Study of stability against deep sliding of gravity dam based on JC method

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Abstract. In this study, stability against deep sliding of gravity dam is analysed in order to provide guidance for ensuring stability against sliding of gravity dam during the construction and operation periods. Based on checking-point method (JC method), the study proposes an analysis model for stability against deep sliding of gravity dams. In addition, the rigid body limit equilibrium method is utilized to construct the limit state function of deep anti-sliding stability. With sliding plane anti-shear friction coefficient and anti-shear cohesion calculated as random variables, safety degree of stability against deep sliding is analysed. With JC method employed, the calculation program is compiled to calculate the stability against deep sliding of gravity dam and analyse the non-overflow section of gravity dam under normal storage level. According to calculation program, the reliability index β of stability against deep sliding of gravity dam is 4.36, and the failure risk P_f is approximately zero. Additionally, based on the traditional rigid body limit equilibrium method, the safety factor K is 3.23, which meets the requirements of the design specification. According to the above methods, stability against deep sliding of the dam section meets the requirement. In conclusion, JC method provides a calculation and analysis model for gravity dam's stability against deep sliding, serving as references in the design of gravity dams based on reliability theory.

1. Introduction

In recent years, intensive study have been conducted by domestic and foreign researchers and important progress has been made concerning the application of reliability theory in the design of gravity dams. For instance, Lan [1] delivered a preliminary discussion on the use of reliability theory to calculate the deep anti-sliding risk of gravity dams, and proposed the limit state equation as a calculation formula. Based on this structural reliability theory, Li [2] developed a reliability calculation formula for gravity dam's stability against deep sliding in relation to a deep single sliding plane within the foundations and verify its correctness. Taking advantage of equal safety coefficient method, Yang [3] established a mechanical model which considered lateral resistance in calculating the reliability index of a gravity dam's deep anti-sliding stability. Besides, Wang [4] analysed the reliability index of a gravity dam's stability against sliding along the foundation plane, and made a sensitivity analysis of several random variables affecting the reliability index. Furthermore, Duncan [5] figured out the structure reliability index with the traditional safety coefficient calculation method and variable probability values employed. Based on a real dam section, Su [6] set a detailed example to illustrate the proposed risk analysis approach while Ji [7] worked out structural reliability of gravity dam using artificial neural networks method. Zhang [8] devised a system reliability analysis method for multiple failure patterns of gravity dam on top of the Copula function. Built on the weighted regression response surface method, the reliability indexes of gravity dam foundation faults were computed [9]. Xu [10] deduced the upper limit of failure probability for failure mode and system reliability of dam on the foundation of Bayes formula and Cauchy-Schwarz inequality. Built on the previous research achievements, this study introduced the use of JC method to calculate the stability against deep sliding of the deep double sliding surface in the foundation of a gravity dam, with the aim of introducing reliability theory in the design of gravity dams.

2. Reliability analysis method

In the analysis of the structural reliability, the limit state of a structure is described by performance function when there are n random variables (x_1, x_2, Λ, x_n) that affect the reliability of the structure, the performance function of the structure can be expressed as follows:

$$Z = g(x_1, x_2, \Lambda, x_n) \quad (1)$$

When $Z > 0$, the structure is in a reliable state; when $Z = 0$, the structure is in the limit state and when $Z < 0$, the structure is in a state of failure [11].

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If the random variables x_1, x_2, Λ, x_n obey a normal distribution, with means and standard deviations being $\mu_{x_1}, \mu_{x_2}, \Lambda, \mu_{x_n}$ and $\sigma_{x_1}, \sigma_{x_2}, \Lambda, \sigma_{x_n}$ respectively, then the performance function of the structure $Z = g(x_1, x_2, \Lambda, x_n)$ should also be a normal random variable, with mean μ_z and standard deviation σ_z being expressed as

$$\mu_z = g(\mu_{x_1}, \mu_{x_2}, \Lambda, \mu_{x_n}) \tag{2}$$

$$\sigma_z = \sqrt{\sum_{i=1}^{n} \left(\left. \frac{\partial g}{\partial x_i} \right|_{\mu_{x_i}} \right)^2 \sigma_{x_i}^2} \tag{3}$$

respectively, and probability density of Z being expressed as

$$f(z) = \frac{1}{\sqrt{2\pi}\sigma_z} \exp(-\frac{(z-\mu_z)^2}{2\sigma_z^2}) \tag{4}$$

When normal distribution of Z is converted into a standard normal distribution, the failure risk of Z shall be expressed as follows:

$$P_f = \Phi(-\mu_z/\sigma_z) = \Phi(-\beta) = 1 - \Phi(\beta) \tag{5}$$

In the formulas above, β is the reliability index of a structure, $\beta = \mu_z/\sigma_z$.

However, in most cases, many of the variables in an actual project do not obey a normal distribution. Therefore, it is necessary to transform non-normal random variables into normal random variables through equivalent normalisation, during which the advanced checking-point method (JC method) is generally recommended by JCSS. The service condition of JC method for equivalent normalisation is when the distribution function value and the probability density function value of the equivalent normal random variable $X_i^{'}$ and the non-normal variable X_i are the same, respectively, at the design checking point, x_i^*, which can be expressed with the following formulas:

$$F_{X_i}(x_i^*) = F_{X_i^{'}}(x_i^*) = \Phi(\frac{x_i^* - \mu_{X_i^{'}}}{\sigma_{X_i^{'}}}) \tag{6}$$

$$f_{X_i}(x_i^*) = f_{X_i^{'}}(x_i^*) = \frac{1}{\sqrt{2\pi}\sigma_{X_i^{'}}} \exp(-\frac{(x_i^* - \mu_{X_i^{'}})^2}{2\sigma_{X_i^{'}}}) \tag{7}$$

In the above formulas, $F_{X_i}(\bullet)$ and $F_{X_i^{'}}(\bullet)$ are the distribution function of the normal distribution variable X_i and the non-normal distribution variable X_i respectively; with $f_{X_i}(\bullet)$ and $f_{X_i^{'}}(\bullet)$ being the probability density function of the normal distribution variable X_i and non-normal distribution variable X_i, respectively.

If the mean μ_{X_i} and standard deviation σ_{X_i} of the non-normal random variable X_i are replaced by the mean $\mu_{X_i^{'}}$ and standard deviation $\sigma_{X_i^{'}}$ of the equivalent normalised random variable $X_i^{'}$, the following formulas shall be obtained:

$$\mu_{X_i^{'}} = x_i^* - \Phi^{-1}[F_{X_i}(x_i^*)]\sigma_{X_i^{'}} \tag{8}$$

$$\sigma_{X_i^{'}} = \phi\{\Phi^{-1}[F_{X_i}(x_i^*)]\} / f_{X_i}(x_i^*) \tag{9}$$

In the formulas above, $\Phi^{-1}[F_{X_i}(\bullet)]$ is the inverse function of $F_{X_i}(\bullet)$ while $\phi(\bullet)$ is the cumulative probability density function of the standard normal variable.

The reliability index β represents the length of the design checking point P^* to O (origin of the coordinates) along the normal direction of a limit state surface tangent plane in a standard normal space coordinate system. The direction cosine of the normal line, OP^*, to the coordinate vector can be expressed as follows:

$$\cos\theta_{X_i} = -\frac{\left. \frac{\partial g}{\partial X_i} \right|_{P^*} \sigma_{X_i}}{\sqrt{\sum_{i=1}^{n} \left[\left. \frac{\partial g}{\partial X_i} \right|_{P^*} \right]^2 \sigma_{X_i}^2}} \tag{10}$$

Converting the above-mentioned relationship into the original coordinate system, the coordinate of P^* in the original coordinate system can be expressed with following formulas：

$$x_i^* = \mu_{X_i} + \overline{x_i^*}\sigma_{X_i} = \mu_{X_i} + \beta\sigma_{X_i}\cos\theta_{X_i} \tag{11}$$

In the formulas above, $\overline{x_i^*}$ represents the coordinate of the design checking point x_i^* to the non-normal variable X_i in the standard normal space coordinate system.

Since the point P^* is on the limit state surface, it meets the limit state of formula (1) and can be expressed as follows:

$$g(x_1^*, x_2^*, \Lambda\ x_n^*) = 0 \tag{12}$$

The values of β and x_i^* ($i = 1,2,\Lambda, n$) can be calculated instantly by solving the formulas (10)–(12) simultaneously.

3. Calculation process of stability against deep sliding of a gravity dam (based on JC method)

(Fig. 1) shows the calculation process used to solve the stability against deep sliding of a gravity dam by using JC method, which can be briefly introduced as follows [12]:

1) Set the initial structure reliability index β_0;

2) Assume the initial design checking point $x_i^{1*} = \mu_{X_i}$;

3) Equivalent normalise the non-normal random variables through formulas (8) and (9), then obtain $\mu_{X_i^{'}}$ and $\sigma_{X_i^{'}}$;

4) Determine the partial derivatives of the limit state function to random variables, by calculating $\frac{\partial Z}{\partial f_1^{'}}$, $\frac{\partial Z}{\partial f_2^{'}}$, $\frac{\partial Z}{\partial c_1^{'}}$, $\frac{\partial Z}{\partial c_2^{'}}$

5) Calculate the sensitivity coefficient α_{X_i}. Sensitivity coefficient α_{X_i} reflects the effect of the random variable X_i on the standard deviation of whole the performance function, $\alpha_{X_i} = \cos\theta_{X_i}$, which can be calculated through formula (10).

6) Calculate the new design checking point x_i^{2*} through formula (11). Repeat steps 3–6 until the differences of two successive design checking point are reduced within the allowable range.

7) Substitute the new design checking point x_i^{2*} into the limit state function (12), and examine whether it can meet the limit state function. If it can't, recalculate the reliability index β through $\beta_{n+1} = \beta_n + g_n \cdot \Delta\beta/\Delta g$, and repeat steps 3–7 until $g(x_1^*, x_2^*, \Lambda\ x_n^*) \approx 0$. The final value of β represents the required structure reliability index.

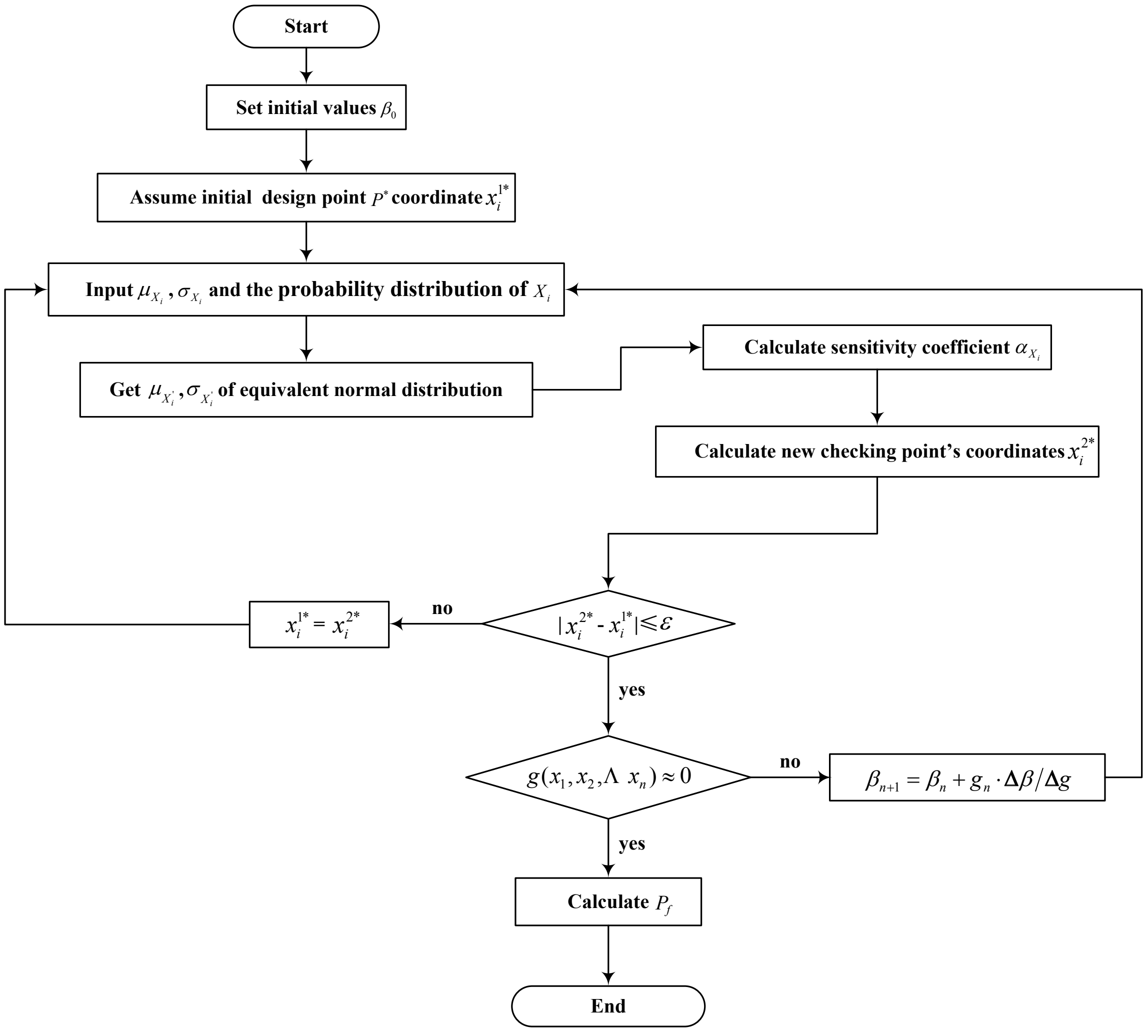

Figure. 1 Flow chart of JC method

4. Examples of Gravity Dam's Stability Against Deep Sliding Calculation

Based on the aforementioned theory and calculation procedure, this study made an analysis of stability against deep sliding in the case of a hydro project non-overflow gravity dam section (Fig. 2).

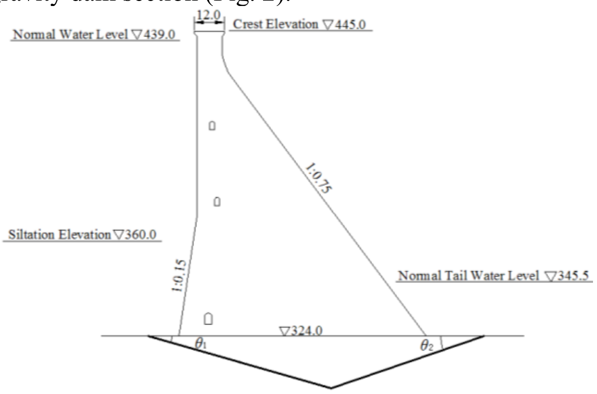

Figure. 2 Sketch of stability against sliding of deep foundations (Unit：m)

The maximum height of the non-overflow section of this dam is 121.0 m, with the foundation plane's elevation being 324.0 m, crest elevation being 445.0 m and the width of dam crest being 12.0m. In accordance with engineering geological data, the angles of the main is θ_1 = 16°while the auxiliary slide plane to the horizontal plane is θ_2 = 19°. In the calculation of stability against deep sliding of a gravity dam foundation, the shear friction coefficient $f_1^{'}$, $f_2^{'}$, as well as shear cohesion $c_1^{'}$, $c_2^{'}$ of the primary and auxiliary slide planes are taken as random variables, whose the characteristics are shown in Table 1. Moreover, bulk density of concrete is 23.8 kN/m^{3}while bulk density of bedrock is 27.0 kN/m^3. The basic combination of the normal storage level is taken as the calculation conditions when upstream water level is 439.0 m and the downstream water level is 345.5 m. The loads to be calculated include the dead weight of the gravity dam, bedrock above the slide plane and the hydrostatic and uplift pressures. Furthermore, the uplift pressure reduction coefficient is taken as 0.25. (The table 1 at the bottom of the page)

According to a self-compiled program, the reliability index β of the gravity dam's stability against sliding is 4.36 and the failure risk P_f is approximately zero. As indicated by the results calculated with reliability theory, gravity dam's stability against deep sliding can meet the requirements. Verified by the rigid body limit equilibrium method, the factor of safety against deep sliding K is found to be 3.23. For basic combination, when safety factor K is greater than 3.0, anti-sliding stability is considered to meet the requirements. Results by both methods suggest that requirements can be met by the stability against deep sliding of the gravity dam.

5. Conclusions

The existence of gently inclined soft layers in the foundations of a gravity dam may result in a deep sliding instability in the foundations. In this study, by taking anti-shear strength parameters, the most sensitive factor to the anti-sliding stability, as random variables, the author established the limit state function using rigid body limit equilibrium method and normalised the non-normal random variables with JC method in order to analyse the stability against deep sliding of the gravity dam's foundation with structural reliability theory. From the calculation results, it can be concluded that the conclusions drawn from the analysis of stability against deep sliding of a gravity dam are the same when using different methods: JC method, and the rigid body limit equilibrium method. Therefore, conclusions can be drawn that JC method can be used in analysing the deep anti-sliding stability of a gravity dam.

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Table.1 Random variables statistical characteristics of gravity dam foundation deep anti-sliding stability

Variable	$f_1^{'}$	$c_1^{'}$ /kPa	$f_2^{'}$	$c_2^{'}$ /kPa
Mean value	0.55	450.00	0.76	500.00
Coefficient of variation	0.25	0.40	0.21	0.35
Standard deviation	0.15	120.00	0.16	180.00
Probability distribution type	Normal distribution	Lognormal distribution	Normal distribution	Lognormal distribution

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