E3S Web of Conferences **25**, 03001 (2017) DOI: 10.1051/e3sconf/20172503001
RSES 2017

Using of sensitivity models of the steady-state of electric power systems for account of failure of elements

Dmitry Krupenev[1*]

[1]Melentiev Energy Systems Institute, 130 Lermontov str., Irkutsk, Russia

Abstract. The article presents an algorithm for obtaining sensitivity models of the first and second orders of the steady-state regime of electric power systems (EPS). The sensitivity models are intended for express calculations of steady-state when estimating the static security of EPS. The use of sensitivity models allows one to simulate failures of EPS elements without calculating new steady-state. To verify the reliability of the sensitivity models obtained, the results of an experiment performed on a 3-node test pattern are presented.

1 Introduction

For minimize of the consequences of electrical equipment failures in electric power systems (EPS), it is necessary to made a set of specific actions, one of which is an operational estimation of the security of EPS. The security estimation of EPS is determine of the indicators that most fully characterize the current steady-state from the standpoint of opposing possible disturbances in EPS. Estimation of the security of EPS is technologically capacious and computationally expensive. There are the static and the dynamic security of EPS. In this paper, we describe static security. The estimation of a static security of EPS is based on the assumption that the transition of EPS to a new state due to an element failure is not accompanied by a violation of dynamic stability.

When we estimate of security of EPS very important parameters are accuracy and time of the estimation, since the estimation should be carried out in real time. Various simplifications when we solve this problem can lead to distortion of the result and, ultimately, acceptance of a non-optimal or incorrect set of control actions.

There are some approaches to the estimation of security of EPS. We can divide such approaches as the approach based on the application of the criterion n-i [1-3], the machine-based approach [4], the Jacobi matrix based approach [5], In [6] to estimate the security of EPS using the Monte Carlo method. In most of the present approaches (methods), one of the main stages of estimation, and the most time-consuming expense, is the stage of modeling the effect of failures of power equipment on the EPS parameters or the calculation of steady-state of EPS at failures of its elements. When we estimate of security of EPS, it is necessary to analyze single and group failures of EPS elements in a minimal time. EPS consists of a many of elements, the number of possible states is large. In this paper, we study the problems of accelerating the calculation of steady-state of such steady-states of EPS. We propose to use steady-state sensitivity models or to replace the exact model of calculating of steady-state of EPS by its approximation.

Models of first-order sensitivity of steady-state of EPS are used in solve deferent electric power problems, among them one can single out the problem of analysis of voltage stability of EPS [7]. In [8] the first-order sensitivity models (matrices) are used to calculate the steady-state of EPS when to change the reactive power at the nodes of the EPS. So, in [7], [8], using the models of the first-order sensitivities, linearly approximate the steady-state and search for the steady-state parameters of the EPS during the introduction of the perturbation. The aim of this paper is to obtain sensitivity models of not only the first, but also the second order for the rapid calculation of the steady-state of the EPS. Models of second-order sensitivity give a quadratic approximation to the new steady-state point, which increases the accuracy of calculating the steady-state and, accordingly, the accuracy of the estimation of the security of EPS.

2 Formation of sensitivity models of the steady-state of electric power systems

The steady-state of EPS can be described by a vector equation of the form [9]:

$$W(X,V)=0, \tag{1}$$

where: W – vector-function, $W:G\rightarrow \mathbb{C}^{n+m-1}$, $G\subseteq \mathbb{C}^{n+m-1}$; X – vector of input parameters of the EPS steady-state, $X\in \mathbb{C}^{n}$; V – vector of output parameters of the EPS steady-state, $V\in \mathbb{C}^{m-1}$, – number of power lines of EPS; n – number of nodes of EPS.

When we deal estimate of security of EPS, we should to find the dependencies of some parameters on others,

* Corresponding author: krupenev@isem.irk.ru

and the set of input and output parameters is not known in advance.

In accordance with the implicit function theorem [10] for Banach spaces [11], to which the space of complex numbers ℂ, which characterize the parameters of steady-state operation of EPS, if the mapping $W : O \to ℂ^{n+m-1}$, defined in the neighborhood of the O point $(x_0,v_0) \in ℂ^{n+m-1}$, $x \in ℂ^n$, $v \in ℂ^{m-1}$, such is that

1. $W \in ℂ^{(1)}(O, ℂ^n)$ (vector function is continuously differentiable),
2. $W(x_0,v_0) = 0$ (the equation has a solution at the point (x_0,v_0)),
3. $W_v'(x_0,v_0)$ – reversible matrix,

then, for any point from the neighborhood (x_0,v_0)

$$W(x,v) = 0 \Leftrightarrow v = f(x)$$

$$f'(x) = -[W_v'(x,f(x))]^{-1}[W_x'(x,f(x))], \qquad (2)$$

The expression (2) is first-order sensitivity matrix of steady-state EPS [12-14]. To determine the change in the vector of output variables v relative to the vector of the input variables x, it is necessary to find the corresponding sensitivity matrix or differential in accordance with relation (2).

Let's write the system of steady-state equations in the vector form to obtain the sensitivity models.

$$\overline{A}I = (\widetilde{U})^{-1}S, \qquad (3.a)$$

$$A^T U = \widetilde{Y}^{-1}I, \qquad (3.b)$$

where: A is the incidence matrix of an EPS graph with size of $m Ч n$; $\overline{A}$ is the truncated incidence matrix by the basis node; I is a vector that composed of the components which are the power flows along the power transmission lines of EPS, $I \in ℂ^n$; S is the vector whose components are the power in the nodes of EPS (the residual between generation and demands) $S \in ℂ^{m-1}$; U is a vector that composed of the components which are the nodes voltage of EPS, $U \in ℂ^{m-1}$; $\widetilde{U}$ is a diagonal matrix with $m-1$ order of the EPS nodes voltage; $\widetilde{Y}$ is a diagonal matrix with n order of EPS lines conductivity.

The system (3.a), (3.b) will be called as the initial model. This record of the vector equations system of EPS steady-states is not complete, because of these equations are nonlinear and there are solution set that appear in case of the solution this system. For example in determining the voltages in EPS nodes, the number of solutions is 2^{m-1}. Therefore, to make the system with a single solution, it is necessary to supplement this system of equations by an additional condition:

$$Re\ U > 0. \qquad (4)$$

Moreover, to obtain a sensitivity model of 1st dimension the vector equations system (3) should be differentiated by variable that change own value during the process of reliability assessment. This variable depend on current problem. It is worth to be noting that the EPS elements failures and random deviations of consumers demand are simulating during the process of the system reliability assessment. Hence, it is required to differentiate the system (3.a), (3.b) by the Y (vector of lines conductivity) for the EPS lines failure simulating (modeling) and by S for the simulating (modeling) of random deviations of consumers demand and failures of generating blocks. In this work we will focused on the differentiation by Y. With the aim of uniquely defined the differentials $d_Y I$ (differential of I vector by Y variable) and $d_Y U$ (differential of U vector by Y variable) it is necessary to fix the diagonal matrix $\widetilde{U}$ at the solution point, before differentiation of the vector equations system (3.a), (3.b). The values of voltages in nodes will be located on the diagonal of this matrix.

on the right-hand side.

$$\overline{A}d_Y I = 0, \qquad (5.a)$$

$$A^T d_Y U = -\widetilde{Y}^{-1}(d\widetilde{Y})\widetilde{Y}^{-1}I + \widetilde{Y}^{-1}d_Y I, \qquad (5.b)$$

where: $d_Y I \in ℂ^n$; $d\widetilde{Y} \in ℂ^n$; $d_Y U \in ℂ^{m-1}$.

The differentiation of the inverse matrix function was carried out according to [15]: $d(\widetilde{Y}^{-1}) = -\widetilde{Y}^{-1}d\widetilde{Y}\widetilde{Y}^{-1}$.

Next, let us convey from equation (5.b):

on the right-hand side.

$$d_Y I = \widetilde{Y}A^T d_Y U + (d\widetilde{Y})\widetilde{Y}^{-1}I. \qquad (6)$$

Replace the $d_Y I$ from (6) to (5.a):

$$\overline{A}\widetilde{Y}A^T d_Y U = -\overline{A}(d\widetilde{Y})\widetilde{Y}^{-1}I, \qquad (7)$$

as follows from (7) equation we will take:

$$d_Y U = -(\overline{A}\widetilde{Y}A^T)^{-1}\overline{A}(d\widetilde{Y})\widetilde{Y}^{-1}I = -(\overline{A}\widetilde{Y}A^T)^{-1}\overline{A}\widetilde{Y}^{-1}\widetilde{I}dY, \qquad (8)$$

Where $\widetilde{I}$ is the diagonal matrix, where the power flows along the power transmission lines of EPS located on the diagonal, $I \in ℂ^n$.

The matrix permutation was carried out in the mathematical expression (8) that is possible for diagonal matrices.

The linearized dependence of the voltages in the nodes of the EPS on the conductivities change in the elements of the EPS can be represented in the form $U \approx U_0 + d_Y U$, where U_0 is a column vector whose components are values of the voltages in the nodes of the EPS at the solution point $U_0 \in C^{m-1}$. The column vector ΔY will be used instead dY for defining the differential $d_Y U$. That column vector composed of the components which are increments of the conductivities on the corresponding lines $\varDelta Y \in C^n$.

Let's substitute $d_Y U$ (8) into (6) for to get $d_Y I$ .Then

$$d_Y I = \widetilde{Y}A^T(\overline{A}\widetilde{Y}A^T)^{-1}\overline{A}(d\widetilde{Y})\widetilde{Y}^{-1}I + d\widetilde{Y}\widetilde{Y}^{-1}I. \qquad (9)$$

The linearized dependence of the change of power flows along the EPS power transmission lines with depending on conductivities change can be represented and look like $I \approx I_0 + d_Y I$, where I_0 is column vector, whose components are of power flow values along the power transmission lines in the solving point, $I_0 \in C^n$.

Besides, to get the sensitivity models of 2nd order it is necessary to get the second differentials. Hence, the first differentials (8) and (9) must be differentiated. In summary, the second order differentials will have the next form:

$$d^2_{YY}U = 2(\overline{A}\widetilde{Y}A^T)^{-1}\overline{A}d\widetilde{Y}A^T(\overline{A}\widetilde{Y}A^T)^{-1}\overline{A}\widetilde{I}\widetilde{Y}^{-1}dY, \qquad (10)$$

$$d^2_{YY}I = d\widetilde{Y}A^T d_Y U + \widetilde{Y}A^T d^2_{YY}U - \widetilde{Y}^{-2}\widetilde{I}d\widetilde{Y}dY. \qquad (11)$$

In this case the 2nd order sensitivity models, namely the dependence of voltage in EPS nodes on conductivity change in EPS elements can be represent as: $U \approx U_0 + d_Y U + \frac{1}{2} d^2_{YY}U$, the dependence of power flows along the EPS power transmission lines on EPS elements conductivity change could be can be represent as: $I \approx I_0 + d_Y I + \frac{1}{2} d^2_{YY}I$.

In event of EPS lines failure the disturbance of voltages and streams along the power transmission lines of EPS will be determined as follows:

$$\varDelta_{YY}U \approx d_Y U + \frac{1}{2} d^2_{YY}U, \qquad (12)$$

$$\varDelta_{YY}I \approx d_Y I + \frac{1}{2} d^2_{YY}I \qquad (13)$$

where: $\varDelta_{YY}U$ is a vector of voltages change in EPS nodes, $\varDelta U \in C^{m-1}$, $\varDelta_{YY}I$ is a vector of EPS power flows change, $\varDelta I \in C^n$.

Thus, vector $\varDelta Y$ that characterizes the values of conductivities that equivalent to switching off the transmission line in equations (8-13) remains undefined. However, the conductivity Y_i power transmitting line i has a definite value during the normal functioning. In case of power transmission line i failure the value of its conductivity become equal 0, $0 = Y_i + \varDelta Y_i$. In summary the value of conductivity that equivalent of power transmission line i disconnections will be defined as:

$$\varDelta Y_i = 0 - Y_i = -Y_i \qquad (14)$$

During the use sensitive models for the steady-states of EPS calculation by one iteration it is possible to get the vector of voltage changes in EPS nodes in case of single and non-single disconnections of power transmission lines. It could be done if represent the $\varDelta Y$ as a matrix $\varDelta\widetilde{Y}$ (instead vector) with size nЧn and values of $-Y_i$ by the main diagonal, of course if it is necessary to get the assessment of single failures of EPS power transmission lines, and other zero elements. If the assessment of multiple failures required then it is necessary to fill matrix $\varDelta\widetilde{Y}$ in a certain way.
preparing them.

3 Case study

The sensitivity models of the first and second orders applications are demonstrated for steady state calculation of the three-node EPS test scheme shown in fig.1. The experiment includes power line I capacity changing, with its final shutdown.

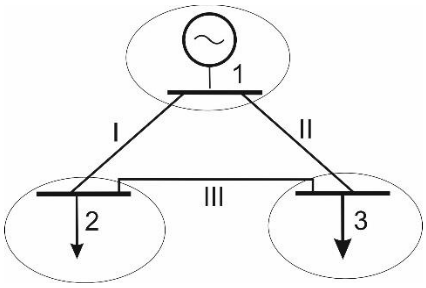

Fig. 1. EPS test scheme.

The main characteristics of EPS necessary for steady state calculation are shown in table 1.

Table 1. The initial data of the investigated EPS.

Power line	Impedance, Ω	Node	Load, MVA	Voltage, kV
I	12,1+ i43,5	1	0	220,0+ i0
II	12,1+ i43,5	2	329,0-i114,0	-
III	12,1+ i43,5	3	669,0-i173,0	-

As a result of the steady-state calculation of the initial model (3.a), (3.b), the following voltages: U_2 = 234,94 - i7,73 kV; U_3 = 213,67 - i71,69 kV, and power lines flows: I_{12} = 0,08 + i0,36 kA, I_{13} = 1,57 + i0,29 kA, I_{23} = 1,49 - i0,07 kA were obtained at the nodes of the EPS.

Further, the voltages at nodes 2 and 3 were determined using the obtained sensitivity models of the first and second orders for taken power line I capacity perturbations increments. In table 2 calculation results are demonstrated when power line I capacity changing on the levels: 1; 5; 9; 34; 48; 67; 86; 90; 95 % of the initial power line capacity. Calculation result when power line I is disabled (capacity perturbation is 100%) is shown in the last row of table 2.

DOI: 10.1051/e3sconf/20172503001

Table 2. Voltage deviation in test scheme nodes on power lines I-II capacity perturbation.

Power line capacity perturbation		U (init.mod.), kV U_2 = U_3 =	$\Delta_Y U$, kV $\Delta_Y U_2$ = $\Delta_Y U_3$ =	U *(lin.mod.)*, kV U_2 = U_3 =	$\Delta_{YY} U$, kV $\Delta_{YY} U_2$ = $\Delta_{YY} U_3$ =	U *(quad.mod.)*, kV U_2 = U_3 =
% of initial.	absolute value					
1%	$-5{,}87*10^{-5}$ + $i2{,}11*10^{-4}$	235,02 - i7,78 213,72 - i71,7	0,1 - i0,05 0,05 - i0,03	235,04- i7,78 213,72 - i71,72	0,13 - 0,06i 0,06 - 0,03i	235,17 -7.81i 213,75 -71,73i
5%	$-2{,}83*10^{-4}$ + $i0{,}10*10^{-2}$	235,32 - i8,08 213,89 - i71,8	0,47 - i0,25 0,24 - i0,12	235,41- i7,98 213,91 - i71,81	0,6 - 0,31i 0,3 - 0,16i	235,72 -8.13i 214,06 -71,89i
9%	$-5{,}4*10^{-4}$ + $i0{,}19*10^{-2}$	235,75- i8,22 214,1 - i71,81	0,91 - i0,47 0,45 - i0,23	235,85 - i8,2 214,12 - i71,92	1,15 - 0,59i 0,57 - 0,3i	236,42 -8.5i 214,41 -72,07i
34%	$-1{,}98*10^{-3}$ + $i0{,}71*10^{-2}$	238,41 - i9,9 215,5 - i72,21	3,32 - i1,72 1,66 - i0,86	238,26 - i9,45 215,33 - i72,55	4,22 - 2,18i 2,11 - 1,09i	240,37 -10,54i 216,38 -73,09i
48%	$-2{,}97*10^{-3}$ + i0,01	240,84- i11,51 216,79 - i72,62	4,98 - i2,58 2,49 - i1,29	239,92 - i10,31 216,16 - i72,98	6,32 - 3,27i 3,16 - 1,64i	243,08 -11,94i 217,74 -73,8i
67%	$-3{,}96*10^{-3}$ + i0,014	244,03 - i13,74 218,49 - i73,21	6,64 - i3,43 3,32 - i1,72	241,58 - i11,16 216,99 - i73,41	8,43 - 4,36i 4,22 - 2,18i	245,79 -13,35i 219,1 -74,5i
86%	$-4{,}95*10^{-3}$ + i0,018	248,4- i17,02 220,86 - i74,13	8,3 - i4,29 4,15 - i2,15	243,24 - i12,02 217,82 - i73,84	10,54 - 5,45i 5,26 - 2,73i	248,51 -14,75i 220,45 -75,2i
90%	$-5{,}4*10^{-3}$ + i0,019	250,98- i19,08 222,27- i74,74	9,05 - i4,68 4,53 - i2,34	243,99 - i12,41 218,22 - i74,03	11,5 - 5,95i 5,74 - 2,97i	249,74 -15,39i 221,07 -75,52i
95%	$-5{,}65*10^{-3}$ + i0,02	252,68 - i20,5 223,21 - i75,17	9,48 - i4,91 4,74 - i2,45	244,43 - i12,64 218,41 - i74,14	12,04 - 6,23i 6,02 - 3,12i	250,45 -15,75i 221,42 -75,7i
Disabled	$-5{,}94*10^{-3}$+ i0,021	254,79 - i22,31 224,38 - i75,73	9,96 - i5,15 4,98 - i2,58	244,9 - i12,88 218,65 - i74,27	12,65 - 6,54i 6,32 - 3,27i	251,22 -16,15i 221,81 -75,9i

Graphical interpretation of the results is shown on the fig.2, namely the values of the real part of the voltage in the second node. The values of the imaginary part of the voltage in the second node are shown on the fig.3. Fig. 4 and 5 show similarly values for third node.

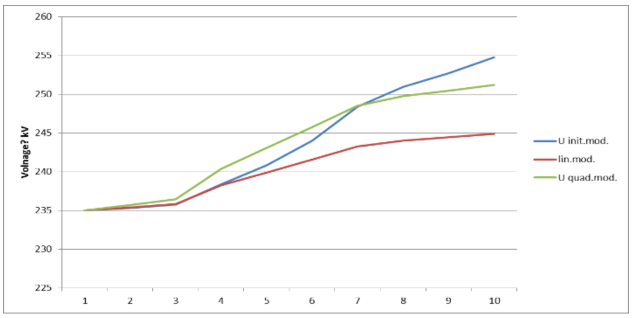

Fig. 2. The values of the real part of the voltage in the second node.

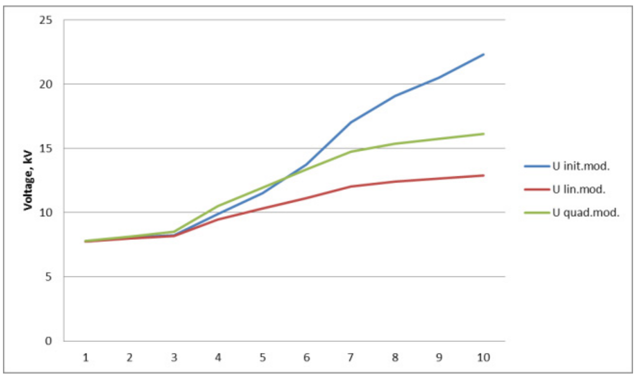

Fig. 3. The values of the imaginary part of the voltage in the second node.

 DOI: 10.1051/e3sconf/20172503001

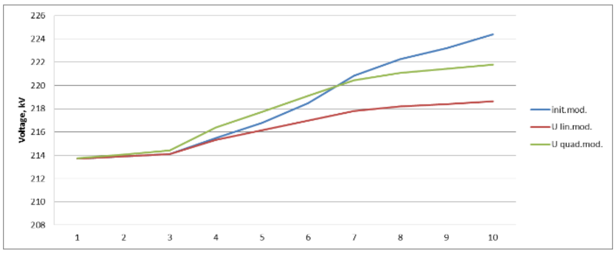

Fig. 4. The values of the real part of the voltage in the second node.

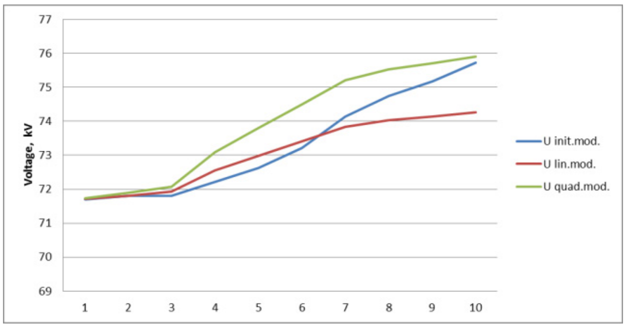

Fig. 5. The values of the imaginary part of the voltage in the third node.

As can be seen from the results, steady state linearized model using for investigated EPS with disabled power line 1 gives voltage deviation in comparison with calculation of steady state with disabled power line in 9.89kV.

When sensitivity model of second order was used deviation was 3,57 kV (1,4%) on the active part of power and 6,16 kV (27,6%) on the reactive part. The voltage deviation by modulus in the second node was 1.6%, in the third node it was 0.8%.

4 Conclusion

In this paper we consider the approximation task of the postfault EPS steady state calculation while assess its static security. Sensitivity models based on first-order and second-order differentials of EPS steady state are suggested for doing that. When security assess, it becomes necessary to calculate the set of EPS steady states because of equipment failures and deviations of customers load. Therefore, when using sensitivity models changings of powerlines capacity, customers load and generating capacity are taken as input parameters and voltages in EPS nodes and powerlines overflows as output. Some differentials of the first and second orders of the steady-state EPS are obtained.

Within the framework of a numerical experiment, investigations were carried out on a three-node EPS scheme where one of the powerlines was disabled. The values of the voltages in the EPS nodes were obtained as a result of steady state calculating on the initial model and on the sensitivity model, which characterizes the voltage deviation in the EPS nodes from the powerlines capacity changing.

The reported study was funded by RFBR according to the research projects No. RFBR 16-38-00312 мол_а

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