Search method of the parameters network elements based on three-dimensional calculations

Dmitry V. Boykov [2,*], and *Aleksandr* A. Dekterev [1,2]

[1] Kutateladze Institute of Thermophysics SB RAS, Novosibirsk, Russia
[2] Siberian Federal University, Krasnoyarsk, Russia

Abstract. The paper proposes a method to identify a pipeline system segment, containing several piping assemblies and branches, based on three-dimensional calculations or experiments, as well as proves the applicability of proposed approach to search the local resistance coefficients of the section elements of the pipeline system, which describes the complex spatial structure, based on the known values of pressure drops and flow rates.

Introduction

Practically each industrial enterprise has one or another pipeline system in which gas or liquid flow distribution essentially influence technical, economic, and ecological characteristics of production in general. In some cases, objects under study can be represented as a hydraulic network [1]. The network may contain structural elements, whose local resistance significantly affects the nature of flow distribution. Sometimes, determining the resistance coefficients of these elements by existing empirical and analytical dependencies[2,3] is impossible.

Applying spatial simulation [4,5] to these structural elements allows defining hydraulic characteristics and using them in network design model. Earlier, the authors in [6,7] mentioned the difficulty of integrating models with a large number of independent inputs. Building a certain network analog of the element was proposed as one of the possible ways to solve this problem. Although this approach has limited applicability and accuracy as compared to the option with direct integration of the three-dimensional element into the network model [8], it allows quickly constructing the hydraulic network model and obtaining the results of calculations.

As an example of the use of proposed approach, the authors consider the problem on simulation of a multi-tiered scrubber (Fig.1). Scrubber is a complex object with five independent inlets. Using a full spatial model or hybrid model with directly embedded three-dimensional element into a network model [6,7] results in excessive computational costs. The authors used a simplified network model of this object (Fig. 2). To determine the parameters of the network elements, several three-dimensional calculations were carried out.

* Corresponding author: dimkadim@yandex.ru

In consequence of these calculations, knowing flow rates at the inlet, we determined pressure drops between the scrubber inlets and outlets (Table. 1).

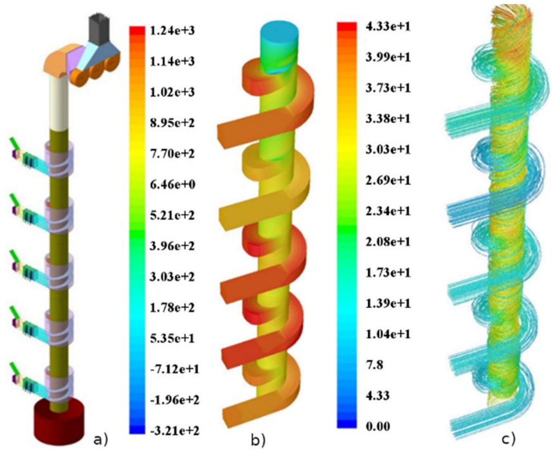

Fig. 1. Scrubber: a) geometry; b) static pressure (Pa); c) streamlines.

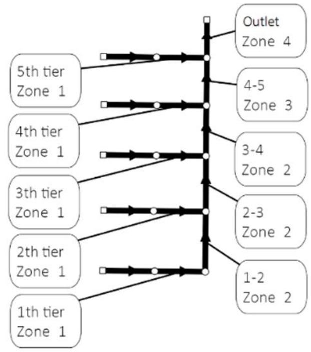

Fig. 2. Network model of scrubber.

The network analogue of the spatial scrubber model (Fig. 2) was divided into several zones:

1. the zone of all inlets, whose geometric similarity suggests the existence of a single local resistance coefficient at all sites in the zone;
2. the zone of a lifting pipe from the 1st to 4th tier, where small velocities allowed suggesting that the influence of the local resistance of these tiers on the flow distribution in the scrubber will be insignificant and, accordingly, local resistance was set to 0 for all sections of the zone;
3. the zone of a lifting pipe from the 4th to 5th tier;
4. the zone of access to the fan.

Accordingly, the problem was reduced to finding three local resistance coefficients for zones 1, 3 and 4 satisfying the condition of minimum difference between the pressure drop in the scrubber for network computations and 3D calculations. Initially, the coefficients were found by the exhaustive search method [6,7]. Unfortunately, such a search method becomes too expensive at a large number of unknown parameters. In this regard, it turned out to be advisable developing a search algorithm to find parameters of the network section based on known data from the results of 3D calculation or experiment.

Table 1. Calculation data of scrubber 3D model

	1st 3D calculation		2nd 3D calculation		3rd 3D calculation	
	Pressure, Pa	**Flow rate, kg/s**	**Pressure, Pa**	**Flow rate,** kg/s	**Pressure, Pa**	**Flow rate, kg/s**
1st tier	799	6.60	2712	13.20	555	6.60
2nd tier	952	7.80	2019	7.80	210	0.10
3rd tier	950	8.12	3263	16.30	196	0.10
4th tier	656	5.14	1529	5.14	383	5.14
5th tier	934	9.86	1566	9.86	836	9.86
Outlet	0	37.52	0	52.27	0	21.80

Flue ducting calculation model

As a mathematical model to describe the fluid flow in the network, a model of steady-state flow of incompressible fluid was adopted. For representation of pipeline system we use the directed graph[9], whose constraint matrix is presented in the following form:

$$\nabla_{il} = \begin{cases} 1, \text{ if } l \in O_i, \\ -1, \text{ if } l \in I_i, \\ 0 \quad otherwise \end{cases} \tag{1}$$

where $l \in O_i$– is the plurality of pipes coming out from the i-th piping assembly; $l \in I_i$– is the plurality of pipes going into the i-th piping assembly. The problem of flow distribution in the network can be reduced to a combination of the mass conservation law in the piping assembly (2), and the flow resistance law in the pipe (3):

$$\sum_{l \in U_j} \nabla_{jl} x_l = 0 \tag{2}$$

$$\sum_{i \in N} \nabla_{il} p_i = h_l \left(x_l \right) \tag{3}$$

where N – is the set of all network piping assemblies; U – is the set of all network branches; x_l – is the flow rate in the branch (kg/s); $h_l(x_l)$ – is the pressure loss (Pa); p – is the pressure (Pa).

Finding a solution to this nonlinear system of equations is carried out by the nodal pressure method. That is, the following flow rate and pressure is sought through the correction:

$$x_l^{K+1} = x_l^K + \delta x_l \tag{4}$$

$$p_i^{K+1} = p_i^K + \delta p_i \tag{5}$$

Substituting equations (4) and (5) in (2) and (3) it is easy obtaining a system of linear algebraic equations (SLAE) for the pressure correction:

$$\begin{gathered} \sum_{i \in N} \Delta_{ij} \left(a \right) \delta p_i = -\sum_{l \in U_j} \nabla_{il} x_l, \, j \in N_{inner} \\ \delta p_j = 0, \; j \in N_{bdr} \end{gathered} \tag{6}$$

$$\Delta_{ii}\left(a\right)=\sum_{l\in U_l}a_l,\ i\in N_{inner}$$

$$\Delta_{ij}\left(a\right)=-\sum_{l\in U_l}a_l,\ i\in N_{inner},j\in N,i\neq j$$

$$a_l=\begin{cases}\left(\dfrac{\partial h_l}{\partial x_l}\right)^{-1},l\in U_{inner}\\ 0,l\in U_{bdr}\end{cases}$$

here N_{bdr}, N_{inner} – are sets of boundary nodes and computational nodes, respectively; U_{bdr}, U_{inner} – are sets of boundary branches and computational branches, respectively. Pressure is fixed in the boundary nodes, while flow rate is fixed in the boundary branches.

Search for local resistance coefficients

For the identified branches, we determine the correction of the analyzed value in the form of [10]:

$$C^{K+1}=C^K+dC \tag{7}$$

where C can be, for example, diameter or local resistance.

Substituting (5) and (7) in (2) and (3) we obtain SLAE, which associates the correction of some magnitude in the branches with the resulting pressure change in the nodes.

$$\begin{gathered}\sum_{i\in N}\Delta_{ij}\left(a\right)dp_i=\sum_{l\in U}\nabla_{il}\left(\frac{\partial h_l}{\partial x_l}\right)^{-1}\left(\frac{\partial h_l}{\partial C}\right)dC,\ j\in N_{inner}\\ dp_j=0,\ j\in N_{bdr}\end{gathered} \tag{8}$$

Where dp_i- is the pressure change; dC - is the correction. The left part of equation (6) at the final iteration coincides with the left part of the SLAE of the correction (8).

It is easy to show that the resulting pressure change for the sum of the different impacts is equal to the sum of the pressure changes of these impacts. Using the computation results of the SLAE (8) this allows determining the SLAE for correction of the variable:

$$\frac{dp_I^m}{dC_m}\cdot\delta C_m=P_I-p_I,\ m\in U_{ident},I\in M_{control} \tag{9}$$

here δC_m – is the correction of the identified value, $m\in U_{ident}$ – is the set of identifiable objects, P_I^m – is the required pressure in the control points, p_I^m – is the pressure in the specified nodes (control points) of the network model for which the model is adapted, $I\in M_{control}$ - is the set of control points.

The coefficient dp^m_I/dC_m is defined as the ratio of pressure change in the control node dp_I to the corresponding change of the identified value $dC_m$and is found through the solution of the equations system (8) for all options of control actions. The object being identified can be understood as both a single branch and a group of similar branches (by example 1, Fig. 2).

In this example, the control points are the inlets to the scrubber (Table 1). The required pressure P_I is taken from the corresponding 3D calculation (three 3D calculations, 5 nodes in each, 15 calculation points in total). The resulting system of equations is over-defined [11]. A fixed flow rate is given on boundary branches.

The coefficient of local resistance in the formula for pressure loss is the identifiable value: $h_l\left(x_l,\xi_m\right)=\xi_m\dfrac{\left|x_l\right|\cdot x_l}{2\cdot\rho_l\cdot s_l^{\ 2}}$ where ξ_m – is the local resistance coefficient; s_l – is the cross-sectional area of the pipeline (m^2), and ρ_l – is the flow density (kg/m^3). The right part of SLAE (8) can be represented as:

$$\left(\frac{\partial h_l}{\partial x_l}\right)^{-1}\left(\frac{\partial h_l}{\partial \xi_m}\right)d\xi_m=\frac{x_l}{2\xi_m}d\xi_m. \tag{10}$$

The above allows proposing the following algorithm for searching the local resistance of identifiable branches: The network model (2, 3) is calculated for each three-dimensional calculation, in which the flow rate is fixed at the boundary branches. To calculate matrix columns $dp^m{}_l/dC_m$, SLAE (8) is solved for each three-dimensional calculation and for each identifiable object. New local resistance coefficients are calculated after solving resulting SLAE (9), after which the cycle repeats over again.

The result of solving the test task

Table 2. The local resistance coefficient

	Search method	Option 1	Option 2	Option 3
1st tier	2.8	2.4	2.5	2.7
2nd tier	2.8	2.0	2.5	2.7
3rd tier	2.8	2.3	2.5	2.7
4th tier	2.8	2.2	2.5	2.7
5th tier	2.8	2.6	2.5	2.7
1st and 2nd tiers	0	0	1.8	0
2nd and 3rd tiers	0	0	0	0
3rd and 4th tiers	0	1.5	1.2	0
4th and 5th tiers	2.8	2.1	1.8	2.5
Outlet	2	2.2	2.3	2.1

The proposed algorithm was applied to calculate the multi-tier scrubber. Three grouping options of local resistance coefficients were considered: 1– all branches are considered to be independent; 2 – inlets at tiers are united into a single block (zone 1 in Fig. 2); and 3 – the option fully corresponds to the problem statement as in the exhaustive search method. Table 2 shows the local resistance coefficients for all three options. The exhaustive search method and the method proposed in option 3 gave similar results.

Table 3 shows the comparison of the models with respect to the pressure drop. In all options, pressure drops are close to the data obtained through 3D modeling, while the error does not exceed the accuracy of calculations accepted for engineering practice.

Conclusion

We proposed a method for determining the parameters of the network branches based on to the three-dimensional calculation. The local resistance coefficients of several branches of the network model based on the spatial simulation are determined on the example of calculations of a multi-tier scrubber.

Table 3. Comparison of network models

No. of 3D caculation			1st calculation		
	3D calculation	Exhaustive search method	Option 1	Option 2	Option 3
1st tier	799	990	1003	995	966
2nd tier	952	1118	1053	1095	1086
3rd tier	950	1154	1113	1127	1121
4th tier	656	854	778	788	837
5th tier	934	1087	1003	1059	1076
No. of 3D calculation			**2nd calculation**		
	3D calculation	**Exhaustive search method**	**Option 1**	**Option 2**	**Option 3**
1st tier	2712	2714	2680	2678	2619
2nd tier	2019	1817	1915	1901	1819
3rd tier	3263	3385	3215	3221	3255
4th tier	1529	1595	1454	1446	1556
5th tier	1567	1431	1459	1464	1447
No. of 3D calculation			**3rd calculation**		
	3D calculation	**Exhaustive search method**	**Option 1**	**Option 2**	**Option 3**
1st tier	555	502	460	481	489
2nd tier	210	179	189	188	182
3rd tier	196	178	188	188	181
4th tier	383	375	332	351	367
5th tier	836	844	800	773	816

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