An account of the foundation in assessment of earth structure dynamics

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Abstract. An assessment of the dynamic behavior of a plane earth structure with account of its foundation is considered in the paper. A structure with a foundation is considered as an inhomogeneous system, the material of its certain parts is considered elastic or viscoelastic. To assess the effect of the foundation on dynamic behavior of the structure, a finite domain is cut from the foundation and conditions are set at the boundary of this domain that provide energy entrainment from the structure to infinity in the form of the Rayleigh wave. To describe the internal dissipation in material, a linearly hereditary theory of viscoelasticity with the Rzhanitsin kernel is used. A mathematical model, method and algorithm have been developed to assess the dynamic behavior of the structure-finite foundation system. To ensure the adequacy of the mathematical model and to assess the accuracy of the calculation, model problems have been solved when describing the process under consideration. Dynamic behavior of inhomogeneous viscoelastic system of earth dam-foundation with non-reflecting boundary conditions on the boundary of the final domain of the foundation is investigated. In the process of studying the dynamic behavior of inhomogeneous viscoelastic "structure-foundation" systems, a number of mechanical effects.

1. Introduction

When assessing the dynamics of a structure, the effect of earth foundation on the dynamic behaviour of a structure is often not taken into account, despite the fact that the effect of the foundation may in some cases be significant. Usually, when taking into account the structure-foundation interaction, the Winkler foundation model is used, which, despite its simplicity in calculation, does not take into account a number of physical effects associated with the inertia properties of earth foundation. The elastic half-space model is devoid of this disadvantage; however, due to mathematical complexity, it does not allow to obtain an analytical solution in a closed form, with the exception of a number of particular static problems.

Recently, the final model of the foundation, cut out from the half-space, is widely used to account for the joint operation of a structure-foundation system. When using a finite

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foundation model, there appear some parasitic eigenfrequencies of the “structure-final foundation” system, which under forced oscillations can lead to parasitic resonance phenomena. To eliminate this, it is necessary to set artificial boundary conditions at the boundary of the final domain, excluding the occurrence of parasitic resonances. To ensure this, on the border of the final domain of the foundation, the conditions are set that ensure energy entrainment from the structure to infinity, i.e. wave entrainment of energy. Many existing models of the “structure-finite foundation” system do not allow one to adequately describe the dynamic process of energy entrainment to infinity.

To reliably assess the dynamic behavior of the “structure-foundation” system, it is necessary, along with taking into account the wave entrainment of energy, to account the inhomogeneous features of the structure and dissipative (viscoelastic) properties (an internal dissipation) both in structure material and at the foundation; this also complicates the solution of dynamic problems for structure-foundation system.

For an equivalent replacement of an infinite foundation by a finite one considering the above factors, it is necessary to use non-reflecting boundary conditions on fictitious (artificial) boundaries of the computational finite domain [1, 2].

There are numerous published papers [3–10], which propose the use of non-reflecting conditions at the boundary of the finite domain of the foundation, which provide energy entrainment from the structure to infinity [3–10]; a detailed review of other papers related to this problem is given in [11-13].

Here are listed just some of the papers devoted to the problem of studying the dynamic behavior of the “structure-foundation” systems with the use of artificial non-reflecting conditions at the boundary of the finite domain of the foundation that provide energy entrainment.

Fundamental studies [11, 12] are devoted to the problem of statement of exact boundary conditions on the artificial boundaries of the computational domain, mathematical justification, analysis and their effectiveness in solving specific problems; a lot of published papers are analyzed there which use artificial boundary conditions; the results are obtained.

Hence, it follows that the problem of estimating the dynamic behavior of inhomogeneous viscoelastic systems “structure-foundation-base”, with account of internal dissipation and wave entrainment of energy across the boundaries of the finite earth foundation, is far from the final solution and is an urgent problem that needs to be solved.

2. Method

A plane inhomogeneous system (structure + foundation + base) is considered here, consisting of a deformable body occupying a volume $V=V_1+V_2+V_3+V_4$ and a deformable half-space (figure 1). The material of a deformable inhomogeneous body and a half-space is, in general case, viscoelastic one, and physical properties of their components differ from each other.

At the interfaces of the elements of the system, the displacement and stress components, normal and tangent to the interface, are continuous. The structure under consideration is a massive structure; therefore, the mass forces $\vec{f}$ are taken into account as well as various force effects applied to an arbitrary surface Σp.

The task is to determine dynamic characteristics, displacements and stresses in an inhomogeneous system (figure 1) under dynamic effect.

The considered problems are set for a finite domain (figure 1) of a volume $V+V_5$ (V_5 is the volume cut out from the half-space) and bounded surfaces $\Sigma_1^- + \Sigma_1^+ + \Sigma_2^+$ on which non-reflecting conditions are set.

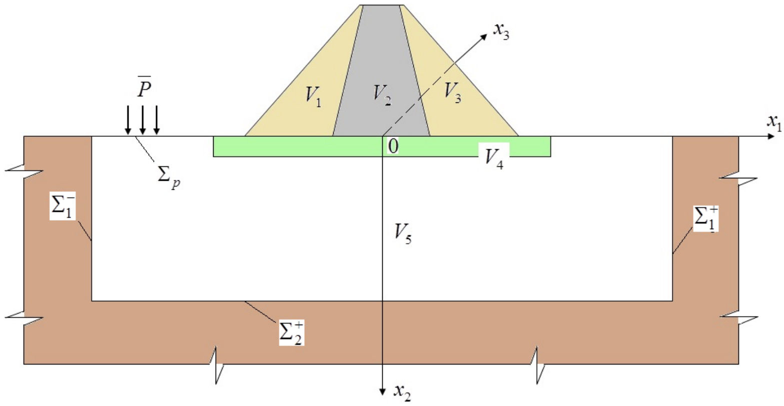

Fig. 1. Calculation model of inhomogeneous system.

To describe the dynamic processes occurring in the system (figure 1), the principle of possible displacements is used, according to which the sum of the work of all active forces, including inertial forces, on virtual displacements is zero:

$$\delta A = -\int\limits_{V+V_5} \sigma_{ij}\delta\varepsilon_{ij} dV - \int\limits_{V+V_5} \rho_n \ddot{\vec{u}}\delta\vec{u}\, dV + \int\limits_{\Sigma_1^- + \Sigma_1^+ + \Sigma_2^+} \sigma_{ij}\nu_j \delta\vec{u} d\Sigma + \\ + \int\limits_{V} \vec{f}\delta\vec{u}\, dV + \int\limits_{\Sigma_p} \vec{p}\delta\vec{u}\, d\Sigma = 0 \tag{1}$$

Further:

- to describe the internal dissipation in material of both structure and foundation, a linear hereditary theory of viscoelasticity is used which connects the components of the stress tensor with the strain tensor [14]

$$S_{ij} = \mu_m\left[e_{ij} - \int\limits_0^t \Gamma_m(t-\tau)e_{ij}(\tau)d\tau\right], \\ \sigma = K_m\theta, i = 1,2 \tag{2}$$

- with the Rzhanitsin's kernel [15]

$$\Gamma_m(t) = Ae^{-\beta t}t^{\alpha-1}\,(0<\alpha<1), \tag{3}$$

- Cauchy relations are used, connecting the components of the strain tensor ε_{ij} with the components of the displacements vector $\vec{u}$:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), i = 1,2 \tag{4}$$

- and non-reflecting conditions at the boundary of the finite domain of the foundation [1, 2, 11, 13] are

$$\vec{x} \in \Sigma_1^{\pm}: \quad \frac{\partial u_i}{\partial x_1} \pm \frac{1}{\bar{c}_R} \frac{\partial u_i}{\partial t} = 0, \qquad \vec{x} \in \Sigma_2^{+}: \quad u_i = 0, i = 1,2 \tag{5}$$

providing energy entrainment from the structure to infinity in the form of the Rayleigh wave across the boundaries of the finite domain V_5.

Here: $\vec{u}$, ε_{ij}, σ_{ij} - are the components of the displacement vector $\vec{u} = \{u_1, u_2\}$, of the strain and stress tensors, respectively; $\delta\vec{u}$, $\delta\varepsilon_{ij}$ are the isochronous variations of displacements and strains; ρ_m is the density of material of the m-th element of the system; $\vec{f}$ is the vector of mass forces; $\bar{p}$ is the vector of external loads; Γ_m is the relaxation kernel; ν_j are the guide cosines of the outer normal; $\bar{c}_R$ is the propagation velocity of the Rayleigh wave in the half-space (when viscoelastic properties of the foundation material are taken into account, these quantities are complex values); Km, μm are the instantaneous modulus of volume and shear strains; S_{ij}, e_{ij} are the components of the stress and strain deviator; σ is the spherical part of the strain tensor; θ is the volume strain; A, α, β are the kernel parameters determined from experiment [16, 17]; m=1,2,3,4,5 – are the numbers of the system elements; i,j=1,2.

Natural, steady-state and unsteady forced oscillations of an inhomogeneous system are considered (figure 1). All considered problems are solved by the finite element method (FEM) with the discretization of the domain $V+V_5$ on different types of finite elements. When solving specific tasks, the discretization of the domain $V+V_5$ (figure 1) on finite elements is carried out taking into account the design features and the physicomechanical properties of material of different parts of the system.

2.1 Natural oscillations

The problem of natural oscillations of the system (figure 1) using the FEM procedure is reduced to solving the eigenvalue problem of an algebraic equation with complex coefficients:

$$\left(\left[\bar{K}\right] - i\omega\left[\bar{C}\right] + \omega^2 [M]\right)\left\{\bar{X}\right\} = 0 \tag{6}$$

Here $[M]$ is the mass matrix; $\left[\bar{K}\right]$ is the stiffness matrix and $\left[\bar{C}\right]$ is the matrix, taking into account the wave entrainment of energy across the boundary of the finite domain. Matrix elements ($\bar{k}_{ij}, \bar{c}_{ij}$) are complex values. ω =ωR-iωI is the eigenfrequency and $\left\{\bar{X}\right\} = \{X_R\} - i\{X_I\}$ is the eigenvector.

In equation (6), complex matrices appear when taking into account the viscoelastic properties of the material and replacing the Voltaire integral operator (2) with complex relations [11, 14].

The system under consideration (figure 1) is non-conservative, even if only the elastic properties of material are taken into account using condition (5); therefore the natural frequencies and vibration modes (ω and $\left\{\bar{X}\right\}$) are the complex values. The real part ωR of the complex parameter ω (of natural frequency) is in its physical sense the frequency of free damping oscillations of the system, while the virtual partωIcarries information about the oscillation damping velocity and, up to a sign, is equal to the damping coefficient, which is a quantitative characteristic of the oscillation damping velocity and determines dissipative properties of the systems in general. This explains the legitimacy of the use of the wave dissipation term, i.e. energy entrainment across the boundary of the finite domain V_5.

To find the roots of complex algebraic equations (6), a special algorithm and a software computer program using the Muller method have been developed, and the Gauss method was used to determine the eigenvectors.

2.2 Steady-state forced vibrations

At long-term harmonic effect, the initial conditions do not affect the motion of the system. In this case, the dissipative properties of the system are manifested mainly in resonant modes. Resonance amplitudes of displacements and stresses are used as a quantitative estimate of the intensity of dissipative processes [18].

The problem of steady-state forced oscillations of a system (figure 1) using the FEM procedure is reduced to solving a system of inhomogeneous algebraic equations with complex coefficients, i.e.:

$$\left([\bar{K}]-i\Omega[\bar{C}]-\Omega^2[M]\right)\{u\}=\{F\}+\{f\} \tag{7}$$

Here $[M]$, $[\bar{K}]$, $[\bar{C}]$ are the same notations as above; Ω is the given actual frequency of external effect; $\{\bar{X}\}$ is the vector of the sought for complex amplitudes; $\{f\}$ is the amplitude vector of periodic effect; $\{F\}$ is the total vector of external loads (mass forces, hydrostatic pressure of water, etc.).

When forming equations (7), the Volterra operator (2) is replaced by complex relations [11, 14], which take into account the infinite lower limit of the integral in (2).

The algebraic equation with complex coefficients (7) is solved by the Gauss method.

2.3 Unsteady forced oscillations

At short-term dynamic effect in the system (figure 1), there occur unsteady forced oscillations, the study of which allows to determine the maximum values of displacements and stresses of the structure during the entire impact process and to identify the most stressed areas in the system taking into account various inhomogeneous material parameters and structural features of the structure.

The problem of unsteady forced oscillations of the system (figure 1) using the FEM procedure is reduced to solving a system of linear integro-differential equations

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=\{F\}+\{f(t)\}+\int_0^t \Gamma(t-\tau)[K]\{u(\tau)\}d\tau \tag{8}$$

With initial conditions

$$\{u(0)\}=\{u_0\},\ \{\dot{u}(0)\}=\{\mathrm{v}_0\} \tag{9}$$

Here matrices [M], [K] are the matrices of mass and stiffness of the system; $[C]$ is the matrix, accounting the wave entrainment of energy; $\{u(t)\}$ is the vector of the sought for displacement amplitudes; $\{f(t)\}$ is the vector of dynamic load; $\{F\}$ is the total vector of static loads (mass forces, hydrostatic pressure of water, etc.).

The solution of the system of integro-differential equations (8) with initial conditions (9) is obtained by the Newmark method [19].

3. Results

Test problem 3.1

The tasks of determining the natural frequencies and amplitude-frequency characteristics (AFC) of longitudinal forced oscillations of a viscoelastic rod of finite length (with different means of ends fixing) with A.R.Rzhanitsin kernel (3) and the oscillations of piecewise homogeneous semi-infinite elastic rod are considered in the paper. Analysis of the results has shown that the presence of an infinite domain in an elastic oscillatory system leads to the occurrence of dissipation related to the wave entrainment of energy. The real part of the natural vibration frequencies of a piecewise homogeneous semi-infinite rod coincides with the frequencies of an elastic rod of finite length with a fixed end, while for a viscoelastic rod of finite length this frequency is less than the frequency of the corresponding elastic rods; the dissipation related to the viscoelastic properties of material weakly depends on the natural vibration frequency, and the wave entrainment of energy in a piecewise-homogeneous semi-infinite elastic rod is most intensely manifested at the first vibration frequencies.

The study of steady-state forced oscillations of these rods at different frequencies of external effect and the obtained amplitude frequency characteristics confirmed the conclusions related to the manifestation of dissipation.

Test problem 3.2

The solution of the axisymmetric Lamb problem for an elastic half-space with a rigid round stamp installed on the surface which performs harmonic oscillations in the vertical direction is investigated in the paper. When solving the problem, a finite axisymmetric domain of volume V_5 is cut out of the half-space, on the boundary of which condition (5) is set. Experimental data corresponding to this problem are available in [20], where the propagation of a wave in soil initiated by the hammer impact on the foundation is investigated. Comparison of the obtained results led to the possibility of using the condition of energy entrainment when solving a dynamic problem for a finite domain. The accuracy of the solution depends on the choice of the volume V_5 of the cylindrical body.

Results of the studies have shown that solving a dynamic problem when replacing an infinite foundation with a finite domain without setting conditions that ensure energy entrainment in the form of a wave across the boundaries of the finite domain leads to parasitic resonances caused by oscillations of a finite-dimensional body unrelated to the problem in question and the use of conditions providing the energy entrainment in the form of a wave across the boundaries of the finite domain, allows one to get rid of the resonance that does not take place in reality.

Problem 3.3

Natural oscillations of the Pachkamar earth dam are considered taking into account the viscoelastic properties of material and the inhomogeneous features of the “structure-foundation” system with and without taking into account the wave entrainment of energy at the boundary of the finite domain of the foundation (5).

This dam has: height H = 70.0 m; the coefficients of the laying of the upstream slope m_1= 2.0; of downstream slope m_2= 2.0. Shear modulus (μ), Poisson's ratio (ν) and specific

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weight (γ) of the dam material are: for the core (loam soil) - ν = 0.35, μ = 2900 kgf/cm^2, γ = 0.0018 kgf/cm^3; parameters [16] of the relaxation kernel (3): A = 0.0146, α = 0.2, β = 0.0000057; for retaining prisms (sand and pebble) - ν = 0.35, μ = 3580kgs/cm^2, γ = 0.00225 kgf/cm^3; for facing (monolithic concrete) - ν = 0.25, μ = 84000 kgf/cm^2, γ = 2.4 tf/m^3; for the foundation - ν = 0.3, μ = 3790kgs/cm^2, γ = 0.0026 kgf/cm^3; parameters of the relaxation kernel (3) are: A = 0.034, α = 0.25, β = 0.00036.

If to consider the dam without regard to the foundation, i.e. located on a rigid foundation, then an account of the foundation leads to a slight decrease in natural frequencies of the dam, although only elastic properties of structure material are taken into account.

For the “structure - foundation” system at the boundary of the finite domain of the foundation with account of the wave entrainment of energy, the natural frequency of the system is complex, even if the material of the “structure-foundation” system is elastic. The virtual part of the frequency, i.e. ω_I, means the manifestation of dissipation in the system related to the wave entrainment of energy (i.e., wave dissipation) from the structure to infinity.

The real part ω_R of natural frequency $\omega = \omega_R - i\omega_I$ of the system oscillation is slightly reduced compared with the frequency ω in elastic case, obtained without considering the wave entrainment of energy and internal dissipation.

If to estimate the value of the logarithmic decrement of oscillations of the system δ, (when only the wave entrainment of energy is taken into account at the boundary of the finite domain), then we can see that the values δ, corresponding to the first natural frequency have the largest values, compared to otherδ,corresponding to second, third and other frequencies (figure 2).

a) b)

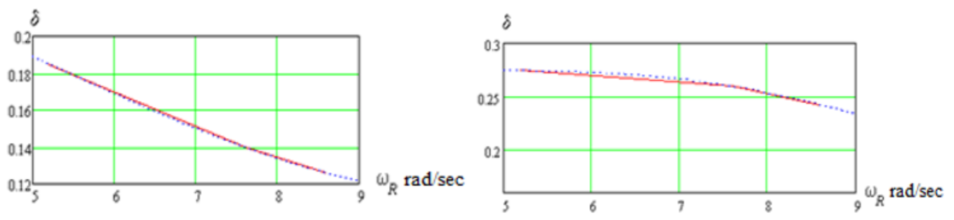

Fig. 2. Change in logarithmic decrement of oscillations depending on the value of the natural frequencies of the Pachkamar dam with wave entrainment of energy (a) and with internal dissipation in material (b)

If, in exactly the same way, to evaluate the manifestations of internal dissipation in material of the structure-foundation system, then we can see a picture similar to the one described above. The only difference is that the values of the logarithmic decrement of oscillations δ, in different natural frequencies of oscillations of the system differ slightly (figure 2b).

The results obtained (figures 2a,b) for evaluating the manifestation of dissipation indicate that when assessing the dynamic strength of earth dams, it is necessary to take into account both types of dissipations; this can provide adequate results corresponding to the actual strain of the structure.

Problem 3.4

Next, using the above method, we have studied the dynamic behavior of the above-considered inhomogeneous system (figure 1) in a plane statement. The foundation is considered elastic, and the structure - viscoelastic. As an external load, a non-stationary effect is used, changing according to the law:

$$\vec{x} \in \Sigma p: \quad P(t) = \begin{cases} 100000 & t = 0 \\ -250000t + 100000 \quad at & 0 \le t < 0{,}4 \text{ sec} \\ 0 & t \ge 0{,}4 \text{ sec} \end{cases} \tag{10}$$

The load *P(t)* in *kN* is also plane and applied at a distance of 25 m from the foot of the dam on the surface of the foundation, i.e. on the site Σ_p (figure 1). It is necessary to determine the fields of displacements and stresses in the body of the dam at different points in time under instantly applied loads (10).

In calculations the following values have been taken:

- for the dam: the height H = 168.0 m, coefficients of the upstream and downstream slopes m_1=m_2=2.2m; the crest width b = 10.0 m; material properties: modulus of elasticity E = 3000.0 MPa; Poisson's ratio ν = 0.3; the specific gravity of soil = 2.2 tf/m^3. To take into account the viscoelastic properties of soil, the A.R. Rzhanitsin's kernel (3) is used with the parameters [16]: A=0.0146; α=0.2; β=0.0000057.

-for the foundation: the modulus of elasticity E = 3600.0 MPa; Poisson's ratio ν = 0.3; the specific gravity of soil = 2.8 tf/m^3.

The solution of this problem with the given parameters has revealed that the waves initiated by the applied load *P(t)* create an irregular field of displacements in the dam body. The beginning of the motion of each point of the structure corresponds to the time the wave front approaches it, determined by the distance of the point from the point of load application and the velocity of wave propagation in soil.

Figure 3 shows the isolines of the distribution of horizontal displacements in the cross section of the dam at different points in time. A wave from a source located in relative proximity to the foot of the dam, propagating along the foundation, first causes a displacement of the foot of the upstream slope (figure 3a), and over time covers more remote areas of the structure (figures 3c, d). In this case, the lower region of the upstream slope, bounded by the isoline "1", remains motionless as a result of wave diffraction at the foundation-slope junction [21]. An isoline with the same index on the downstream slope (figure 3b) corresponds to the position of the wave front, in front of which there is an undisturbed (at t = 0.46 sec) area of the dam (the right side of the figure). In the subsequent time, the disturbance from the load *P(t)* completely covers the dam body and the propagation of horizontal displacements in it is represented by the isolines in figures 3a-d. After wave propagation, the strain state of the dam gradually stabilizes.

The values of horizontal displacements on isolines (figure 3) increase with an equal interval of 0.005 *m* beginning from 0.0 m - on the isoline "1". The maximum displacement is 0.042 *m* and is observed in the area bounded by the line with index "9"; on the line the displacement is 4 *cm*.

The stress state of the dam, represented by the principal stresses σ_1 at different points in time: at the beginning, in the middle and at the end of the process is shown in figure 4. The dimensionality ofstress is MPa.

At the initial time of the process the lower part of the upstream slope is strained in the dam, where a tension zone with positive stresses σ_1 appears (line "2" in figure 4a), later, as the wave propagates, this zone extends upwards along the slope (figures 4b, c) covering the entire internal area of the dam (figures 4c, d). The value of stresses σ_1 on isolines (figure 4) changes with the same step of 0.05 MPa: beginning from 0.0 MPa - on the line "1" up to 0.3 MPa - on the line "6".

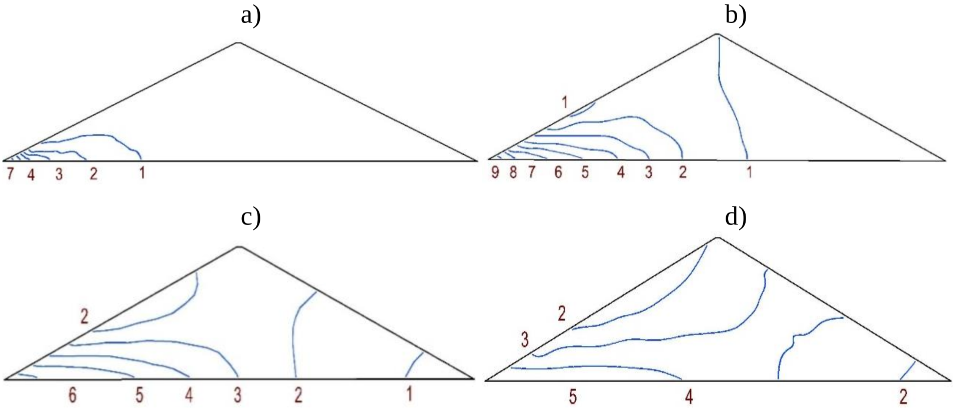

Fig. 3. Isolines of the distribution of horizontal displacements (m) in the section of the dam at different points in time *t*: (a) - 0.2 sec, (b) - 0.32 sec, (c) - 0.52 sec, (d) - 0.60 sec

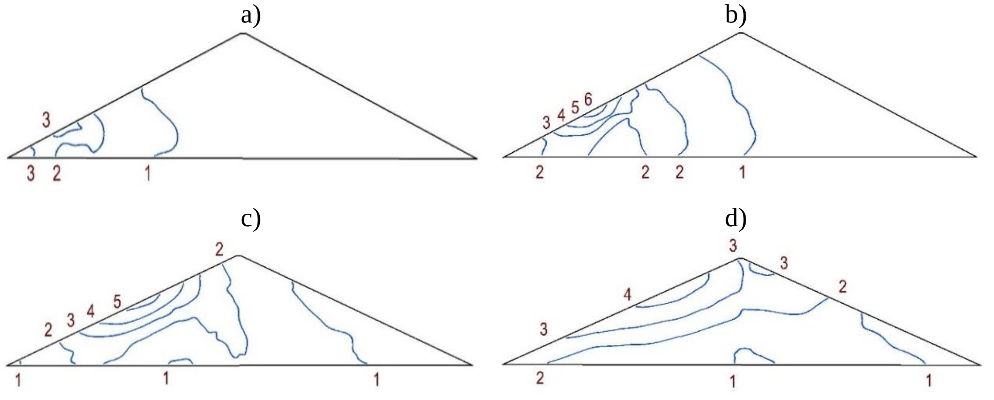

Fig. 4 . Isolines of the distribution of the principal stresses σ_1 in the dam section at different points in time t: (a) - 0.2 sec, (b) - 0.32 sec, (c) - 0.52 sec and (d) - 0.60 sec

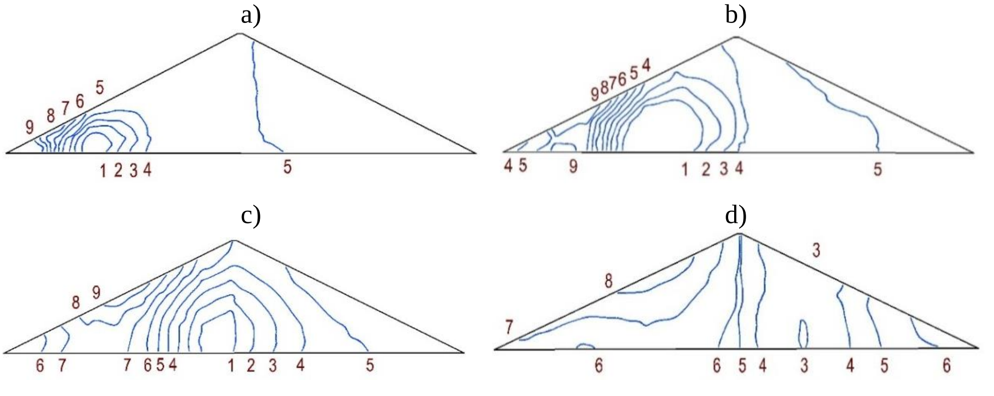

Fig. 5. Isolines of the distribution of tangential stresses σ_{12} in the section of the dam at different points in time t: (a) - 0.2 sec, (b) - 0.32 sec, (c) - 0.52 sec and (d) - 0.60 sec.

Maximum tangent stresses (σ_{12}) arise on the surface of the upstream slope (figure 5): first at its foot, and then over its height, which is fraught with the possibility of a landslide on the slope (figure 5).

The magnitude of the stresses σ_{12} on isolines (figure 5) varies with a step of ± 0.025 MPa from 0.0 MPa on the “5” line to ± 0.1 MPa on the “1” and “9” lines.

4. Conclusion

1. A mathematical model, method and algorithm have been developed for evaluating the dynamic behavior of an inhomogeneous structure–foundation system, taking into account non-reflecting conditions at the boundary of the finite domain of the foundation. A linearly hereditary theory of viscoelasticity is used to describe the internal dissipation; as for the wave dissipation the conditions are used to ensure the energy entrainment in the form of a Rayleigh wave across the boundaries of the finite domain of the foundation.

2. Various model problems have been solved to assess the adequacy and accuracy of models and calculation methods.

3. The study of the dynamic behavior of inhomogeneous viscoelastic “dam-foundation” system with non-reflecting boundary conditions on the boundary of the finite domain of the foundation has shown that:

- the maximum principal stresses σ_1, arising in the lower part of the upstream slope, gradually propagate over the entire slope and the central area of the dam;

- the maximum principal stresses σ_2 are reached near the foot of the dam and as the wave propagates, they move along the foundation directly behind the wave front;

- maximum values of tangential stresses σ_{12} are reached on surface of the upstream slope, first at the foot of the dam, then over the surface of the slope. There are no tangential stresses in the center of the dam;

- at the wave propagation in the dam the symmetric pattern of stress state caused by the static effect of gravitational forces is broken; there occurs an asynchronous motion of its parts, which decays due to the energy entrainment and viscoelastic properties of material of the system.

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