Theoretical Analysis for Local Buckling of Corrugated Steel Plate

Bai Jian LI, Liang Sheng ZHU, Xin Sha FU

School of Civil Engineering and Transportation, South China University of Technology, Guangzhou, Guangdong, 510640, China

Abstract: To study local buckling of Corrugated Steel Plate under concentrated loads. Through experimental tests and theoretical analysis, bearing capacity and failure form of Corrugated Steel Plate were discussed. Bearing capacity of Corrugated Steel Plate associated with local buckling, which can be assumed to be composed of three parts: buckling of plane rigid frame caused by concentrated loads, buckling of roof and web caused by bending stress. These three parts were unified by buckling relevant equations, then local buckling calculation formula was obtained. Comparing with experimental results, the loads obtained by local buckling calculation formula agree with test results very well. Since the buckling calculation is independent of the material strength, the calculation formula of local buckling is reliable, it can be used to evaluate local buckling of Corrugated Steel Plate.

1 Introduction

Corrugated Steel Plate (CSP) is pressed or bent corrugated by steel plate, which has high bearing capacity and high stability, has been widely used in bridge and culvert engineering [1-3] and PC box girder with corrugated steel webs [4-11]. In recent years, researchers have developed a new type of structure on the basis of the application of steel-concrete composite deck called corrugated steel-concrete composite bridge deck. This type of structure is first found in a bridge reinforcement project located in Pingnan County, Xunjiang River. The corrugated steel bridge deck was used to improve the shear bearing capacity [12]. Experiments of the corrugated steel-concrete composite bridge deck by Prof. Xu Haiyan and his team obtained load-displacement curves and stress-strain state, which indicated that the structure had characteristics of high bearing capacity and good ductility. Bearing capacity could be calculated by composite structure theory [13]. Prof. Su Qingtian also conducted laboratory experiments, confirming that the corrugated steel-concrete composite bridge deck had the characteristics of light weight, high capacity and high cost; compared with the steel deck, it had the characteristics of low cost, easy to pave, high fatigue strength and high weight; compared with the ordinary steel plate-concrete deck, it had the characteristics of low weight, high shear strength. It is a feasible option in the bridge deck of medium span bridges [14-15].

In summary, the application of corrugated steel plate in multiple fields was studied deeply, but there were more problems need to be solved, such as local stability of the corrugated steel-concrete composite deck, which affects the mechanical properties of composite deck during construction, the present study mainly focus on the shear buckling performance of corrugated steel webs and the buckling of corrugated steel plate arch. There is no research on local buckling of CSP. Therefore, this paper focuses on solving local buckling problem of CSP.

2 Structural Form and Experimental Test

2.1 Structural Form

Compared with the steel platforms, CSP platform will reduce the number of supporting ribs, greatly increase the span of the structure, and reduce the number of steel reinforced beams. The ends of CSP are usually supported on steel beams or concrete beams with setting connecting bars.

*Corresponding author: Email: BJian_LI@163.com

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This kind of platform is mainly applied in the architectural engineering called Composite Floor, the thickness of steel plate used in Composite Floor is less than 1.5mm, but the thickness of CSP is usually from 3mm to 10mm, maybe thicker. The increasing of the thickness will supply higher bearing capacity.

2.2 Experimental Test

In this paper, the experimental test of CSP platforms will be conducted with the following three kinds of dimensions: 400×50×3mm (Width, Height and Thickness), 500×50×5mm and 500×86.6×5mm. The span of the structure is 2390mm, the distance of the support of the distribution beam is 800mm, and CSP section characteristics are shown in Figure 2 and table 1.

![Figure 2 Graphic Presentations of CSP: a) CSP1; b) CSP2; c) CSP3](image)

<table>
<thead>
<tr>
<th>Components</th>
<th>Moment of inertia I/mm⁴</th>
<th>Area A/mm²</th>
<th>Plastic modulus S/mm³</th>
<th>Height of centroid y/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSP1</td>
<td>611676</td>
<td>1324</td>
<td>26471.9</td>
<td>20</td>
</tr>
<tr>
<td>CSP2</td>
<td>1043717</td>
<td>2707</td>
<td>58875.3</td>
<td>27.5</td>
</tr>
<tr>
<td>CSP3</td>
<td>4382812</td>
<td>3000</td>
<td>108189.7</td>
<td>45.8</td>
</tr>
</tbody>
</table>

Testing loads are controlled by the variation of load-displacement law, CSP1: the load step is 3kN, and break for 1 minute at the end of each load step, until reaching the ultimate bearing capacity; CSP2, CSP3: the load step is 5kN, and break for 1 minute at the end of each load step, until reaching the ultimate bearing capacity.

![Figure 3 Loading Plan](image)

3 Result Analysis and Discussion

The load-displacement curves obtained by experimental tests are drawn in Figure 4. According to the bearing capacity curve of Figure 4, it is seen that the ultimate bearing capacity of CSP1 is 30kN, CSP2 is 67.1kN and CSP3 is 125kN. All CSPs showed good elastic performance, load-displacement curve is basically linear before yielding.

![Figure 4 Load - Displacement Curves](image)

The deformations of the loading position are shown in Figure 5, both CSP1 and CSP3 show as elastic-plastic buckling, which occurred after yielding with unrecoverable deformation; CSP2 shows as elastic buckling, the plate deformed in the process of loading and deformation recovered when the load was removed. The deformations of the loading point illustrates that the ultimate bearing capacity of the components is relevant to the local buckling.

![Figure 5 Deformation of Loading Position: a) CSP1; b) CSP2; c) CSP3](image)
4 Analysis of Local Buckling

The mechanical behavior of local compressive position of CSP is very complicated. It contains vertical concentrated loads and bending stress in the roof and web. Furthermore, since the web skews cross the roof, the concentrated loads will be decomposed into two component forces along the direction of the web and roof. In addition, the relationship between the roof and the web buckling must be considered. Combining these factors above, local buckling is very difficult to be analyzed. The calculation diagram is shown in Figure 6.

![Figure 6 Analysis Diagram of Local Buckling](image)

The bending stress σ in Figure 6 can be solved easily according to bending theory. The parameters a and b both are half wavelength, which should be calculated by the differential equation of plate buckling [17].

4.1 Half Wavelength

Due to the skew between the web and the roof, the horizontal component force of the X direction will be generated in the roof under the vertical load P, and the roof will be in a state of biaxial compression. In which, \( p_y \) is produced by the bending stress of the roof, and \( p_x \) is produced by the horizontal component force of P.

![Figure 7 Simply Supported Plate of Two-way Compression](image)

The solutions of buckling modes of biaxial compression can be used to solve the half wavelength a and b.

![Figure 8 Analysis Diagram of Local Buckling](image)

1) Buckling of plane rigid frame in X direction

The stability analysis is carried out by using buckling theory of rigid frame [18].
When the left side is $\infty$, the right side will become the following:
$$
\frac{2 - 2\cos k_1l - (k_1l)\sin k_1l}{\sin k_1l - k_1l \cos k_1l} = \infty
$$

$k_1l = \pi/0.7$ is obtained by solving the equation above.

Make $\pi/\mu = k_1l$, $\mu$ will be change from 0.5 to 0.7 with different tangent values.

It is difficult to solve the equation above exactly, which needs to be constantly repeated to get the buckling load $P$.

2) Influence of Y Direction

The buckling analysis above is based on the considerations of buckling of plane frame in X direction, but the slab in Y direction will produce restraint to the buckling in X direction, which will increase the buckling loads. The constraint function in the Y direction can be solved by bending differential equation of plate with 4 sides simply supported [19].

Table 2 Load Conversion Coefficient of Buckling Considering Constrains in Y Direction

<table>
<thead>
<tr>
<th>$l/l_2$</th>
<th>0.5</th>
<th>0.55</th>
<th>0.6</th>
<th>0.65</th>
<th>0.7</th>
<th>0.75</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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</tr>
</tbody>
</table>
When the buckling problem of plane rigid frame and the constraint of Y direction is solved, the stiffness of the frame I_b and I_c, which are relevant to the width of plane rigid frame, needs to be solved, the key parameter to solve I_b and I_c is a', shown in Figure 6 and Figure 8.

The following equation will be obtained after all expression introducing into equation (2).

\[
P + \frac{\sigma}{\sigma_{cr}} + \frac{\sigma_{cr}}{\sigma_{cr}} = 1
\]

(2)

The following equation will be obtained after all expression introducing into equation (2).

\[
\frac{P}{\pi^2EI_y} + \frac{M}{T_y} + \frac{M}{T_x} = 1
\]

(3)

Notice: the relationship between P in equation (3) and P in load-displacement curve in Figure 4 is that the value of P in equation (3) is equal to 1/4 of the value of P in Figure 4, which is the distribution load of the lifting jack. So to make these two unified, the equation (3) becomes the following form:

\[
\frac{P}{\pi^2EI_y} + \frac{P \mu L_{yb} y_c}{2 c t_b \pi^2 E} \frac{y_c^2}{12(1-v^2) \left( \frac{h}{b} \right)^2} = 1
\]

(4)

Where: \( \mu l \) - the height of the web, \( L \)-height of CSP; \( y_c \)-distance from centroid to the roof; \( k \)-elastic modulus of steel; \( \mu \)-calculating factor of length; \( h \)-height of CSP; \( v \)-Poisson’s ratio; \( \chi \), \( \chi_2 \)-the elastic restraint coefficient; \( b \)-width of the roof; \( t \)-thickness of the plate; \( k_1, k_2 \)-local buckling coefficient; \( I_1 \)-moment of inertia of CSP; \( y_c \)-distance from centroid to the roof; \( L_2 \)-distance from loading point to support point.

When buckling load of the plane rigid frame is calculated, buckling of the roof and web (Figure 8) needs to be considered, which affect the buckling of local area. Buckling of the roof and web can be solved by the methods mentioned in reference [21].

If the height of web is \( l \), the distribution width of the bottom will be \( 2l \). Assuming the width with \( l \) or \( 2l \) in the plane rigid frame is not suitable obviously, because the stress distribution is not uniform. So, a relatively concentrated area need to be found, and assuming the stress distribution is uniform in this area, load P is mainly borne by the width of the strip in this area.

In the theory of elastic stability, \( \mu l \) is used to calculate the length of the compression column according to the different supporting conditions. The largest deformation and the maximum internal force is in 0.5\( \mu l \) position of the component. So, the stress distribution width of the web in 0.5\( \mu l \) position (Figure 11) is \( a'=\mu l \) (when \( \mu l=a \); if \( \mu l>a \), \( a'=a \)). With the assumption that the load P is uniformly distributing along the width, see Figure 11.

4) Buckling relevant equation

When buckling load of the plane rigid frame is calculated, buckling of the roof and web (Figure 8) needs to be considered, which affect the buckling of local area. Buckling of the roof and web can be solved by the methods mentioned in reference [21].

Figure 11 Determination Method of a’

According to “Code for Design of Steel Structures”, when the load is applied to the top of the web, it will diffuse along the web with 45 degree angle, and the stress is gradually decreasing along the structural height direction.

**Figure 12 Simplified Model of Buckling; a) Buckling of the roof; b) Buckling of the web**

When the three parts decomposed in Figure 8 are solved. The buckling relevant equation is used to estimate whether buckling happens in local area. If \( P>P_y \), it is elastic-plastic buckling, otherwise, it’s elastic buckling.

The buckling relevant equation can be expressed as:

\[
\frac{P}{P_{cr}} + \frac{\sigma}{\sigma_{cr}} + \frac{\sigma_{cr}}{\sigma_{cr}} = 1
\]

(2)

When \( P \) in equation (3) and \( P \) in load-displacement curve in Figure 4 is that the value of \( P \) in equation (3) is equal to 1/4 of the value of \( P \) in Figure 4, which is the distribution load of the lifting jack. So to make these two unified, the equation (3) becomes

<table>
<thead>
<tr>
<th>Category</th>
<th>Critical stress σ_{cr} = \frac{\pi^2 E c^2}{12(1-v^2) (b/t)^2}</th>
<th>4</th>
<th>1.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buckling of the roof</td>
<td>\sigma_{cr} = \frac{\pi^2 E c^2}{12(1-v^2) (b/t)^2}</td>
<td>23.9</td>
<td>1.0</td>
</tr>
</tbody>
</table>

When \( \mu l \) is the direction of the local area of the \( \mu l \) direction.

### Table 3 Calculation Formula of Local Buckling

When \( \mu l \) is the direction of the local area of the \( \mu l \) direction.

### Table 4 Results of Buckling Calculation

<table>
<thead>
<tr>
<th>Component</th>
<th>( \mu l=0.5 )</th>
<th>( \mu l=0.7 )</th>
<th>( P_{cr} )</th>
<th>Conclusions</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSP1</td>
<td>37.6kN</td>
<td>36.6kN</td>
<td>30kN</td>
<td>yielding</td>
</tr>
</tbody>
</table>

### 4.3 Method validation and discussion

The calculation results of CSP1, CSP2 and CSP3 are summarized in Table 4.
Pcr calculated by formula (4) can only give the distribution range, when the stiffness of the roof from 0 to ∞, which correspond to the situation of μ=0.5 and μ=0.7, it is very difficult to give an exact result. According to table 4, the buckling load range of CSP1 is 36.6kN - 37.6kN, which is greater than the experimental load, the ultimate bearing capacity is controlled by the bending strength; The buckling load range of CSP2 is 67.08kN - 67.8kN, the experimental load is between the range, the ultimate bearing capacity is controlled by the buckling strength of the structure; The buckling load range of CSP3 is 140kN - 144.7kN, which is greater than the experimental load, the ultimate strength is controlled by the bending strength. The results above agree with the experimental results, which illustrate the formula (4) is reasonable. For checking general buckling under concentrated loads, formula (3) is required to be changed to establish the relevant equations of the rigid frame, the roof and the web.

Thickness ratio of the roof of CSP1 is 100/3=33.3, and the web is 70.71/3=23.57, which are close to or less than b/t=30. Thickness ratio of the roof of CSP2 is 200/5=40, and the web is 70.71/5=14.14, which are greater or less than b/t=30; thickness ratio of the roof of CSP3 is 200/5=40, and the web is 100/5=20, which are greater or less than b/t=30. It is not accurate to judge if buckling of CSP takes place or not according to thickness ratio of the plate. The “code for design of steel structures” [21] stipulates that: when the width-thickness ratio b/t is less than or equal to 30, local buckling won’t take place, experimental phenomena does not conform to the provisions in the code obviously, so the width-thickness ratio can’t be used to determine whether buckling of CSP take place or not.

According to the analysis above, the buckling load calculated by formula (4) is reliable. Since the buckling calculation is only relevant to the stiffness of the components and not relevant to the strength of materials, which does not need to be obtained by the material stress-strain relationship. The buckling capacity calculated by the stiffness agrees with the experimental phenomena and results very well, so the formula is reasonable.

The boundary condition of the plane rigid frame is simplified to fixedly connect with the foundations, but the actual boundary condition of the structure is elastic boundary. The bottom plate is partly tensile and partly compressive, the tensile stress will increase the buckling capacity in compressive direction. The concentrated force acts on the roof, which will be transferred to the bottom plate, the transferred pressure is less than that of the roof. The buckling of the web will only take place near the roof. When these factors were considered, the fixed connected boundary condition of plane rigid frame is reasonable.

Experiment used one waveform of CSP, which is not fully reflect the actual structure, but due to the bottom plate of CSP is pulled and is not prone to buckling, so the whole section of actual structure can be divided into a number of single waveform by bottom plates. Buckling loads of actual structure and one waveform of CSP are similar in this paper, so the analysis method and the experimental method are reasonable.

5 Conclusion

The following conclusions can be drawn by the analysis above:

Load-displacement curve of CSP is basic linear before yielding which shows these structures have good elastic properties; the final failure mode has two kinds: one is yielding with the plastic deformation; the other is the elastic buckling with the unstable elastic deformation.

Checking if local buckling takes place at the loading point or not, formula (4) can be used. In general cases, formula (3) can be used. The formula includes three parts: buckling of the simplified plane rigid frame, buckling of the roof and the web. Buckling loads calculated by the formula agree with the experimental results very well, which can be used to determine the local buckling loads.

References

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<table>
<thead>
<tr>
<th>CSP2</th>
<th>67.8kN</th>
<th>67.08kN</th>
<th>67.1kN</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSP3</td>
<td>144.7kN</td>
<td>140kN</td>
<td>125kN</td>
</tr>
</tbody>
</table>

bucking | yielding


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