third equation in Eqs. (1)). The velocity magnitude $u(t)$ temporarily assume constant $u_0 > 0$. Then u_0 can be interpreted as the magnetic Reynolds number Re_m.

Now applying the Galerkin method yields the system

$$\begin{aligned}
\frac{dg_k}{dt} &= \mathrm{Re}_m \sum_i W_{ki} g_i + \mathrm{R}_\alpha \sum_i A_{ki} g_i - \eta_i g_i, \\
W_{ki} &= \int_{r_i \le r \le r_o} \mathbf{rot}\,(\mathbf{v}_0 \times \mathbf{B}_i)\,\mathbf{B}_k dV, \qquad (2) \\
A_{ki} &= \int_{r_i \le r \le r_o} \mathbf{rot}\,(a(r)\cos\theta \mathbf{B}_i)\,\mathbf{B}_k dV,
\end{aligned}$$

where η_i is an eigenvalue of $\mathbf{B}_i$.

In fact, we have obtained a small-mode approximation in kinematic dynamo with six convective cells. Let us denote by λ_i the eigenvalues of the matrix of the system (2). The dynamo works if $\max \Re\mathfrak{e}\lambda_i > 0$. The mode with its eigenvalue is the leader, it grows faster than others. It is important that if the eigenvalue of the leading mode is complex, the magnetic field will oscillate.

So, we will gradually increase the number of magnetic modes until an oscillating dynamo appears. Of course, the eigenvalues depend on the parameters Re_m and R_α and on the radial profile $a(r)$ of the α-effect. We used two expressions for the profile, $a(r) = 1$ and $a(r) = r$. It has turned out that the results do not differ qualitatively: the para- meters varied over a logarithmic scale in the ranges $[10^{-1}; 10^3]$. Upon obtaining the oscillating dynamo, we discarded some modes, if this did not qualitatively change the situation and did not increase greatly the generation threshold. As a result, the magnetic modes were the following $\mathbf{P}_{01}^0$, $\mathbf{P}_{03}^{\pm 2}$, $\mathbf{P}_{11}^0$, $\mathbf{T}_{04}^{\pm 2}$, and $\mathbf{P}_{05}^{\pm 2}$.

The areas of oscillatory and non-oscillatory dynamo on the parameter plane $(\mathrm{Re}_m, \mathrm{R}_\alpha)$ are shown in Fig. 1, where $a(r) = r$. Case $a(r) = r$ different small details.

In this kinematic dynamo, an unlimited growth of the field is only possible. To obtain a stable generation of the bounded field, the mechanism of quenching is necessary. In this regard, let us consider the areas in Fig. 1 marked by ellipses. Let us fix $\mathrm{R}_\alpha \sim 15$. Let the point $(\mathrm{Re}_m, \mathrm{R}_\alpha)$ fluctuate along the bigger axes of the ellipse due to the changes of Re_m. It is possible to obtain a stable bounded field generation. Note that in our models Re_m is a fixed value of the magnitude $u(t)$ of the velocity mode $\mathbf{v}_0$. Therefore, we consider a model of magneto-convection with a velocity variable magnitude. To do this, we substitute the considered decompositions for the velocity, temperature and magnetic field in all Eqs. (1) and apply the Galerkin method.

A non-linear dynamic system is obtained, describing the self-consistent six-jet magneto-convection in the Earth's core:

$$\begin{aligned}
&\frac{\mathrm{E}}{\mathrm{Pm}}\frac{du(t)}{dt} = -\mathrm{E}\mu u(t) + \mathrm{RaPm}S\theta_0(t) + \sum_{i,j} L_{ij}g_i(t)g_j(t),\\
&\frac{d\theta_0(t)}{dt} = -Fu(t)\theta_1(t) + Hu(t) - \frac{\mathrm{Pm}}{\mathrm{Pr}}\zeta_0\theta_0(t),\\
&\frac{d\theta_1(t)}{dt} = Fu(t)\theta_0(t) - \frac{\mathrm{Pm}}{\mathrm{Pr}}\zeta_1\theta_1(t),\\
&\frac{dg_k(t)}{dt} = u(t)\sum_i W_{ki}g_i(t) + \mathrm{R}_\alpha \sum_i A_{ki}g_i(t) - \eta_i g_i(t),
\end{aligned} \tag{3}$$

where ζ_i is the eigenvalues temperature mode T_i, $\mu > 0$, and $\mu > 0$, S, L_{ij}, F, H are the Galerkin coefficients.

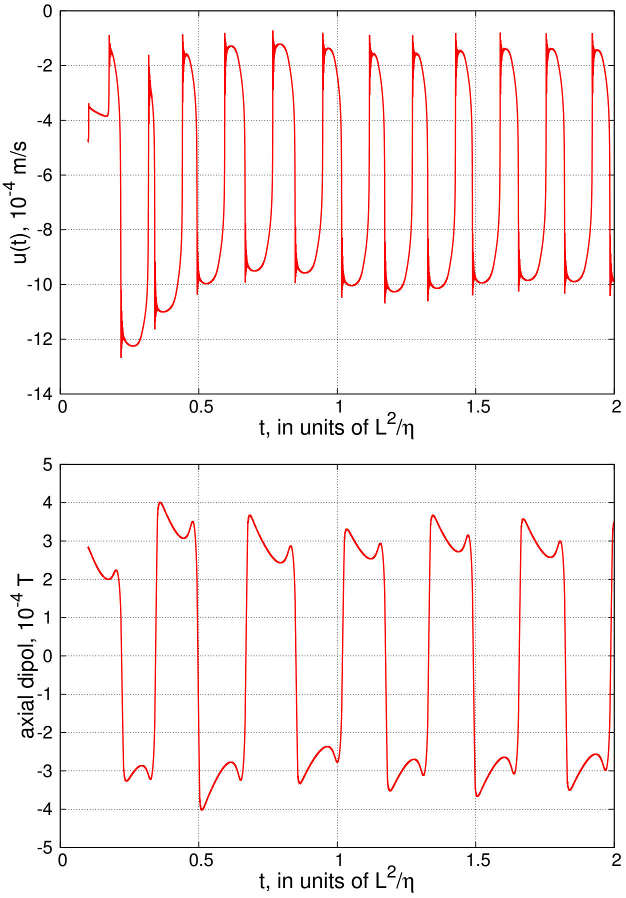

Figure 2. The magnitude of the velocity and axial dipole: Ra = 5000.

3 Simulation results

When performing a numerical simulation with the model Eqs. (3), we used turbulent values of the dissipation coefficients $\nu = 10^2$ m^2/s, $\kappa = 10^{-2}$ m^2/s, $\eta = 20$ m^2/s [7]. The outer radius of the core $r_o = 3480$ km and the angular velocity $\Omega = 7.29 \times 10^{-5}$ rad/s. The relevant parameter values areE $= 10^{-7}$, Pr $= 10^4$, Pm $= 5$ Also, we used $\mathrm{R}_\alpha \sim 15$, following the estimations made in [8]. Different Rayleigh number values were used in the simulation, in particular, Ra $= 10^2 \div 10^4$.

We found stable dynamo regimes with reversals at Ra $\sim 10^3$. Fig. 2 shows the graphs for the magnitude of the velocity and dipole part of the magnetic field at Ra $= 5000$. It is seen that the magnetic field and the velocity are oscillating. However, if the magnetic field undergoes reversals regularly, then the sign of the velocity does not change. Thus, in the model, the reversals have been realized, without convection regime change.

Of course, the numerical results in this simple model should be treated cautiously. However, a characteristic convection velocity value illustrated in Fig. 2 is 5×10^{-4} m/s. This agrees well with the known estimates of the actual velocity of convection $\sim 10^{-4}$ m/s [1]. The characteristic value of the field in the model 5×10^{-4} T is also in good agreement with the extrapolation of the real magnitude of the geomagnetic field at the boundary of the outer core [9]. The polarity interval in the model is 3200 years, which can correlate with mean polarity intervals on the geomagnetic polarity scale [9].

So, the numerical simulations have shown that a stable regime of the magnetic field generation with reversals can be realized with this model. It is important that with these reversals there is no change in regime of convection. The characteristic value of the model convection velocity is in good agreement with the available estimates of the real velocity value in the western drift. The dipole and toroidal components of the model field have the same order of magnitude as the real geomagnetic field.

The reversals of the field in the model have a regular nature in contrast to the real chaotic sequence. Since it is likely impossible to realize chaotic reversals in such simple dynamic model, random perturbations are required for their simulation.

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