

Drag determination of an array of square cylinders subjected to shear flow in a compound channel

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Abstract. Overbank flow in rivers threatens integrity of built elements located in the floodplain. Elements of infrastructure close to the interface between main channel and floodplain are subjected to complex hydrodynamic actions resulting from the obstruction of the shear flow that develops in that interface. In the current paper, the drag forces and the drag coefficient of building-like structures positioned in the interface are investigated. The experimental setup in Laboratório Nacional de Engenharia Civil (LNEC) involves the placement of an array of square cylinders on the floodplain of a straight compound channel, next to the interface with the main channel. Three-component instantaneous-velocity recordings were performed by means of Acoustic Doppler Velocimetry (ADV) within the boundaries of a considered fluid-control volume encompassing the array, while uniform-flow conditions were established in the channel. The equation of momentum conservation was applied in its integral form in the fluid control-volume towards estimation of the time-averaged drag force at a certain elevation from the floodplain. The drag coefficient is estimated accounting for the typical shear layer at the main-channel/floodplain interface and is compared with coefficients strictly valid for isolated cylinders.

1. Introduction

The retardation of flow caused by an obstacle inside a moving fluid is expressed by the drag force. The drag caused by arrays of emergent cylinders in open-channel flows has gained attention by researchers the past decade [1,2,3]. In most cases, flow through the array is considered uniform and the drag coefficient C_d involves the parameters that refer to a single cylinder of the array, that are: the force exerted on it, the cylinder's diameter and a streamwise velocity that characterizes the mean flow through the array. The assessment of this array-averaged drag coefficient often requires the validity of the following assumptions: existence of an infinite array and steady energy slope of a uniform flow along the channel.

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Drag of emergent cylinders in compound-channel flows was poorly investigated. The particular case where bluff bodies are positioned on the floodplain, at the interface with the main channel is of special interest. The obstacles experience the effects of a shear layer that develops in the mixing region of the two fluids with abrupt velocity differences. Strong streamwise-velocity gradients in the lateral direction [4] and 3-D processes like secondary currents and vortices [5,6] mainly characterize this region, blurring the well-known image of cylinder wakes in flows with spatially uniformly-distributed bulk velocities and negligible turbulence effects.

This paper presents an analytical drag-force estimation of a finite array of emergent square cylinders placed at the main-channel/floodplain interface of a laboratory compound-channel. The combined drag-force of the array emerges from the assessment of the counteracting force of the array on the flow. The momentum-balance equation is applied in its integral form in a fluid control-volume encompassing the cylinder array. Mean-flow and turbulence parameters were experimentally evaluated in the near region of the array and fed into the equation. The drag coefficient is estimated based on the measured drag force and the flow velocity which characterizes the shear layer. Its value is compared with the commonly-used drag coefficients for isolated cylinders. Comparison is also made with the coefficients estimated experimentally for an isolated cylinder under a) uniformly-distributed flow, in the middle of the floodplain and b) shear flow at the interface, respectively.

2. Theory

2.1. Equation of momentum conservation and drag force

A control volume of fluid is considered that encompasses an array of bluff bodies. The integral equation of conservation of time-averaged momentum is written as follows:

$$\int_{S_c \setminus S_0} \rho U_i (U_j n_j) dS = \int_{S_c \setminus S_0} -P n_i dS + \int_{V_c} \rho g_i dV + \int_{S_c \setminus S_0} -\overline{\rho u'_i u'_j} n_j dS + \int_{S_c \setminus S_0} T_{ij} n_j dS + \left(\int_{S_0} -P n_i dS + \int_{S_0} T_{ij} n_j dS \right) \quad (1)$$

where n_i is the outward pointing normal unit-vector, U_i is the time-averaged velocity vector, P is the time-averaged pressure, T_{ij} is the time-averaged viscous-stress tensor, $-\overline{\rho u'_i u'_j}$ is the Reynolds-stress tensor, g_i the acceleration of gravity, ρ the density of the fluid, V_c the control volume, S_c the total surface of the control volume and S_0 the part of S_c that is bounded by the cylinders walls. The last two terms of the right-hand part compose the acting force on the flow by the cylinders. Therefore it is valid that:

$$L |R_i| = \left| \int_{S_0} -P n_i dS + \int_{S_0} T_{ij} n_j dS \right| \quad (2)$$

where L is the wet length of the cylinders and R_i is the averaged within-the-length- L force on the array per unit submerged length.

Considering Equations (1) and (2) and assuming negligible viscous effects, the momentum-balance principle is applied in the fluid control-volume of Figure 1, in the streamwise direction x as:

$$hR_x = \int_V \rho g_x dV + \sum_{k=1..6} \left\{ - \int_{S^{(k)}} \rho U_x^{(k)} (U_j^{(k)} n_j^{(k)}) dS + \int_{S^{(k)}} -P^{(k)} n_x^{(k)} dS + \int_{S^{(k)}} -\rho \overline{u'_x u'_j} n_j^{(k)} dS \right\} \quad (3)$$

where h is the height of the control volume.

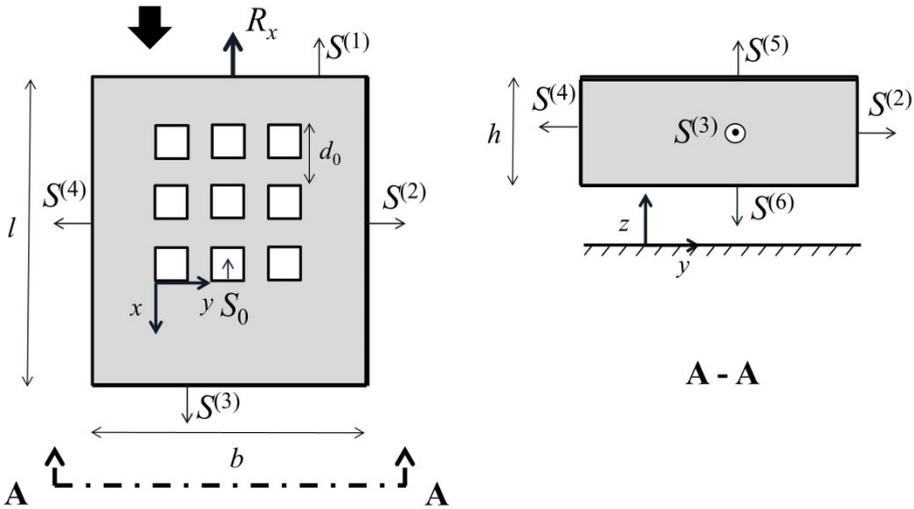


Fig. 1. Orthogonal fluid control-volume, assumed for application of the momentum balance.

2.2. Drag coefficient in shear flow

In most drag-assessment issues in open channels, a 2-D (or 2-D depth-averaged) uniform velocity-field is assumed upstream an emergent cylinder in a steady flow, so that the space-average velocity of the wake equals the undisturbed-flow velocity upstream the obstacle U_0 . In the case of infinite arrays of cylinders, although U_0 is not defined, the consideration of negligible shear stresses, lateral momentum-fluxes and pressure gradients integrated at the boundaries of the flow allow for the assumption that U_0 equals to the spatially-averaged velocity $U_{av}=Q/A$, where Q is the flow rate of the steady flow and A the total cross-sectional area. Therefore the drag coefficient in each of the case of isolated cylinders and cylinders in arrays is defined respectively as:

$$C_d = \frac{2R_x}{\rho U_0^2 d} \quad (4a)$$

$$C_d = \frac{2\langle R_x \rangle}{\rho U_{av}^2 a d} = \frac{2\langle R_x \rangle}{\rho U_p^2 d} \quad (4b)$$

where d is the width of the cylinder and U_p is the time-and-space-averaged pore velocity of the flow through the array. Momentum balance necessitates that $a=1/(1-\lambda)^2$, where λ is the solid-volume fraction. Angle brackets denote averaging among the n cylinders of the array.

In any case, U_0 , actually defined or represented, characterizes a uniformly distributed 2-D flow towards the obstacle, but it is not representative for its use in Equations (4a) and (4b) when this hypothesis is not applicable. Instead, a fluid control-volume encompassing

all obstacles in the flow may be considered, like the one described in 2.1, with this specification: The positions of its boundaries ($S^{(1)}$ - $S^{(6)}$) are chosen as such, so that the assumptions regarding the shear stresses, the fluxes and the pressure gradients described in the beginning of 2.2 are valid. Then, the equivalent velocity to U_0 , U_{av} , emerges from space averaging across the width b (Figure 1) of this control volume.

When the described method is applied in the basic case of single obstacle within a uniformly distributed 2-D flow, b is equal to the width of the obstacle. In the case of single obstacle within a well-defined 2-D shear layer, b equals to the width of the shear layer. Herein a regular finite array is examined that is found on the floodplain of a symmetrical straight compound-channel, next to the floodplain/main-channel interface, as shown in Figure 2. The one boundary of the array is out of the shear layer. Considering the assumptions made, drag estimation should refer to elevations well above the bottom, where 2-D flow may be approached. The corresponding control volume would theoretically feature a length $l \rightarrow \infty$ (Figure 1), $h \rightarrow 0$ (to avoid effects of 3-D processes and neglect vertical-flux imbalance) and width b equal to the effective width Δ , which coincides with the distance between the edge of the array in the inner floodplain and the center of the main channel. The relative longitudinal position of the control volume would again-satisfy the boundary conditions. Equation (4b) now may be applied with λ now referring to the solid-volume fraction of the array in the control volume and with $a \rightarrow 1$. Depth-averaged velocity U_a is given by:

$$U_{av} = \frac{1}{\Delta} \int_{\Delta} U_x dy \tag{5}$$

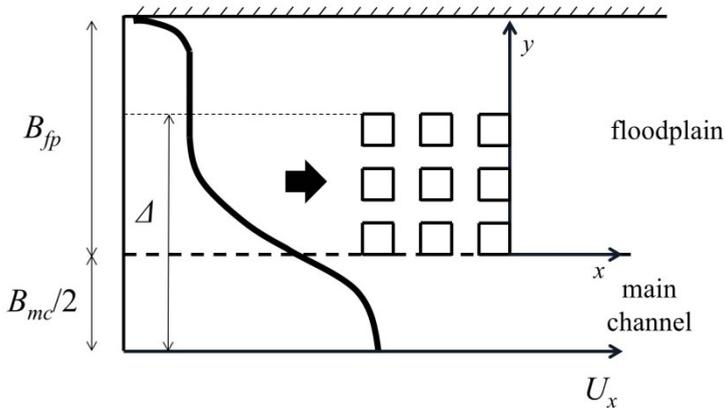


Fig. 2. Plan view of an orthogonal array next to the interface of main-channel and floodplain in a straight symmetrical compound channel. The upstream typical streamwise-velocity profile in the lateral direction is also shown.

3. Experimental configuration and methods

3.1. Experimental facility and setup

The experimental test was run in the 10m-long straight symmetrical compound channel at the National Laboratory for Civil Engineering in Portugal (LNEC). The channel consists of a trapezoidal main channel with width $B_{mc}=0.6$ m and two symmetrical floodplain-sections with width $B_{fp}=0.7$ m each. The longitudinal slope is constant and equal to 0.0011. The

bottom of the main channel is hydraulically smooth (polished concrete) while that of the floodplain is artificially roughened by a layer of synthetic grass, the detailed characteristics of which are described in Fernandes, Leal, and Cardoso [7]. Separate-inlet configuration for the main channel and the floodplain, allows for establishing uniform-flow conditions within an adequate length in the channel. Detailed description of the facility can be found in Fernandes [8].

Nine square cylinders were placed in the channel, according to the configuration depicted in Figure 2. The cylinders are placed at equal distances among them d_0 (Figure 1) forming a 3x3 regular orthogonal array. Uniform-flow conditions were established in the compound channel, by distributing total discharge Q_{tot} in the main channel (Q_{mc}) and in the floodplain (Q_{fp}), according to the Weighted Divided Channel Method, proposed by Lambert and Myers [9]. Under these conditions the cylinders were emergent. A summary of the main experimental parameters is given in Table 1. The relative flow-depth h_r is defined as the ratio between the floodplain flow-depth h_{fp} and the main-channel flow-depth h_{mc} .

Table 1. Experimental parameters.

h_r	h_{fp}	Q_{tot}	Q_{mc}	Q_{fp}	d	d_0
(-)	(m)	($l s^{-1}$)	($l s^{-1}$)	($l s^{-1}$)	(m)	(m)
0.31	0.045	58.9	42.3	16.6	0.045	0.1

3.2. Experimental procedure

Three dimensional (3-D) instantaneous-velocities were acquired with a side-looking Nortek Vectrino Acoustic Doppler Velocimeter (ADV) at a rate of 200 Hz. The measuring grid covers the horizontal area of the control volume depicted in Figure 1 with $l=0.625$ m ($\sim 14d$) and $b=0.27$ m ($=6d$). Duration of recordings was set at three minutes. The measuring grid was applied at 10 horizontal lines along the streamwise direction containing typically 15 measuring points each. The same grid was applied at two elevations from the floodplain bottom, $z = 0.015$ m ($h_{fp}/3$) and $z = 0.012$ m, yielding a height of the control volume $h=3$ mm. The acquired velocity time-series were subject to the phase-space despiking filter proposed by Goring and Nikora [10]. Measurements of the free-surface elevation at the boundaries of the assumed control volume were conducted with ultrasound recorders. The recordings had duration of two minutes and were performed at a rate of 10 Hz.

4. Results and discussion

4.1. Drag parameters

The drag force at elevation $z=h_{fp}/3$ was estimated by Equation (3) applied in the control volume described in 3.2. The average velocity of the incoming undisturbed flow U_{av} is estimated by Equation (5), after the assessment of the effective width Δ according to 2.2 for the following cases with the same flow conditions: a) Δ_0 for single cylinder in the floodplain, equal to the width of the cylinder, b) Δ_1 for single cylinder at the interface, and c) Δ_a for cylinder array at the interface. The case with the cylinder in the floodplain represents a typical 2-D flow with uniform velocity-distribution. In Figure 3 the relevant distances are presented relatively to the geometry of the lateral profile of the mean streamwise-velocity upstream the obstacles. Table 2 summarizes the values of Δ and the corresponding Reynolds numbers defined as $Re_d=U_{av}d/\nu$, where ν is the kinematic viscosity

of the fluid. It is expected that U_{av} for the single cylinder is greater than the corresponding one for the array, since the latter occupies more space and its width expands at a greater distance towards the inner floodplain. Higher Re_d for the array configuration is due to different mean water-temperatures during the experimental campaign.

Table 2. Shear-flow parameters.

Tests	Re_d	Δ
	(-)	(m)
Isolated cylinder at floodplain (“0”)	8571	0.045
Isolated cylinder at interface (“1”)	20888	0.420
Cylinder array at interface (“a”)	22127	0.545

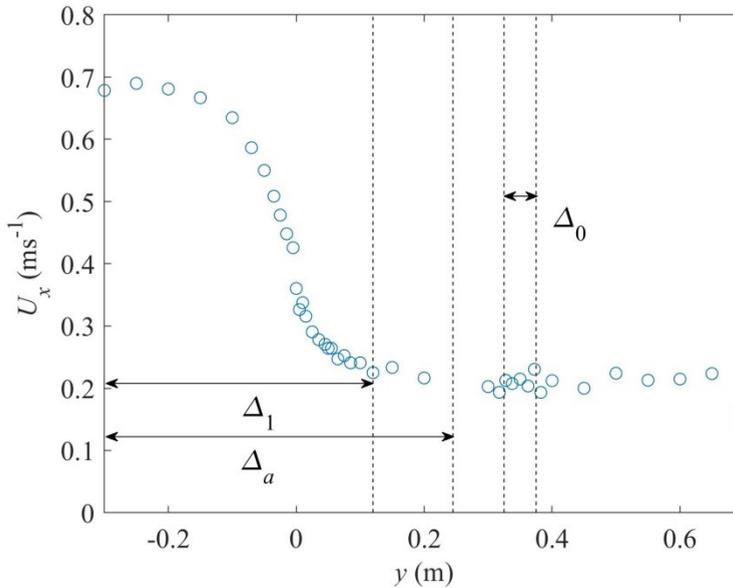


Fig. 3. Lateral distribution of the streamwise velocities.

4.2. Mean drag-coefficient

In Table 3 the corresponding drag-coefficients of the two cases of isolated cylinders, in the floodplain and at the interface, are compared to the ones found in studies that regard an open-channel flow ($Re_d \sim 10000-22000$) [11], a closed-water channel flow ($Re_d \sim 21400$) [12] and an air flow ($Re_d \sim 24000$) [13]. This comparison is made in order to assess the accuracy of the followed method. The estimated C_d values are found within the range of estimations of the other authors. However, it seems that the presence of the shear layer reduces the drag coefficient of the cylinder. This observation may be attributed to the occurrence of 3-D interfacial processes that increase shear-stress in the near-wake region, opposing to the drag on the cylinder.

The drag coefficient produced from the mean force exerted on the array of cylinders at the aforementioned elevation is equal to 0.37. The array does not demonstrate similar behavior to that of both investigated cases of isolated cylinder, since its C_d is significantly

lower than the isolated-cylinder C_d . In an attempt to understand the drag-reduction mechanisms, the coefficients of cylinder pairs found by Robertson [11] ($Re_d \sim 5600-12800$) are presented in Table 4. These pairs regard configurations of two tandem or side-by-side square cylinders with different ratios of d/B , where B is the width of the channel, and flow depth $H > 4d$. The relevant coefficients refer to tandem cylinders with streamwise distance d_x , within the range $2d < d_x < 3d$, and side-by-side cylinders with lateral distance between them d_y , within the range $2d < d_y < 3d$.

The array C_d is lower than that of all configurations of cylinder pairs (Table 4). The coefficient that is found closer to the array's C_d is that of the tandem cylinders for the higher d/B ratio. This observation could justify someone's assumption that the array's influence on the flow resembles that of a tandem-cylinder pair, but the flow processes related with drag reduction are more intense and effective. If the 3x3 examined array is considered as a side-by-side configuration of three columns of consequent tandem-cylinder pairs, and assuming that the number of side-by-side cylinders does not affect significantly their C_d (see Table 4 for 1x2 side-by-side), it may be concluded that the existence of the third cylinder-row contributes to a further reduced drag-coefficient comparatively to the case of the 2x1 tandem arrangement (Table 4).

Moreover, the reduced C_d may be also attributed to the relative position of the array in the shear layer. Since U_{av} in Eq. (4b) characterizes a flow area mainly affected by the shear layer, the estimated C_d expresses the effect of the array on the shear flow, bounded within the width Δ , and not only the width of the frontal area of the array. This implies that the estimated C_d represents the additional non-uniformity induced by the array, in a flow regime posed by the high velocity-differences between the floodplain and the main channel.

Table 3. Drag coefficients of square isolated cylinders.

Case	C_d
	(-)
Measured in uniformly distributed flow	2.06
Measured in the shear layer	2.00
Uniformly distributed flow in [11]	2.11
Uniformly distributed flow in [12]	2.10
Uniformly distributed flow in [13]	1.96

Table 4. Drag coefficients of pairs of cylinders.

Case	C_d
	(-)
Pairs side-by-side: $d_x=0$, $2d < d_y < 3d$, $d/B=6.3\%$ in [11]	2.59 – 2.73
Pairs side-by-side: $d_x=0$, $2d < d_y < 3d$, $d/B=12.7\%$ in [11]	3.22 – 3.28
Pairs tandem: $d_y=0$, $2d < d_x < 3d$, $d/B=6.3\%$ in [11]	1.07 – 1.15
Pairs tandem: $d_y=0$, $2d < d_x < 3d$, $d/B=12.7\%$ in [11]	0.92 – 1.13

5. Conclusions

Drag on a finite array of square cylinders subject to shear flow in a compound channel was assessed by considering momentum balance in the wake region. Estimation of the mean drag-force at a certain elevation from the floodplain was based on laboratory velocity-measurements. The drag coefficient was found significantly decreased compared to the common-cited drag-coefficients of isolated cylinders in uniformly distributed flows.

Some of the advantages of the presented method would feature:

- its compatibility in determining analytically the combined drag-force of any system of bluff bodies in a random flow. It can be applied in order to estimate the force at a given elevation, or the entire length of submergence.
- its adequacy in assessing drag indirectly considering the counter-effects in the flow (mean flow and turbulence), when means of direct measurement are not available, efficient or appropriate.

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