A Short-baseline Dual-antenna BDS/MIMU Integrated Navigation System

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Abstract. This paper puts forward a short-baseline dual-antenna BDS/MIMU integrated navigation, constructs the carrier phase double difference model of BDS (BeiDou Navigation Satellite System), and presents a 2-position initial orientation method on BDS. The Extended Kalman-filter has been applied for the integrated navigation system. The differences between MIMU and BDS position, velocity and carrier phase information are used as measurements. The experiment results indicate that the position error is less than 1m, the pitch angle error and roll angle error are less than 0.1 °, and the heading angle error is about 1 °. It shows that the new integrated navigation system has good performance and can be applied in various fields including USV and UAV.

1 Introduction

Integrated navigation is a kind of navigation technique, integrating two or more navigation systems and using a complementary algorithm which obviously improves the system's navigation precision. This paper develops an integrated system based on the BeiDou System, China's domestically designed navigation system, which provides accurate positioning information within transmission range. However, it can't continuously navigate when the satellites are shielded as is the case with GPS [1]. Inertial navigation system based on MIMU is a highly independent and invisible navigation technology [2]. Although MIMU has the advantage of being small, light-weight and low-cost, its precision is low; there is a significant error divergence limiting the working hours of positioning. Therefore, it's necessary to integrate BDS and MIMU, which helps achieve better navigation effects.

Concerning single-antenna GNSS/MIMU integrated system, the significant noises of MIMU make it difficult to define heading angle in initial alignment [3], so some assistant sensors are needed. In conventional solutions, magnetic compass is usually mentioned, but it brings along another problem in that the compass is not stable since it's easily interfered by external magnetic fields. With the development of satellite orientation determination technology, positioning and attitude measuring using multiple antennas is becoming more advanced [4-5], so it's beneficial to implement a rotary dual-antenna BDS/MIMU integrated navigation system. Scholars have made some advancements in the field, for example, Russian researchers integrate rotary GNSS dual antennas and MIMU. The system uses an alternative heading determination algorithm and results show that the orientation error is less than 1 ° [6-7]. Researchers from the University of New Orleans put forward an IMU/GPS compass, with Kalman-filter fusing data and the compass outputs roll angle whose calculation accuracy is 2.12 ° [8].

Usually the dual-antenna GNSS integrated system is based on the double difference carrier phase model, which eliminates or diminishes satellite ephemeris error, clock error, ionosphere error, troposphere error to achieve millimeter-scale precision. Compared to the integration based on pseudo range model, the double difference carrier phase model is much more accurate [9-10]. The key problem of GNSS orientation is determining an ambiguity solution. The main algorithms include ambiguity mapping function, least ambiguity de-correlation adjust [11]. Researchers using these methods can achieve high accuracy, but usually the length of baseline is over 1m [12-13]. Currently the research focus is on a method to shorten the baseline length and simultaneously maintain accuracy. The longer baseline length can surely improve the precision, but also makes the system larger and more expensive.

In this paper, we design a novel method to determine the integer ambiguity double difference based on a rotary short-baseline. By this method, the dual-antenna BDS/MIMU integrated navigation system with 0.3m baseline is able to provide precise position and heading angle continuously, which has practical value in short-baseline dual-antenna integrated navigation technology.

2 Description of Hardware

The system is composed of a rotary module, a navigation computer, a Micro Inertial Measurement Unit (MIMU), dual-antenna BDS receiver and electronic circuit, as described in Fig. 1. The OEM BDS board provides the system with position, velocity, carrier phase and satellite ephemeris at a frequency of 5HZ. MIMU,

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model STIM300 adopted specifically, includes a 3-axis gyroscope, a 3-axis accelerometer and a 3-axis inclinometer, providing data at a frequency of 125HZ. The rotary module drives the OEM BDS board and MIMU rotating. The navigation computer consists of a DSP and a FPGA, as Fig. 2 shows. The FPGA connects peripherals and DSP, and sends navigation results to PC; the DSP runs navigation algorithm. The two units are connected through EMIFA interface.
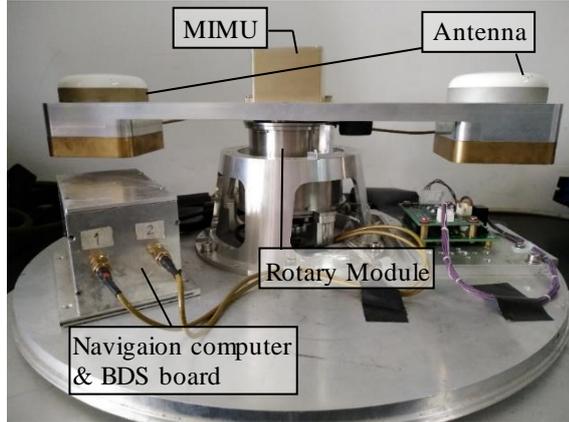

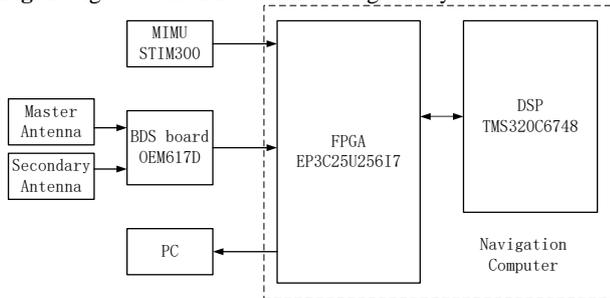**Fig. 1.** Figure of the BDS/MIMU integrated system.

Fig. 2. Graphical description of the hardware structure.

3 Mathematical Model

3.1. Double difference carrier phase model

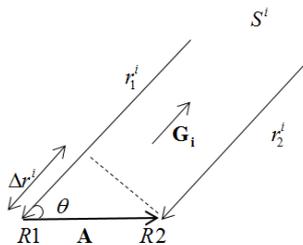The geometric relation of BDS satellite S^i and dual antennas is shown in Figure 3.

Fig. 3. Geometric relation graph of BDS satellite and dual antennas.

As Fig. 3 shows, S^i represents BDS satellite i, a baseline vector A is composed of two antennas R1 and R2, r_1^i and r_2^i represent the actual distance between satellite and antennas, Δr^i is the range difference of the two antennas and satellite, G_i is regarded as a unit direction vector since the baseline is far shorter than the distance between satellites and antennas. Δr^i can be written as

$$\Delta r^i = G_i^T A = |A|\cos\theta, \qquad (1)$$

where G_i^T is the transpose of the unit direction vector, θ is the angle between baseline vector A and G_i. The observation equation of BDS carrier phase is

$$\lambda\varphi_k^i = r_k^i + c[\delta t_k(t) - \delta T^i] + \lambda N_k^i + \Delta_{trop}^i + \Delta_{ion}^i + bias_k^i, \qquad (2)$$

where φ_k^i is the carrier phase measurement of antenna k, λ is wavelength of BDS B1 carrier, r_k^i is the actual distance between satellite S^i and antenna k, c is speed of light, $\delta t_k(t)$ is the clock bias of receiver, δT^i is the clock bias of satellite S^i, N_k^i is integer ambiguity, Δ_{trop}^i is tropospheric delay between satellite S^i and receiver, Δ_{ion}^i is ionosphere delay between S^i and receiver, $bias_k^i$ is bias of carrier phase measurement, such as multipath error and noise.

In a short-baseline dual-antenna system, the influence caused by tropospheric delay and ionosphere delay can be regarded as the same. So the difference between two antennas' carrier phase measurement is

$$\lambda\Delta\varphi^i = -\Delta r^i + c\Delta t + \lambda\Delta N^i + \Delta bias^i, \qquad (3)$$

where $\Delta\varphi^i = \varphi_2^i - \varphi_1^i$ is the measurement difference, $\Delta r^i = r_1^i - r_2^i$ is the range difference between S^i and two antennas, $\Delta t = \delta t_2(t) - \delta t_1(t)$ is the clock error between two receivers, $\Delta N^i = N_2^i - N_1^i$ is the single difference between carrier phase integer ambiguity of two antennas, $\Delta bias^i = bias_2^i - bias_1^i$ is the difference between measurement bias of two antennas.

From equation (1) and (3), the following equation can be written as

$$G_i^T A = -\lambda(\Delta\varphi^i - \Delta N^i) + c\Delta t + \Delta bias^i. \qquad (4)$$

Equation (4) shows that the single difference can eliminate the satellite clock error, but the BDS receiver clock error still exists. To eliminate the receiver clock error, the carrier phase's difference between different satellites is needed. Supposed R1 and R2 receives 4 or more satellites simultaneously, the satellite $S^j(i \neq j)$ is the referenced satellite, then the double difference carrier phase model can be written as

$$(G_i^T - G_j^T)A = -\lambda[\nabla\Delta\varphi^{ij} - (\nabla\Delta N^{ij} - \nabla\Delta bias^{ij}/\lambda)], \qquad (5)$$

where $\nabla\Delta\varphi^{ij} = \Delta\varphi^i - \Delta\varphi^j$ is carrier phase's double difference of two antennas, $\nabla\Delta N^{ij} = \Delta N^i - \Delta N^j$ is integer ambiguity double difference, $\nabla\Delta bias^{ij} = \Delta bias^i - \Delta bias^j$ is carrier phase measurement bias double difference.

3.2. Fast orientation method with short-baseline

Rotating the dual-antenna baseline by 180°, the double difference carrier phase of the two positions, 0° and 180°, can be calculated through Equation (5). Then the float ambiguity double difference can be calculated by adding the double difference at two positions,

$$\nabla\Delta F^{ij} = (\nabla\Delta\varphi_1^{ij} + \nabla\Delta\varphi_2^{ij})/2. \qquad (6)$$

where $\nabla\Delta F^{ij} = \nabla\Delta N^{ij} - \nabla\Delta bias^{ij}/\lambda$ is float ambiguity double difference, $\nabla\Delta\varphi_1^{ij}$ and $\nabla\Delta\varphi_2^{ij}$ are carrier phase's double difference at position 0° and 180°. Then we can

get the integer ambiguity double difference by rounding up Equation (6).

Suppose 4 satellites are observed, and satellite 1 is the base satellite, the Equation (5) can be written as

$$G^T A = -\lambda B, \qquad (7)$$

where $G^T = \begin{bmatrix} G_2^T - G_1^T \\ G_3^T - G_1^T \\ G_4^T - G_1^T \end{bmatrix}$, $B = \begin{bmatrix} \nabla\Delta\varphi^{21} - \nabla\Delta F^{21} \\ \nabla\Delta\varphi^{31} - \nabla\Delta F^{31} \\ \nabla\Delta\varphi^{41} - \nabla\Delta F^{41} \end{bmatrix}$. Baseline

vector A is calculated according to the least square method,

$$A = -\lambda(G^T G)^{-1} G^T B, \qquad (8)$$

where A is denoted as $A = (x, y, z)^T$. At last heading angle in rotating frame is calculated through the Equation (9),

$$\Psi = \arctan\left(\frac{x}{y}\right). \qquad (9)$$

3.3. Error model

There are 12 errors in the inertial navigation system, that is, 3 attitude errors (φ_E, φ_N, φ_U), 3 velocity errors (δV_E, δV_N, δV_U), 3 position errors (δL, $\delta\lambda$, $\delta\lambda$), 3 bias errors of gyroscope (δG_x, δG_y, δG_z) and 3 bias errors of acceleration (δA_x, δA_x, δA_z).

Mathematical platform error angle equation is

$$\begin{cases} \dot{\varphi}_E = \omega_N\varphi_N - \omega_E\varphi_U + \frac{\tan L}{R}\delta V_E + \left(\omega_{ie}\cos L + \frac{V_E}{R\cos^2 L}\right)\delta L + \varepsilon_1 \\ \dot{\varphi}_N = -\omega_N\varphi_E + \omega_U\varphi_U - \frac{\delta V_N}{R} + \varepsilon_2 \\ \dot{\varphi}_U = -\omega_E\varphi_E - \omega_U\varphi_N + \frac{\delta V_E}{R} - (\omega_{ie}\sin L)\delta L + \varepsilon_3 \end{cases} \qquad (10)$$

In the equation, subscript E, N, U represents eastward, northward, upward direction in geographic coordinate system respectively. The parameters ω_E, ω_N, ω_U are system angular velocity, and R is the radius of earth. The transformation matrix C_o^n can be written as

$$C_o^n = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}. \qquad (11)$$

$$\begin{cases} \varepsilon_1 = T_{31}\delta G_x + T_{32}\delta G_y + T_{33}\delta G_z \\ \varepsilon_2 = T_{11}\delta G_x + T_{12}\delta G_y + T_{13}\delta G_z \\ \varepsilon_3 = T_{21}\delta G_x + T_{22}\delta G_y + T_{23}\delta G_z \end{cases} \qquad (12)$$

Velocity error equation is

$$\begin{cases} \delta\dot{V}_E = f_N\varphi_E - f_U\varphi_U + T_{11}\delta A_x + T_{12}\delta A_y + T_{13}\delta A_z \\ \delta\dot{V}_N = -f_E\varphi_E + f_U\varphi_U + T_{21}\delta A_x + T_{22}\delta A_y + T_{23}\delta A_z \\ \delta\dot{V}_U = -f_N\varphi_N + f_E\varphi_U + T_{31}\delta A_x + T_{32}\delta A_y + T_{33}\delta A_z \end{cases}, \qquad (13)$$

where f_E, f_N, f_U are measured acceleration.

Position error equation is

$$\begin{cases} \delta\dot{L} = \frac{\delta V_N}{R} \\ \delta\dot{\lambda} = \frac{\delta V_E}{R\cos L} + \frac{V_E\sin L}{R\cos^2 L}\delta L \\ \delta\dot{h} = 0 \end{cases}. \qquad (14)$$

There are 3 groups of observation in the filter, that is, velocity observation, position observation, satellite double difference carrier phase observation.

Velocity measurement equation is

$$Z_V(t) = \begin{bmatrix} V_{IE} - V_{BE} \\ V_{IN} - V_{BN} \\ V_{IU} - V_{BU} \end{bmatrix} = \begin{bmatrix} \delta V_E + M_E \\ \delta V_N + M_N \\ \delta V_U + M_U \end{bmatrix}, \qquad (15)$$

where V_{IE}, V_{IN}, V_{IU} are velocity output from INS; V_{BE}, V_{BN}, V_{BU} are velocity output from BDS board; M_E, M_N, M_U are error of velocity measurement in BDS.

Position measurement equation is

$$Z_P(t) = \begin{bmatrix} L_I - L_B \\ \lambda_I - \lambda_B \\ h_I - h_B \end{bmatrix} = \begin{bmatrix} \delta L + N_E \\ \delta\lambda + N_N \\ \delta h + N_U \end{bmatrix}, \qquad (16)$$

where L_I, λ_I, h_I are latitude, longitude, height output from INS respectively; L_B, λ_B, h_B are latitude, longitude, height output from BDS board; N_E, N_N, N_U are position measurement error of BDS board in eastward, northward, upward direction.

Satellite double difference carrier phase measurement equation is

$$Z_S(t) = S^o - S - \delta Cf, \qquad (17)$$

where S^o is projection of carrier phase's double difference to baseline, S is carrier phase's double difference, δCf is error of carrier phase's double difference.

The dual-antenna BDS/MIMU integrated system's state equation and measurement equation are expressed as

$$\dot{X}(t) = F(t)X(t) + G(t)W(t), \qquad (18)$$
$$Z(t) = H(t)X(t) + R(t). \qquad (19)$$

$X(t)$ represents the system state model, $W(t)$ represents the system noise, $F(t)$ represents the state transition matrix, and $G(t)$ represents noise excitation matrix. $Z(t)$ represents the system measurement model, $H(t)$ relates the state to the measurement, and $R(t)$ represents measurement noise.

The 16-dimension state vector X can be written as

$$X = \begin{bmatrix} \varphi_E, \varphi_N, \varphi_U, \delta V_E, \delta V_N, \delta V_U, \delta L, \delta\lambda, \delta h, \\ \delta G_x, \delta G_y, \delta G_z, \delta A_x, \delta A_y, \delta A_z, \delta DCf \end{bmatrix}^T. \qquad (20)$$

In addition, the noise vector W can be written as

$$W = \begin{bmatrix} \omega_{gx} & \omega_{gy} & \omega_{gz} & \omega_{ax} & \omega_{ay} & \omega_{az} \end{bmatrix}^T, \qquad (21)$$

where ω_{gx}, ω_{gy}, ω_{gz} are white noise of gyroscope, and ω_{ax}, ω_{ay}, ω_{az} are white noise of accelerometer.

4 Experiment

To verify the result of the algorithm, an experiment has been undertaken. In the experiment, the navigation system was bound to a handcart, as was a Strapdown Inertial Navigation System (SINS) based on laser gyroscope with high accuracy. The two systems were fixed on an aluminium alloy sheet with screws, ensuring the heading directions were the same. The SINS is regarded as a reference. After the initial alignment of the SINS, the handcart was pushed along a specified route.

Fig. 4 shows the trace plot of the system, where the blue solid line is the trace output from the system and red dotted line is the real trace. The plot shows that the position error of the system is less than 1m.
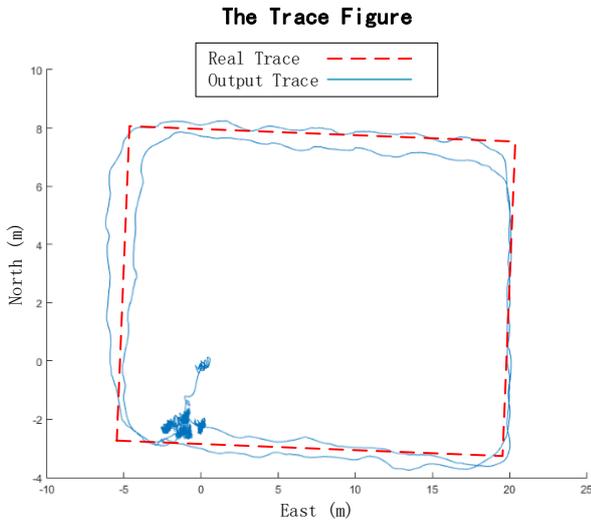
Fig. 4. Figure of experiment trace.

The attitude output from the SINS over a short period of time is very accurate and stable. Therefore, the attitude measurement error of the system is the attitude difference between the SINS and the system. As Fig. 5 shows, the blue line is the pitch angle error, roll angle error and heading angle error of the system. Figure 5 and Table 1 indicate that the system's pitch angle error and roll angle error are less than 0.1°, and heading angle error is about 1°.
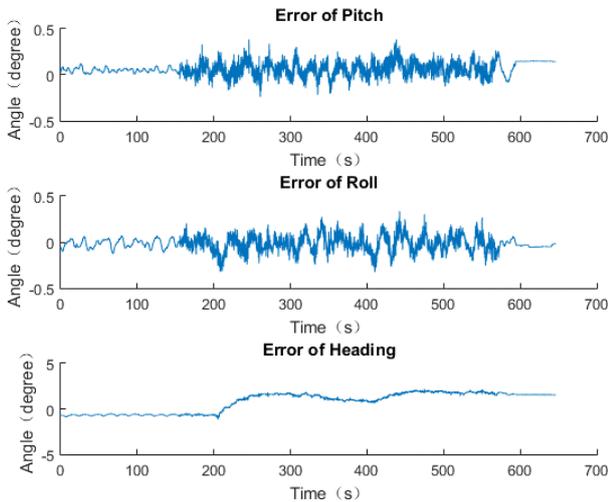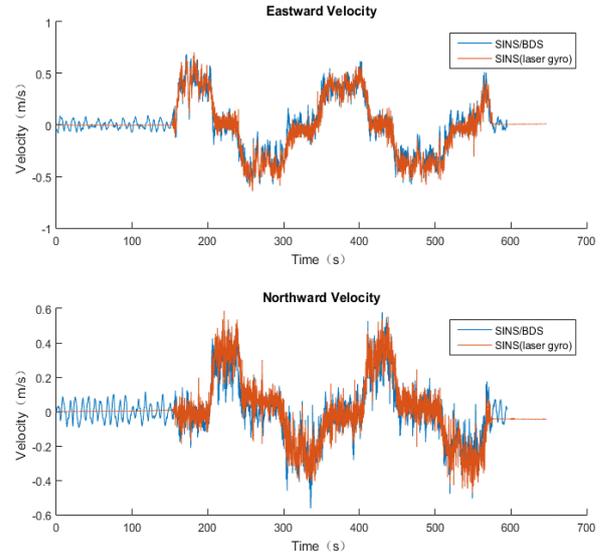

Fig. 5. Figure of attitude error.

Table 1. 1-sigma error of attitude.

	Pitch (°)	Roll (°)	Heading (°)
1σ	0.0639	0.0748	1.0554

In Fig. 6, the blue line is eastward and northward velocity which is calculated by the integrated system. The red line is eastward and northward velocity, which is calculated by the SINS. The figure shows that the two lines are almost overlapping.

Fig. 6. Figure of velocity output from the integrated system and SINS.

Regarding the output from the SINS as a reference, the system's velocity error can be calculated. Fig. 7 and Table 2 indicate that the integrated system's velocity error in eastward and northward directions are both less than 0.07m/s.
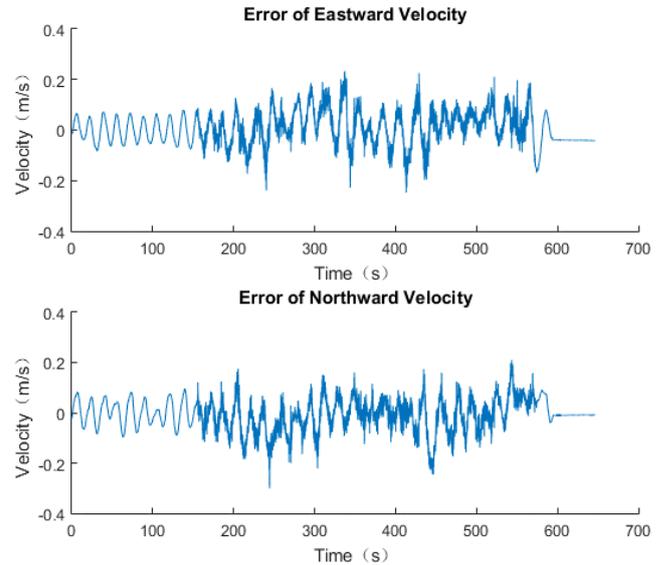
Fig. 7. Figure of error velocity.

Table 2. 1-sigma error of velocity.

	East (m/s)	North (m/s)
1σ	0.0651	0.0620

5 Conclusions

The short-baseline dual-antenna BDS/MIMU integrated navigation integrates the BeiDou Navigation Satellite System and SINS based on MIMU, with extended Kalman-filter fusing data, providing optimum parameters. The Kalman-filter observation vector consists of velocity, position and satellite carrier phase double difference. The experiment conducted on an engineering prototype shows

that the position error is less than 1m, the pitch angle error and roll angle error are less than 0.1°, and the heading angle error is about 1°. The integrated system has an advantage of being compact, low-power, low-cost, and it can be applied in various fields including USV and UAV.

Acknowledgement

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