

# Accuracy analysis of satellite measurements of the measurement geodetic control network on the southern Spitsbergen

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**Abstract.** Geodetic networks are very important in analyses of long-term monitoring deformation of natural objects in polar regions. The ground stability of geodetic reference points can give the quality of the results of geometry measures. Well measures give the purpose to environmental protection of measured natural objects (moraines, talus slopes, debris etc.). The main objective of this paper is to compare the adjustment of the geodetic network by the Least Square Method based on one (ASTRO) or two (ASTRO and NYA1) reference points with full rank and robust adjustment. The main adjustment steps with important numerical results are presented for both methods. The method how to detect the presence of the used wrong approximate coordinates of network points is addressed, and the detection approaches are given for both adjustment procedures. The results of both adjustment procedures summarized in the Conclusion indicate that the combination of these procedures is not suitable way of detecting errors in a geodetic network.

## 1 Introduction

Geodetic measurement networks are very important in the geodetic measurements of geometry of the natural objects. Determination of displacements and changes in surface shape of the natural and artificial objects is carried out based on determined reference points [1]. The measurement control network's points, which are permanently stabilized in the field, constitute such reference points and should be treated as constants during the measurements of geometry of the tested object [2]. Measurement of object geometry and geometry changes is a multi-stage and repeatable process, which is usually stretched over time [3]. Glacial moraines, dunes, sea cliffs, talus slopes, debris natural escarpments in river beds, etc. constitute examples of natural spatial objects that change over time. Changes are of various nature, depending on factors affecting the rate of changes. Observation of changes in the surface shape is possible on the basis of accurate geodetic measurements, where the permanent control network's points are the reference and the guarantee of correctness of the obtained results. Also in the case of examination of industrial objects, such as: chimneys, cooling towers, water dams and bridges, geodetic control networks are the basis for determination and changes of geometry of the measured object [4, 5], development and increase in a number of cartographic documents concerning polar areas. Polar regions, which are very difficult to measure directly due to specificity of these regions, natural conditions, as well as legal-

organizational restrictions, constitute a very interesting area of research [6]. In polar areas, often behind the polar circle, the measurement control networks are established, whose coordinates are determined on the basis of satellite measurements [7, 8]. Due to the lack of reference points, the most commonly applied measurement method is the GNSS measurement, in reference to the GLONASS and GPS positioning system, whose results are developed in the post-processing [9]. This article includes a description of the principle of establishment, measuring and aligning XYZ spatial coordinates, along with the accuracy assessment of two points of the measurement control network [10]. Assumed points constitute the basis for development of the results of direct measurements, as well as the control of measurements and development of results [11]. Analysis of the mutual stability of the measurement control network's points also allows for the assessment of changes, displacements and deformations of the measurement control network, and subsequently changes in the terrain surface shape in the region of development of the direct measurements [12]. The measurement control network was established in the region of the Werenskiöldbreen glacier, in the southern part of the Spitsbergen island, constituting a part of the Svalbard archipelago. The points were established in the area available for direct measurements with very good visibility of the horizon, on the outcrops of crystalline rocks. The control network consists of 2 base points (B1 and B2).

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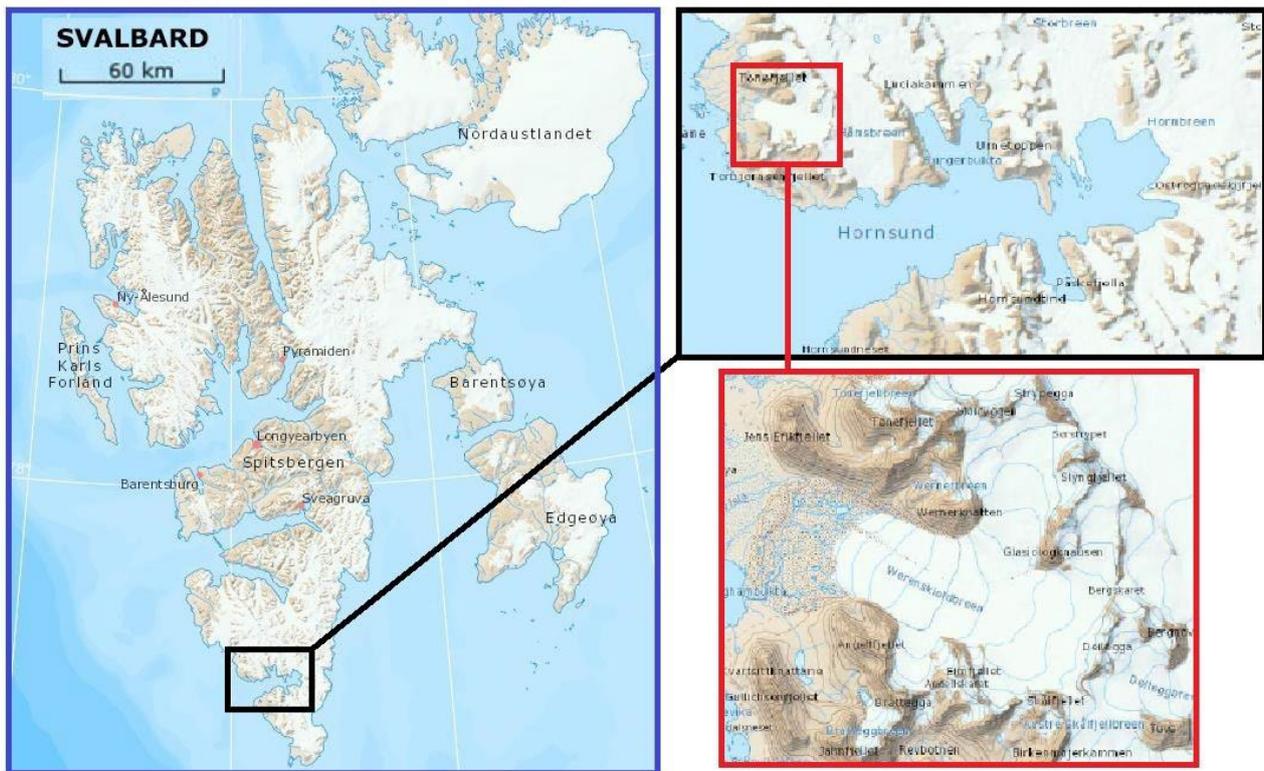


Fig. 1. Spitsbergen with the region of establishment of the measurement control network's points map [13].

## 2 Field measurements

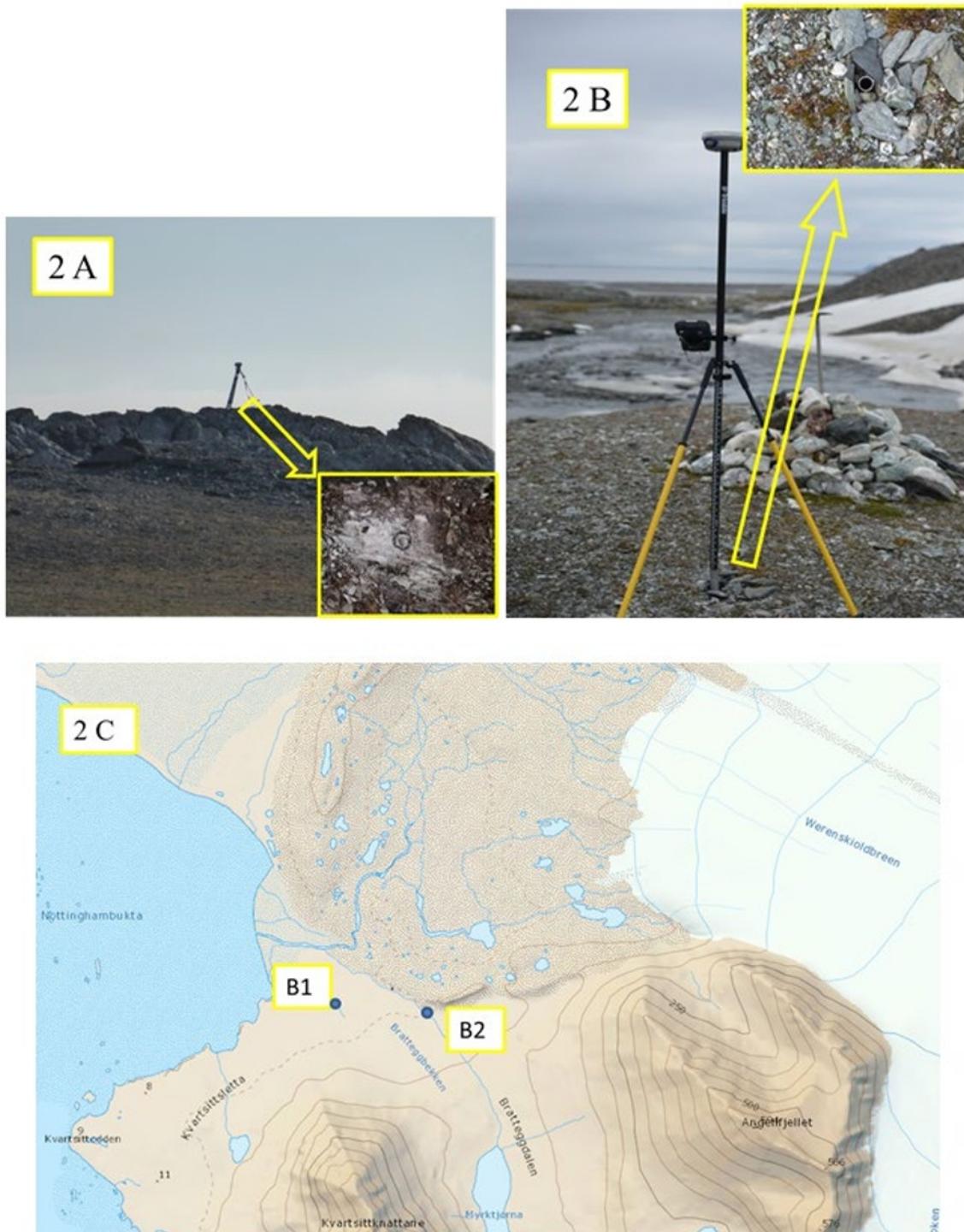
There's only one permanent reference point established in the Svalbard archipelago of the EUREF Permanent GNSS Network in Ny-Alesund with the name of NYA1, in the northern part of the archipelago, on the island of Spitsbergen, in Kongsfjorden Bay, whose corrections can be read continuous mode. The distance from Ny-Alesund to the Nottinghambukta Bay, in the region of which the measurement control network's points were established, amounts to approx. 230 km in a straight line. Near the Polish Polar Station Hornsund, by the Hornsund bay, the ASTRO reference point was established in 1985. Access to corrections is possible only in the post-processing mode. The distance from Hornsund Bay to Nottinghambukta Bay amounts to approx. 11 km in a straight line. At the turn of July and August 2015 and in June 2017, in the area of the Werenskiöld glacier, the controls, repeatable direct satellite measurements were carried out (GPS and GLONASS signals were received) at 2 base points (B1 and B2). Location of the base points was determined every day, at various time intervals (from 5 hours to 30 seconds). B1 point was measured 85 times, while B2 point was measured 110 times.

In the polar regions, for the reasons of nature protection, it's forbidden to establish points that will be permanently connected with the bedrock and it's also forbidden to carry out drillings, concreting and clear marking of control points. In order to establish the points, existing natural faults and rock recesses were used. However, the lack of possibility to mark the points

significantly limits the unambiguous traceability of location of the measurement mark's centre. Location of point B1 is the intersection of natural gaps in the outcrop of crystalline rocks, which however, when unmarked, cause erroneous identification of the correct location of the point. The centring of the antenna was carried out over the scratch and recess in the quartzite, fig. 2A and 2B. Point B2 was located on outcrops of slates in the valley of Bratteg river, on which rock debris of varying granularity was thrown. The place where an old aluminium radio mast was driven into the ground was selected as a point of the measurement control network, Fig. 2C and 2D. The aluminium tube with 3 cm in diameter was deeply driven (approx. 50 cm) into the erratic material of the bedrock. Thanks to such location, the point was easy to find and easy to identify almost unambiguously. However, the large diameter of tube inserted deeply into the ground does not unambiguously indicate the centre of the mark. Photos of the base points B1 (1A) and B2 (1B) are presented above, along with the location on the topographic map 2C, published by the Norwegian Polar Institute.

## 3 Results

GNSS measurements were carried out within 30 measurement days (21 in 2015 and 8 in 2017). The PDOP coefficient of the satellite measurement geometry for each measurement session fluctuated between 1.3 - 3.6. The input mask included satellites located more than 10° above the horizon. During the measurements, a total of 11-21 GPS and GLONASS satellites were visible over



**Fig. 2.** Control points 2A) point B1; 2B) point B2; 2C) location of base points on the topographic map [16].

the horizon, whose signals were used during the determination of spatial location of the control network's points. The errors in the scope of points location obtained during the measurement reached average values of approx. 5 cm in the horizontal plane and approx. 15 cm in the vertical plane, similarly like in literature revue [7, 11]. Similarity of the obtained measurement results was noted in each of the measurement campaigns and the differences appear between the measurement sessions. The sources of occurring measurement errors were

probably the ambiguous markings of control points. The lack of a clearly marked point centre made it difficult to identify and locate the base points. The inaccurate antenna setting of the GNSS receiver (centring error) in these conditions may reached 2-3 cm in the horizontal and vertical plane for each of the points.

In order to determine the most probable values of the coordinates of base points on Spitsbergen, in the region of the Werenskiöldbreen glacier, the strict alignment of all coordinates of the base points with the least squares

method was carried out [14], in accordance with the following principles:

$$x_{sr} = \frac{1}{n} \sum_{i=1}^n x_i \quad (1)$$

where:

$x_{sr}$  - arithmetic mean value,

$x_i$  - result of a single measurement of  $x, y, z$  coordinates of the base point,

$n$  - number of measurements.

Result deviation of the measurement of point  $x$  from the expected value is described by the formula  $Ex_{sr} = \mu$  (2), which can be also written as  $v_i = x_i - \mu$  (3), and it's called the accidental error of the measurement. The expected value is equal to zero, i.e.  $E\varepsilon = 0$  (where  $\varepsilon$  – real error), while the measure of variability  $Ex$  is the covariance matrix of the observation  $C$  [1]:

$$C = Evv^T = E(x - x_{sr})(x - x_{sr})^T \quad (4)$$

and

$$C = \frac{1}{n-1} \sum_{i=1}^n v_i v_i^T = \frac{1}{n-1} \sum_{i=1}^n (x - x_{sr})(x - x_{sr})^T \quad (5)$$

$$= \begin{bmatrix} m_x^2 & m_{xy} & m_{xz} \\ m_{xy} & m_y^2 & m_{yz} \\ m_{xz} & m_{yz} & m_z^2 \end{bmatrix}$$

With the errors  $m_x^2, m_y^2, m_z^2$  as variances of coordinate and values  $m_{xy}, m_{xz}, m_{yz}$  as covariances, known as “mean coordinate errors” and determining coordinate dependencies. From the covariance matrix of the observation  $C$ , the covariance ellipses of mean coordinates  $x$  and  $y$  of a single measuring point were determined. Azimuth  $\alpha$  of the larger semi-axis of (A) ellipse [10]:

$$\alpha = \frac{1}{2} \operatorname{atan}\left(\frac{2m_{xy}}{m_x^2 - m_y^2}\right) \quad (6)$$

Azimuth  $\beta$  of the shorter semi-axis (B) amounts to  $\beta = \alpha + \pi/2$  (7). While the lengths of the semi-axes, respectively [10]:

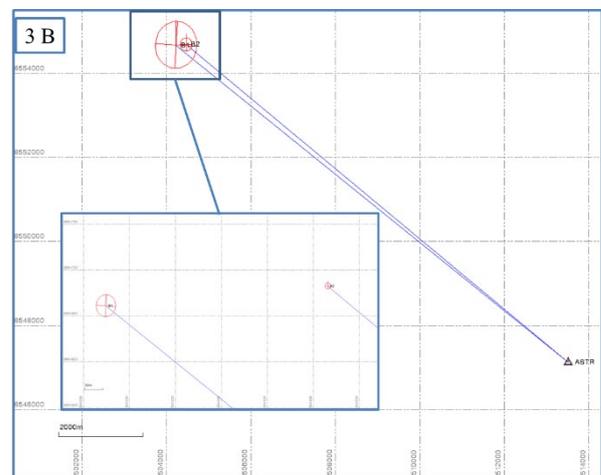
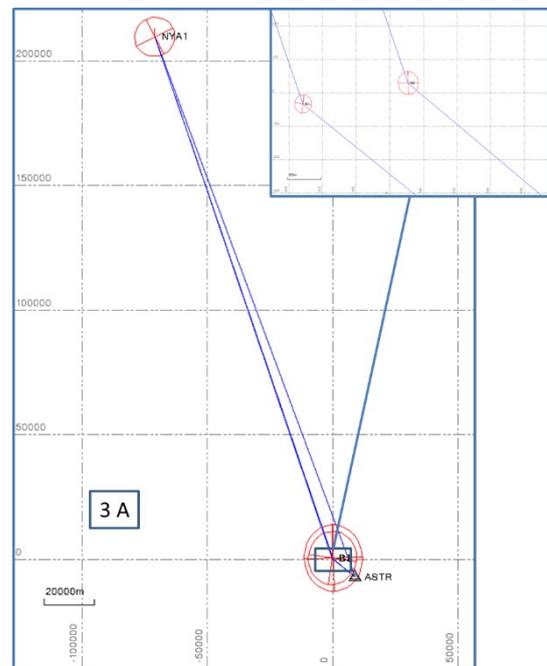
$$A = \frac{\sqrt{m_x^2 m_y^2 - m_{xy}^2}}{\sqrt{m_y^2 \cos(\alpha)^2 - m_{xy} \sin(2\alpha) + m_x^2 \sin(\alpha)^2}} \quad (8)$$

$$B = \frac{\sqrt{m_x^2 m_y^2 - m_{xy}^2}}{\sqrt{m_y^2 \cos(\beta)^2 - m_{xy} \sin(2\beta) + m_x^2 \sin(\beta)^2}} \quad (9)$$

The measure of horizontal accuracy of the point's location is the error  $m_p$ , calculated from the following dependence:

$$m_p = \sqrt{m_x^2 + m_y^2} \quad (10)$$

The calculations were carried out in two variants (every using 1 to 10 formulas). The first one assumed the reference of measurement results to 2 reference points (NYA1 and ASTRO), while the second variant assumed the determination of coordinates only in reference to the ASTRO station in Hornstadt. Results of the analyses are shown in Fig. 3. The calculation of coordinates, due to corrections in the interval 5" from the ASTRO station allow for a more accurate determination of the coordinates of flat base points.



**Fig. 3.** Results of alignment of the base points 3A) in reference to NYA1 and ASTRO, as well as 3B) in reference to ASTRO.

Fig. 3 present ellipses of the location errors of base points, aligned in reference to the point ASTRO (3B), as well as ASTRO and NYA1 (3A). The geometry of GNSS measurements results in the fact that error ellipses are even, elongated in the direction of the X axis, while the

semi-axes A and B have values given in the tables below (Tab. 1 and Tab. 2).

**Table 1.** Error Ellipse Components connection to ASTRO point (formulas 5-9).

Point ID	Semi-major axis A (Meter)	Semi-minor axis B (Meter)	Azimuth of A axis
B1	0.040	0.036	4°
B2	0.033	0.029	177°
NYA1	0.010	0.009	142°

**Table 2.** Error Ellipse Components connection to ASTRO and NYA1 points (formulas 5-9).

Point ID	Semi-major axis A (Meter)	Semi-minor axis B (Meter)	Azimuth of A axis
B1	0.036	0.033	8°
B2	0.012	0.010	178°

During comparison of obtained values of the error ellipses, it can be concluded that tying the measurements to the base point NYA1 does not increase the accuracy of obtained results. The reason for obtained results is the considerable distance between the measurement points and the reference point NYA1 - over 200 km. Moreover, it's not possible for the station NYA1 to obtain reference corrections with an interval smaller than 30". The errors of determining the base point heights, calculated from the same measurement intervals and in reference to the reference station ASTRO amount to 0.071m for point B1 and 0.032m for point B2. In relation to the station NYA1, they are significantly higher and they amount to 0.125m for point B1 and 0.152m for point B2.

After calculations in the post-processing, with the use of reference corrections, in reference to the station ASTRO, average location errors of  $m_p$  points reach the value of 1.1cm in horizontal plane, while  $m_z$  in the vertical plane amounts to 7.1cm. Used reference corrections from the station ASTRO had the intervals of 1" and 5", respectively. The accuracy of determining point B2 is significantly higher than the determination of the location of point B1, which may result from ambiguity in the identification of point B1 of the network, which wasn't unambiguous, located on the outcrop of quartzites. Point B2 is significantly better identified in the field.

## 4 Conclusions

Accurate determination of the location of points in geodetic measurement networks is very important.

Unfortunately, in the case of polar areas, the precise determination of location of the points is difficult to achieve, due to specificity of this area. In the regions, where establishment of permanent points connected with the bedrock is not possible, marking of points on the surface is associated with the accuracy of their determination. The measurement of plane coordinates within 1 measurement period is accurate, however in comparison with other time periods it's divergent. Such situation results from the lack of possibility to precisely identify the control network's points. Application of compensatory variant with the use of estimation methods would probably improve the results of values of the obtained coordinates of base points [15, 16]. However, in the context of global warming and climate change, determination of moraine displacements or changes in the glacier location does not require accuracy at the level of a few or a dozen millimetres, which is required in monitoring or displacement examination of engineering objects. Obtaining the determinability or repeatability of determining the point's location at the level of a few centimetres is sufficient, which was implemented within this study. One of the most important conclusions during establishment of the measurement control network's points is the need to develop accurate and unambiguous topographic descriptions of the points, which are supposed to constitute a geodetic control network in polar regions with high climatic variability, in a difficult and not very accessible area. Polar bases and huts allow to establish the points in locations available for measurements, which are not at risk of being destroyed by nature and animals.

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## References

1. E. Osada, *Osnovy geodezyjne* (Wydawnictwo UxLan, Wrocław, 2013)
2. G. Weiss, K. Bartoš, S. Labant, J. Gašinec, E. Weiss, L. Mixtaj, R. Weiss, J. Zuzik, *Journal of Cleaner Production*, **170**, 789-796 (2018)
3. E. Osada, *Analiza, wyrównanie i modelowanie geodanych* (Wydawnictwo Akademii Rolniczej, 1998)
4. T. Głowacki and Z. Muszyński, *IOP Conference Series: Materials Science and Engineering*, Vol. **365**, No. 4, p. 042075 (IOP Publishing, 2018)
5. Z. Muszynski and W. Milczarek, *IOP Conference Series: Earth and Environmental Science*, Vol. **95**, No. 4, p. 042069 (IOP Publishing, 2017)
6. L. Kolondra, *Old and modern geocosystems of Spitsbergen*, 427-456 (2013)
7. T. Gargula, *Survey Rev.*, **41**, 313, 301-313 (2009)

8. K. Krarup and K. Kubik, Deutsche Geodätische Kommission bei der Bayerischen Akademie Wissen, Reihe A, Heft 98, München (1983)
9. B. Hofmann-Wellenhof, H. Lichtenegger, J. Collins, *Global Positioning System: Theory and Practice* (Springer-Verlag, Wien, 2001)
10. C.D. Ghilani, *Adjustment computations: spatial data analysis* (John Wiley & Sons, 2017)
11. K. Kowalczyk and J. Rapiński, *Geofizika*, **34**(1), 45-65 (2017)
12. A. Leick, *GPS Satellite Surveying* (John Wiley & Sons, New Jersey, 2004)
13. TopoSvalbard, <http://toposvalbard.npolar.no/>, access: 12.06.2018
14. Z. Wiśniewski, *Rachunek wyrównawczy w geodezji (z przykładami)* (Wydawnictwo Uniwersytetu Warmińsko-Mazurskiego, 2009)
15. P.J. Huber, *Robust Statistics* (Wiley, New York, 1981)
16. K.R. Koch, *Parameter estimation and hypothesis testing in linear models* (Springer Science & Business Media, 2013)