

# A New On-The-Move Integer Ambiguity Determination Method for Precise Positioning of Highly Maneuvering Ground Vehicles

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**Abstract.** In the conventional RTK (Real Time Kinematics), carrier phase measurements should be collected for several minutes in stationary state in order to determine the IA (Integer Ambiguity) in carrier phase to get the precise position. To determine the IA in motion, several OTM-RTK (On-The-Move RTK) methods have been proposed using vehicle dynamics or augmenting additional sensors. This paper presents a new OTM-RTK technique to determine the IA without aids of external sensors for precise positioning of highly maneuvering ground vehicles. In the proposed technique, the initial IA is determined fast by estimating precise position change during epochs using dual frequency carrier phase measurements. Therefore, IA determination of the proposed method is not influence by vehicle dynamics. By field experiment, performance of the proposed technique is analyzed including IA determination time according to vehicle dynamics and the number of visible SV.

## 1 Introduction

The accurate position and speed information of the car navigation is needed to control the vehicle precisely. When the precise position associated with road image and road information would be applied to vehicle autonomous control systems such as lane-keeping, robustness of longitudinal/lateral control can be greatly improved. In general, to obtain a high-precision position with an accuracy of several cms using a Global Positioning System (GPS), carrier phase measurements should be used along with code measurements [1]. For precise positioning using carrier phase measurements, the integer ambiguity included in the carrier phase measurement should be determined. The integer ambiguity can be obtained through searching and evaluating integer ambiguity candidates since there is no analytical solution, and many studies have been performed in the past to reduce time and computation of resolving the integer ambiguity such as reducing search range. Among them, Least squares AMBIGUITY Decorrelation Adjustment (LAMBDA) and Ambiguity Resolution with Constraint Equation (ARCE) are the typical methods [2]. Fundamentally, LAMBDA is a method of obtaining a solution of Integer Least Squares (ILS), which has the condition of an integer and has excellent computational advantages in addition to systematic theories, and is applied to various high-precision positioning systems such as Real Time Kinematics (RTK) [2]. In order to resolve the integer ambiguity in RTK, the stationary state must be

maintained for a long time because the integer ambiguity should be determined by collecting measurements for several tens to hundreds of epochs in the stationary state. In addition, it is difficult to apply the conventional method of resolving the integer ambiguity in the stationary state to precision navigation systems of unmanned vehicles because the integer ambiguity must be retrieved again if the satellite signal is disconnected and re-received while the vehicle is moving or if a cycle slip occurs. Therefore, studies on new integer ambiguity resolution techniques are necessary for On-The-Move RTK (OTM-RTK) of moving vehicle.

This study proposed an OTM-RTK technique to resolve integer ambiguity using only Global Navigation Satellite System (GNSS) measurements on the move in order to perform fast and precise positioning of high maneuvering ground vehicles on the move based on low-cost GNSS receivers that can be installed in vehicle navigation systems.

## 2 Floating Ambiguity Estimation

Conventional RTK studies collected measurements for several tens to hundreds of epochs in the stationary state to determine the integer ambiguity included in the carrier phase measurements. Assuming that  $m$  satellites are observed in one epoch, the double differenced code and carrier phase measurements, measured during the  $n$  epoch at the stationary state are as shown in (1) [3].

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$$\begin{aligned} \delta\rho &= \begin{bmatrix} \delta\rho_1 \\ \delta\rho_2 \\ \vdots \\ \delta\rho_n \end{bmatrix} = H\delta x + \varepsilon = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \end{bmatrix} \delta x + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix} \\ \delta l &= \begin{bmatrix} \delta l_1 \\ \delta l_2 \\ \vdots \\ \delta l_n \end{bmatrix} = H\delta x + \lambda N + w = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \end{bmatrix} \delta x + \lambda N + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \end{aligned} \quad (1)$$

Where,  $\delta\rho_i, \delta l_i (i=1 \sim n)$  is a linearized double differenced code and a carrier phase measurement, respectively, and the vector of  $(m-1) \times 1$ ,  $H_i (i=1 \sim n)$  is the line-of-sight differenced vector between the satellites of  $(m-1) \times 3$ ,  $\delta x$  is the base line vector between the base station of  $3 \times 1$  and the vehicle,  $\lambda$  is the length of the carrier wave,  $N$  is the integer ambiguity vector of  $(m-1) \times 1$ , and  $\varepsilon_i, w_i (i=1 \sim n)$  is the double differenced code and carrier phase measurement noise, respectively, and the vector of  $(m-1) \times 1$ . Assuming that measurements are collected at the stationary state and there is no change in the satellites,  $\delta x$  and  $N$  are constant values regardless of the change in epoch, it is assumed that the measurement errors such as ionospheric delay and tropospheric delay are removed by double differencing because the distance between the two receivers is not too far (within 10km) and there is no multipath. The measurement noise  $\varepsilon, w$  is a White Gaussian noise, which is independent from each other, and is assumed to have the characteristics of  $\varepsilon: N(0, Q_p), Q_p = (DD^T)^{-1} \sigma_p^2$  and  $w: N(0, Q_c), Q_c = (DD^T)^{-1} \sigma_c^2$ , respectively. Where,  $D$  is a differentiation matrix.

In order to solve (1), (1) is arranged as (2).

$$\delta y = Aa + Bb + e = \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} + e = M\hat{x} + e \quad (2)$$

Where  $\delta y = [\delta\rho \ \delta l]^T$ ,  $A = [H \ H]^T$ ,  $a = \delta x$ ,  $B = [0 \ \lambda]^T$ ,  $b = N$ ,  $e = [\varepsilon \ w]^T$ ,  $M = [A \ B]$ , and  $\hat{x} = [a \ b]^T$  are represented.

By applying the least squares method to (2), unknown  $\hat{x}$  can be estimated as shown in (3).

$$\hat{x} = (M^T M)^{-1} M^T \delta y \quad (3)$$

It is possible to resolve the integer ambiguity by applying techniques such as LAMBDA and ARCE using the floating ambiguity  $\hat{N}$  estimated from (3) and the covariance of the integer ambiguity, which is a search range of the integer ambiguity, and a high precision positioning is possible using the carrier measurements by obtaining the integer ambiguity [2, 3]. Applying the Wide Lane measurement to the carrier measurement of

(1) allows rapid determination of the integer ambiguity [4].

### 3 The proposed OTM-RTK technique

This paper proposes an OTM-RTK technique to determine the integer ambiguity on the move. The state diagram of OTM-RTK proposed in this study to resolve the integer ambiguity on the move is as shown in Fig. 1.

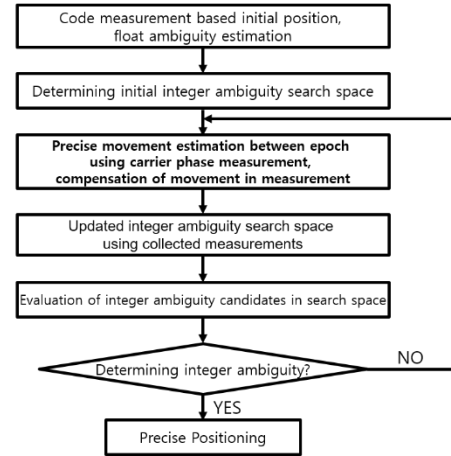


Fig. 1. Proposed OTM-RTK technique

#### 3.1. Code-based Initial Position Calculation, Floating Ambiguity Estimation and Integer Ambiguity Search Range Configuration

The linearized double difference code and carrier phase measurements in the stationary state (0 epoch) are shown in (4).

$$\begin{aligned} \delta\rho_0 &= H_0 \delta x_0 + \varepsilon_0 \\ \delta l_0 &= H_0 \delta x_0 + \lambda N_0 + w_0 \end{aligned} \quad (4)$$

The initial code-based position can be estimated from the code measurements in (4), consisting of a single epoch or several multiple epochs. In the case of a single epoch, code-based estimate of the floating ambiguity is obtained as shown in (5) by substituting the code-based estimated initial position into the carrier phase measurement [1].

$$\hat{N}_0 = \frac{\delta l_0 - H_0 \delta \hat{x}_0}{\lambda} \quad (5)$$

The search range of the integer ambiguity estimated based on the code of a single epoch is expressed as a covariance matrix as shown in (6) [3, 4].

$$\text{cov}(\hat{N}_0) = E[\hat{N}_0 \hat{N}_0^T] \approx \frac{I}{\lambda^2} (\sigma_p^2 + \sigma_c^2) \quad (6)$$

#### 3.2. Position Movement Estimation between Epochs and Carrier Measurement Position Movement Compensation

In LAMBDA,  $\delta x$  is fixed as  $(3 \times 1)$  in (1), because measurements are collected in a stationary state, but  $\delta x$  is updated every epoch in a moving state,  $\delta x$  becomes  $(3n \times 1)$  if the measurements are collected during  $n$  epochs as shown in (7) thus, it is difficult to apply the conventional integer ambiguity resolution technique to measurements of moving vehicles.

$$(7) \quad \begin{bmatrix} \delta \rho_1 \\ \delta \rho_2 \\ \vdots \\ \delta \rho_n \\ \delta l_1 \\ \delta l_2 \\ \vdots \\ \delta l_n \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \\ H_1 \\ H_2 \\ \vdots \\ H_n \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \vdots \\ \delta x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \lambda N \\ \lambda N \\ \vdots \\ \lambda N \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \\ w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

In this step, the precise position movement between epochs is calculated using double differenced Integrated Carrier Phase (ICP) variation between epochs, and this is compensated in the carrier measurement. Estimating the position movement between epochs using ICP variation between epochs is shown in (8) [4].

$$\delta \hat{x}_i = (H_i^T (DD^T)^{-1} H_i)^{-1} H_i^T (DD^T)^{-1} \lambda \delta \Theta_i (i=1, 2, L, n) \quad (8)$$

Where,  $\delta \Theta_i$  is ICP variation between epochs.

By updating  $\delta \hat{x}_i$  from 1 epoch to  $n$  epoch, the measurements can be collected and summarized as shown in (9).

$$(9) \quad \begin{bmatrix} \delta l_1 \\ \delta l_2 \\ \vdots \\ \delta l_n \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \\ M \\ M \\ \vdots \\ M \end{bmatrix} \delta \hat{x}_0 + \begin{bmatrix} H_1 \delta \hat{x}_1 \\ H_1 \delta \hat{x}_1 + H_2 \delta \hat{x}_2 \\ \vdots \\ H_1 \delta \hat{x}_1 + H_2 \delta \hat{x}_2 + L + H_n \delta \hat{x}_n \end{bmatrix} + \lambda \begin{bmatrix} N \\ N \\ \vdots \\ N \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

Although (9) contains the initial position bias  $\delta \hat{x}_0$ , the error between  $\delta \hat{x}_i$  and true value is very small and  $H_i \delta \hat{x}_i (i=1 \sim n)$  becomes the precise ICP correction value between epochs since the subsequent position movement between epochs  $\delta \hat{x}_i$  is continuously estimated from ICP variation between epochs. If accumulated movement of the vehicle is calculated using (8) and then compensated in the carrier measurement, (9) may be rewritten as (10).

$$(10) \quad \begin{bmatrix} \delta l_1 \\ \delta l_2 \\ \vdots \\ \delta l_n \end{bmatrix} - \begin{bmatrix} H_1 \delta \hat{x}_1 \\ H_1 \delta \hat{x}_1 + H_2 \delta \hat{x}_2 \\ \vdots \\ H_1 \delta \hat{x}_1 + H_2 \delta \hat{x}_2 + L + H_n \delta \hat{x}_n \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \\ M \\ M \\ \vdots \\ M \end{bmatrix} \delta \hat{x}_0 + \lambda \begin{bmatrix} N \\ N \\ \vdots \\ N \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

If the left side of (10) is defined as  $\delta l^k$ , (10) is expressed in the same form as the carrier phase measurement of (1), and by applying the process in (2)

and (3) to the following steps, it is possible to estimate the integer ambiguity of the moving vehicle. The integer ambiguity of the moving vehicle can be resolved by applying LAMBDA method using the estimated floating ambiguity and covariance of the integer ambiguity error, which is the search range of the integer ambiguity.

## 4 Experiment Results

The performance of proposed OTM-RTK technique was analyzed using filed driving experiment. The driving test was conducted at a vacant place next to Mokwon University located in Daejeon in March 2018, and compared the results of the navigation solution of proposed OTM-RTK technique and the navigation solution of Virtual Reference System (VRS)/IMU integrated equipment (Novatel SPAN-CPT) for the reference trajectory. In this experiment, a wide lane measurement was used for rapid integer ambiguity determination. The experimental trajectory is shown in Fig. 2.

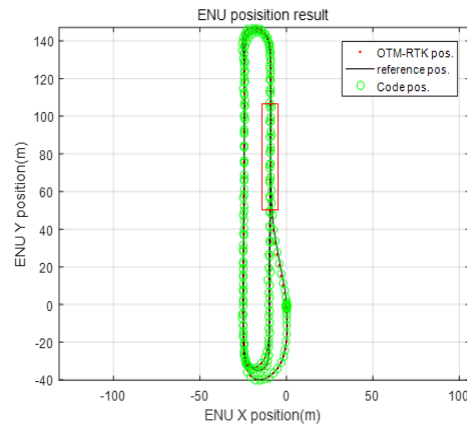


Fig. 2. Result of field driving experiment.

In order to compare the accuracy of the positioning results of proposed OTM-RTK technique, positioning error with the reference position is shown in Fig. 3, and RMS of the positioning error with respect to the reference position is shown in Table 1 in order to compare the accuracy.

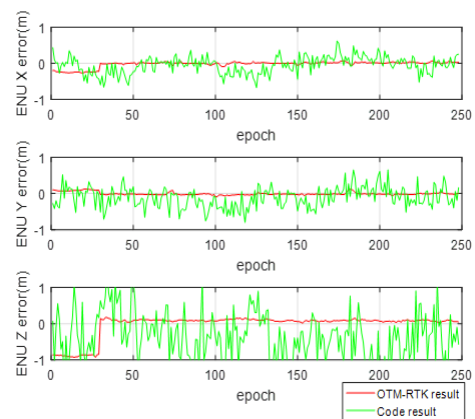


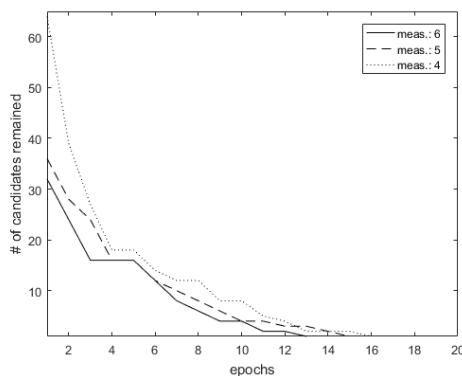
Fig. 3. Position error with reference position.

**TABLE 1.** RMS result of positioning error with reference position (unit: m).

[m]	Code result	OTM-RTK result
x	0.2426	0.0271
y	0.3062	0.0308
z	0.7805	0.0926

As a result of the experiment, although OTM-RTK contains an initial position bias before integer ambiguity determination, continuity of the trajectory was ensured by calculating the precise position movement between epochs using ICP variation between epochs, and accurate positioning was possible as the position bias was corrected after determining the integer ambiguity. In OTM-RTK results of Table 1, RMS results were confirmed to be about 12 times better than RMS results of the code measurement positioning.

Finally, in order to analyze the integer ambiguity resolution performance of proposed OTM-RTK technique, the number of residual integer ambiguities per epoch was compared according to the number of measurements as in the stationary state experiment. Although six measurements were used in the current experiment, elevation angle was adjusted to compare with cases of five and four measurements. The number of residual integer ambiguities per epoch according to the number of measurements is as shown in Fig. 4.



**Fig. 4.** Number of residual integer ambiguities per epoch by number of measurements

The DOP is large if the number of measurements is small, which increases the number of initial ambiguities to be searched as the initial ambiguity search range increases. In the case of four measurements, there were about twice as many initial integer ambiguity candidates than that of six measurements, and it took about 1.2 times longer to determine the integer ambiguity

## 5 Conclusion

This paper proposed an OTM-RTK technique to determine the integer ambiguity in a moving vehicle. The initial code-based measurements were used to resolve floating ambiguity, and the precise movement between epochs was calculated using the carrier phase measurements to correct the measurements, showing that the integer ambiguity can be determined in a moving

vehicle. As a result of the experiment, although proposed OTM-RTK technique contains an initial position bias before integer ambiguity determination, the precise trajectory can be obtained by calculating the precise movement between epochs, and accurate position and trajectory with corrected position biases can be obtained after determining the integer ambiguity. In addition, this study compared the integer ambiguity resolution performance according to the number of measurements. As the number of measurements is influenced by DOP, there was a performance difference in the number of integer ambiguity candidates and integer ambiguity resolution time. Therefore, the proposed OTM-RTK technique can be applied to various vehicle navigation systems such as unmanned vehicles and autonomous vehicles by complementing the RTK constraints.

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