

Statistical modeling of the assembly of single-layer domes

*Evgeny Lebed**

Moscow State University of Civil Engineering, Yaroslavskoe shosse, 26, Moscow, 129337, Russia

Abstract. A special purpose software SBORKA was developed by the author, which is intended for numerical modeling of the assembly process of single-layer lattice domes with respect to random deviations in lengths of individual bars. The dispersion of the inaccuracies in the sizes of bars is consistent with the normal distribution law, and their values are limited by the tolerances corresponding to 3σ . The assembly of a single-layer dome frame is simulated by sequentially calculating the coordinates of its nodes based on the imitation of the connection in these nodes of individual bars. Imperfections in the lengths of the bars lead to the fact that the actual geometric shape of the dome frame will differ from the design shape. Reliable analysis of the nature of the possible errors of the frames of single-layer lattice domes can only be performed by multiple assembly simulations with subsequent statistical processing of the obtained results. For the implementation of numerical statistical simulation of the assembly, the algorithm of the software includes procedures for solving various spatial problems of computational geometry. Computational procedures implemented in the software for modeling the assembly process are described in this paper.

1. Introduction

Single-layer lattice domes consist of a large number of individual elements – straight bars, forming a spatial frame. In the assembly process these bars are consecutively connected to each other through structural joints. The actual dimensions of the bars differ from the nominal values and vary from one bar to another due to random inaccuracies in their manufacturing. Imperfections in the dimensions of bars are regulated by a special system of tolerances [1], but, although the tolerances are satisfied, the accumulation of imperfections during the assembly leads to errors in the geometric scheme of the bar structure. The errors that arise during the assembly of spatial bar structures make it difficult to connect the elements between each other, and reduce the load-bearing capacity of structures due to the appearance of additional internal forces. In addition, the design geometric shape of the structure as a whole is distorted. This leads to various serious implications: change in the analytical model, unpredictable redistribution of forces, initial stresses and strains, and defects in enclosing structures. Therefore, studies of possible errors in spatial bar structures are essential to improve their reliability.

* Corresponding author: evglebed@mail.ru

The author of this paper has developed special-purpose computer software named SBORKA that allows numerical modeling of the assembly process of single-layer lattice domes with respect to the imperfections in lengths of individual bars. The algorithm is based on the Monte-Carlo method [2, 3]. Simulation is carried out repeatedly with subsequent statistical analysis [4, 5] of possible deviations of the nodes from their design positions. The algorithm for modeling of the assembly process is based on special computational procedures for 3-D problems of computational geometry. The software also allows simulation of spatial displacement of an assembled frame and its installation on the prescribed surface.

The actual bar size L_i^* will differ from the nominal size L_i by the amount of random deviation $\delta L(\sigma)_i$ [6]:

$$L_i^* = L_i + \delta L(\sigma)_i . \quad (1)$$

The value of the random deviation $\delta L(\sigma)_i$ is set within the limits of the tolerance ΔL_i in terms of the pseudo-random normally distributed number ζ ($\mu = 0$ and $\sigma = 1$) [7, 8]:

$$\delta L(\sigma)_i = \frac{\zeta}{3} \frac{\Delta L_i}{2} . \quad (2)$$

The accuracy of the distance between adjacent nodes depends on several parameters: accuracy of the dimensions of bar elements, accuracy of the distances from holes to the ends of the bars, accuracy of the distances between the holes in the connecting elements, gaps between the bolts and the holes, etc. That is, there is a linear dimensional chain, and the tolerance of the i -th distance ΔL_i can be calculated as the total (resulting) tolerance using the quadratic addition formula [1].

Studies of possible errors are performed on a computer by simulating the assembly of spatial bar structures using the Monte Carlo method [2, 3]. This method requires the introduction of random variability in the length of the bars in the process of simulating of installation or assembly of bars into a single spatial system [9, 10]. The Monte Carlo method involves statistical analysis of the results obtained, and therefore it is called the statistical computer simulation method.

2. Methodology

The work flow diagram of the SBORKA program consists of three main parts (Fig. 1):

- Input and processing of initial data for the start of statistical modeling;
- Numerical sequential construction of the actual geometric shape of the frame of a single-layer lattice dome of bars with individual random deviations of the lengths of bars from the nominal value;
- Computing and statistical processing of the errors of the actual geometric shape of the dome in comparison with the design shape.

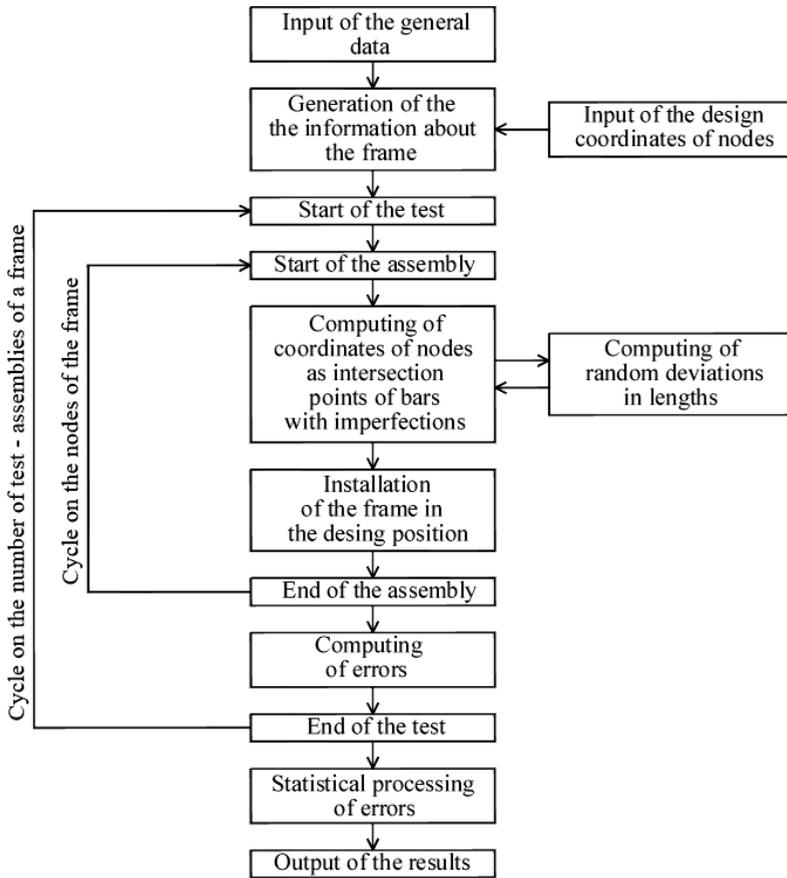


Fig. 1. General work flow diagram of the software SBORKA

The numerical simulation of the actual geometrical shape of the frame of the single-layer dome utilized by the algorithm of the SBORKA software is carried out by sequential calculation of the coordinates of the desired nodes as the intersection points of the actual distances from the previously calculated nodes to the desired node [11, 12].

Consider the features of this algorithm using the example of a small spherical bar system with a radius of curvature R shown in Fig. 2. Calculation of the coordinates of the nodes is reduced to solving a set of problems of computational geometry related to finding the intersection point of various geometric objects.

First, the pivot node is selected, from which the numerical simulation of the construction of the dome frame will begin. In this example the apex node 1 (Figure 2) has been selected. Next, the installation of the first bar is simulated, for example, from node 1 to node 2. Since the actual length of the bar L^* differs from the nominal one, we place this bar in the direction of a straight line between the nodes 1 and 2, aligning the start node 1 of the bar with the previously selected node 1. The required node 2 is the end node of the bar.

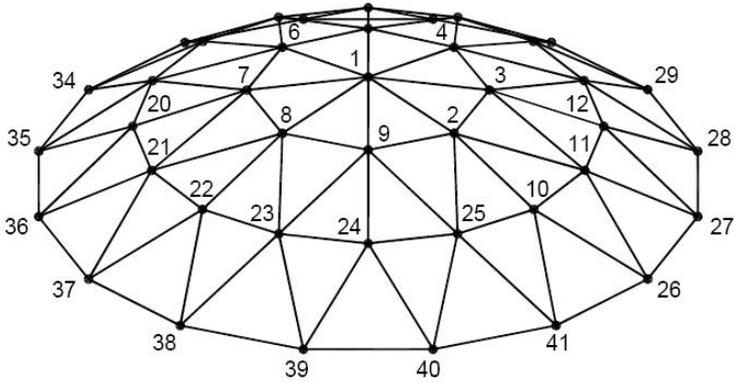


Fig. 2. Fragment of a single-layer dome

This node will be located in the point of intersection of the straight line 1 – 2 and the sphere with the radius L^* and the center at point 1 (Figure 3).

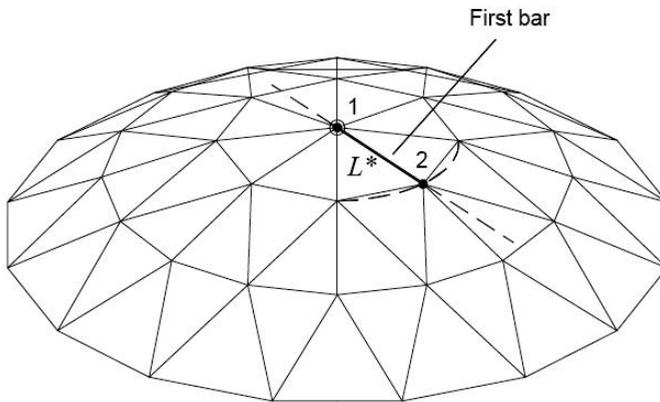


Fig. 3. Location of the first node after the apex

Next, the assembly of the radial bars is simulated, starting from the apex node 1, simultaneously with the circumferential bars from node 2 to node 9. To fix the locations of the nodes in space we need two reference points, two distances L^* , and the plane of the upper ring. The actual dimensions of these radii are computed with the equation (1), and the value of random deviation is taken within the tolerance and is computed with the equation (2). Therefore, each of the sought nodes from 2 to 9 will be located in the points of intersection of the plane of the upper ring and the two spheres with different radii L^* with the centers at node 1 and at nodes from 2 to 8 consecutively (Figure 4).

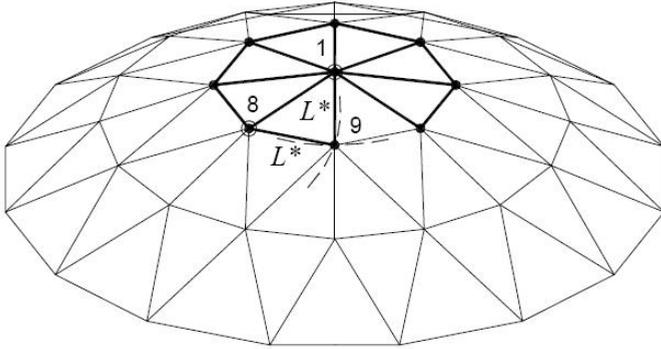


Fig. 4. Location of the last node of the upper tier

Simulation of the assembly of descending bars of the second tier is performed by sequential installation of the bars with random lengths L^* in the neighboring nodes of the upper ring, for example 9 and 2, and connecting them at node 25 lying on the horizontal plane of the second ring from the apex (Figure 5). Therefore, the node 25 will be located at the point of intersection of two spheres with different radii L^* and the centers at nodes 9 and 2, and the plane defined by the design locations of points 13, 18 and 23 of the second ring from the apex (Figure 5). This procedure agrees well with the assembly process from the top down to the bottom.

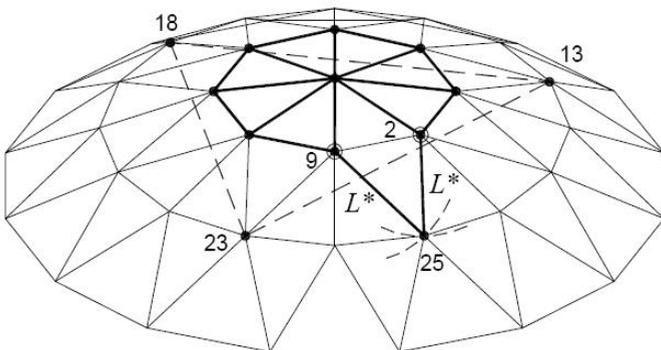


Fig. 5. Location of the first node of the middle tier

After determining the position of the node of the middle ring by attaching the two bars to the first ring, the different procedure is used to model the installation of the bars. One of the bars is connected to the node of the upper ring, and the second – to the node of the middle ring. For example, the sought node 22 will be located at the intersection of the two spheres with different radii L^* and centers at nodes 8, 21, and the plane defined by the design locations of the nodes of the middle ring including nodes 13, 18, and 23 (Figure 6).

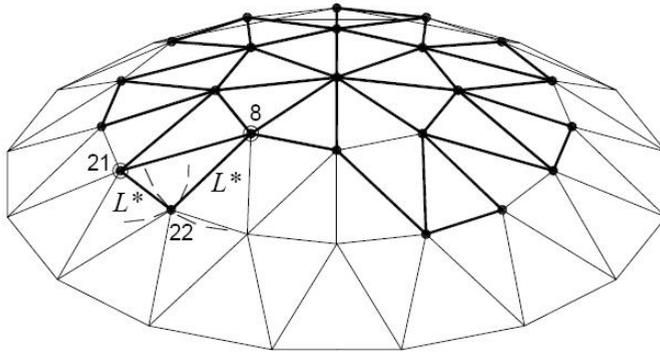


Fig. 6. Completing the assembly of the middle tier

Further process of the assembly simulation brings down to the repetitive procedures, described above. The main one is the procedure of intersection of two spheres with radii L^* and centers at the known nodes, and the plane containing the sought nodes of the dome frame.

The simulation process is demonstrated in Figure 6. It can be observed that some of the bars with lengths L^* remain unused in the process of assembly simulation.

3. Example

As an example, consider numerical simulation of the assembly of the frame of a single-layer dome, shown in Figure 2. The frame is inscribed in a sphere of radius $R = 29$ m and consists of straight bars with a length varying from 2.04 m to 3.09 m [16]. The values of the permissible deviations of the distances between the nodes in the directions of all bars are assumed to be $\Delta L_i / 2 = 2$ mm. A total number of 500 computer simulations of the dome frame were performed. The purpose of the study was to quantify possible deviations in the distances between those nodes where the additional bars should be installed. In the considered fragment of a single-layer dome, the lower tier can be assembled with different sequence of installation of individual bars. Let the bars that are used in the simulation be named assembly bars, and the bars that are not used named bars-inserts. Consider the two different sequences of installation of the bars yielding the two schemes of the assembly of dome framework (Figures 7 and 8).

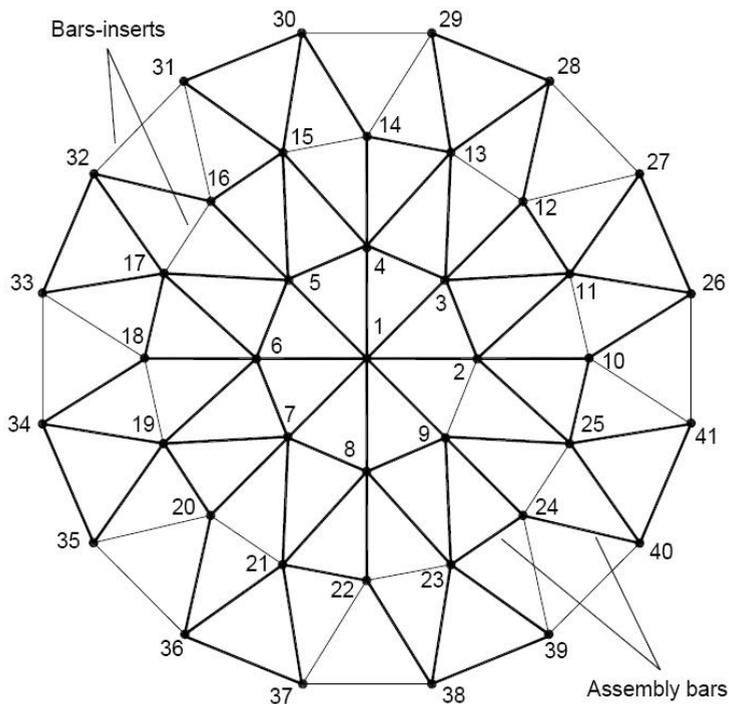


Fig. 7. 1st scheme of the assembly of dome frame

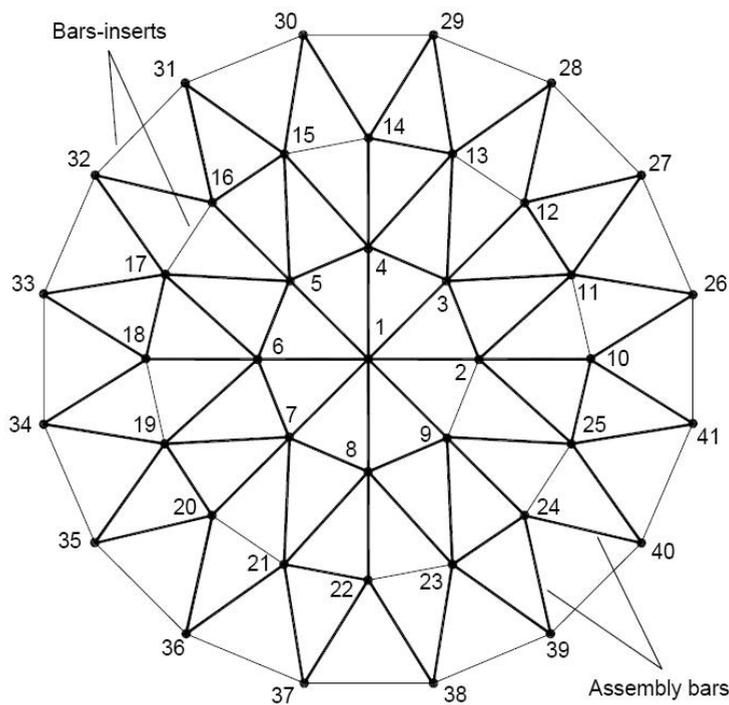


Fig.8. 2nd scheme of the assembly of dome frame.

The results of statistical processing of possible deviations of the distances between the nodes are given in the Table 1 for comparative analysis. Coordinates of the nodes, computed according to the described procedure, will not fully reflect the actual assembly process of the dome frame. The reason is that the bars-inserts, missing in the model, in reality will be installed as the tier is assembled. However, the described procedures are believed to yield the simulated geometric shape of the dome close to the real one.

Table 1. Statistical data on the deviations of the distances between the nodes of the frame of a single-layer lattice dome

№	Start node	End node	$\sigma(\delta_j)$ mm	$\mu(\delta_j)$ mm	$\sigma(\delta_j)$ mm	$\mu(\delta_j)$ mm
			1 st assembly scheme		2 nd assembly scheme	
1	2	9	2,30	-0,03	2,30	-0,03
2	10	11	3,51	-0,04	3,51	-0,04
3	14	15	3,08	0,12	3,08	0,12
4	18	19	2,73	0,10	2,73	0,10
5	22	23	2,77	0,07	2,77	0,07
6	10	41	2,48	-0,04	0,69	0,06
7	14	29	2,40	0,11	0,63	-0,04
8	18	33	2,28	-0,06	0,65	-0,02
9	22	37	2,41	0,01	0,68	-0,05
10	26	41	3,76	-0,10	4,53	0,06
11	29	30	3,83	0,30	3,80	0,17
12	33	34	3,54	0,07	3,47	0,14
13	37	38	3,49	0,01	3,84	-0,08

It can be seen from the table that the mathematical expectations of possible deviations of distances are close to zero. It should be noted that the maximum deviations of the distances (equal to 3σ) between the nodes of the frame where the insert bars are installed, are several times greater than the tolerances for the dimensions of these bars, which are equal to 2 mm.

The software SBORKA allows for the different approaches to the analysis of distortions of frames of single-layer domes in addition to the analysis of deviations of distances between the nodes. For example, the deviations of the nodes from the design spherical shape, or the distances between the support nodes and horizontal plane at the time of installation of the fully assembled dome on the design contour can be studied.

Table 2 contains the data for the deviations of the nodes of the considered fragment of a single-layer dome from their design positions in the radial direction (along the normal to the surface).

Table 2. Statistical data for the deviations of the nodes of a single-layer dome
 in the radial (normal) direction

№	Node number	$\sigma(\delta_j)$ mm	$\mu(\delta_j)$ mm	$\sigma(\delta_j)$ mm	$\mu(\delta_j)$ mm
		1 st assembly scheme		2 nd assembly scheme	
1	4	0,06	-0,00	0,06	-0,00
2	8	0,06	-0,00	0,06	-0,00
3	13	0,13	-0,00	0,13	-0,00
4	14	0,17	0,01	0,17	0,01
5	21	0,15	-0,00	0,15	-0,00
6	22	0,17	-0,00	0,17	-0,00
7	29	0,43	0,03	0,24	0,00
8	30	0,30	-0,01	0,30	-0,01
9	37	0,44	0,01	0,25	-0,00
10	38	0,29	-0,02	0,29	-0,02

Statistical analysis of deviations of the nodes of the dome frame from their design positions, provided for in the SBORKA software, allows determining possible distortions of its geometric shape during the assembly process of a real structure. The study of distortions of the actual geometric shape of the dome frame as a spatial bar system, or of the other errors is carried out in accordance with generally accepted methods of stochastic problems [13–15].

4. Conclusions

The analysis of the basic principles of numerical simulation of the assembly process of a frame of a single-layer lattice dome and the results of numerical studies of errors performed with the author's software SBORKA allow for the following conclusions.

- The SBORKA software allows a fairly accurate analysis of possible errors in the assembly of frames of single-layer domes
- The assembly sequence of the elements has a significant impact on the distribution and magnitude of the errors in the distances between the nodes
- Even strict observance of the distance from the nodes to the center of curvature of the dome cannot ensure the assemblability of a single-layer frame, since the assemblability assumes that the maximum deviations of the dimensions do not exceed the permissible deviations (tolerances).
- To ensure the assembly of the lattice frame, it is necessary to provide special technical solutions or technological means that will allow to compensate for the resulting errors in the frame of the single-layer dome during the assembly.

References

1. Kotlov A.F. Tolerances and technical measurements for the installation of metal and reinforced concrete structures. – M. Stroyizdat, 1988. – 304 p.
2. Sobol I.M. Numerical Methods of Monte-Carlo. – M.: Nauka, 1973. – 312 p.
3. Yermakov S.M. The Monte Carlo method and related questions. – M: Nauka, 1971. – 328 p.
4. Smirnov N.V., Dunin-Barkovskiy I.V. Course of Theory of Probability and Mathematical Statistics for technical applications. – M: Nauka, 1965. – 512 p.
5. Pugachev V.S. Theory of Probability and Mathematical Statistics. – M: Nauka, 1979. – 496 p.
6. Saveliev V.A., Lebed E.V., Shebalina O.V. Mathematical modeling of the assembly of spatial structures // Industrial Construction. 1991. No 1. – P. 18-20.
7. Lebed E.V. The accuracy of statistical computing of the standard deviation of a random variable // IOP Conference Series: Earth and Environmental Science. Vol. 90, 2017. – 012150. – 5 p. EMMFT 2017. IOP Publishing.
8. Lebed E.V. Pseudorandom number generation for computer modeling of actual shapes of spatial bar structures. IOP Conference Series: Materials Science and Engineering, Volume 365, Safety in Construction. 2018. – 042021 – 8 p.
9. Law Averill M. Simulation Modelling and Analysis / Averill M. Law, W. David Kelton. – London: McGraw-Hill, 1991. – 754 p.
10. Larsen Richard J. An Introduction to Mathematical Statistics and its Applications / Richard J. Larsen, Morris L. Marx. – Boston: Pearson Education, 2012 – 757 p.
11. Lebed E.V. On the numerical modeling of the assembly of the frame of single-layer lattice dome. // Journal of the Volgograd State University for Architecture and Civil Engineering. Series: Civil Eng. & Architecture. 2003. No. 29 (48). Pp. 81–86 (in Russian).
12. Lebed E.V. Computer modeling of the assembly of single-layer lattice domes with respect to imperfections in the lengths of the bars / Conference series: Mathematical methods in engineering and technology. Saratov State Technical University, Saratov.2008. Volume4, P. 199-202 (in Russian).
13. Starnes, Yates, Moore. The Practice of Statistics / Daren S. Starnes, Dan Yates, David S. Moore. – New York: W.H. Freeman and Company, 2012 – 898 p.
14. Chai T. and Draxler R.R. Root mean square error (RMSE) or mean absolute error (MAE)? – Arguments against avoiding RMSE in the literature. – USA: Geoscientific Model Development, 7. – 2014. Pp. 1247–1250.
15. Makkonen Lasse, Pajari Matti, Tikanmäki Maria. Closure to "Problems in the extreme value analysis" (Struct. Safety. 2008:30:405-419) / Structural Safety. Elsevier. Vol. 40 (2013) No: January, Pp. 65–67.
16. Ruzhansky I.L. Aluminum dome for the tank with a diameter of 40 m / Construction and Special Works in Civil Engineering. 2002. No. 7. P. 10–16.