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# The design model of reinforced concrete beam formed by the field of cracks directions

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**Abstract**. Now the design of reinforced concrete constructions for static and dynamic loads with regard to the elastoplastic resistance diagrams of materials is widely used. The model of a reinforced concrete beam is proposed, which consists of trapezoidal elements formed by the field of cracks directions. The theoretical angle of inclination of a crack at any point of a beam has been determined on the basis of minimum of external load, necessary for its formation, which has been obtained from the equation of energy balance. The deflections of each point of a beam have been obtained by solving a differential equation of motion at each step the account. The strains in any fiber of normal and inclined sections have been determined according to the hypothesis of bilinear sections. The stresses in concrete and reinforcement have been obtained with the help of the variable elastoplastic stress-strain curves «σ-ε». The failure mechanism of a beam has been determined on a basis of the transverse to longitudinal force ratio in compressed area of concrete. The internal forces have been determined with the help of numerical section height integration of stresses and from the equation of balance of elements above the crack.

The design of structures for high intensity dynamic loads is an important problem. These loads may occur because of emergency situations or seismic impact. In this case the essential elastoplastic deformations develop in constructions.

Now the design of constructions for static and dynamic loads with regard to elastoplastic «σ-ε» diagrams of concrete and reinforcement is widely used [2], [6], [7]. This method enables to obtain full information about the stress-strain state at any time, in particular at the stage of destruction. Besides it makes it possible to take into account different properties of constructions and of loads acting on them. In this case one of the most important problems is the choice of adequate design model.

Usually in design process bending elements are divided in blocks, formed by normal sections. But in fact the field of cracks directions is variable along the span of the beam and submits a certain pattern. A discrete model of a beam is proposed, which elements are formed by the field of cracks directions (see Fig.1). This model more precisely matches the real behavior of constructions and allows determining the mechanism of destruction of normal and inclined sections.

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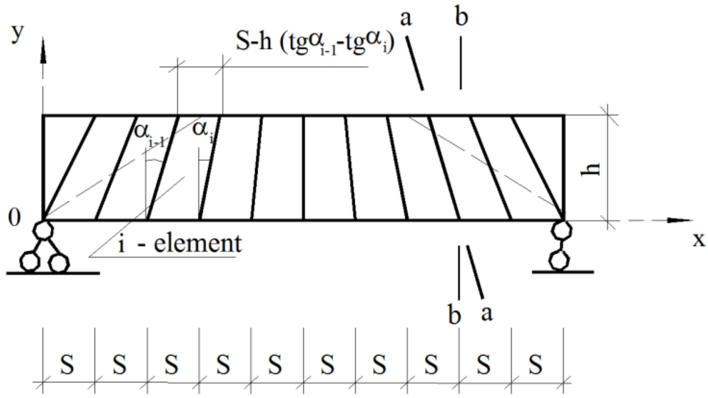


**Fig. 1.** The scheme of division of a reinforced concrete beam into trapezoidal elements

The non-prestressed reinforced concrete beam of finite length of rectangular cross-section under static and dynamic loads is considered. All the geometrical parameters of the beam, boundary conditions, the reinforcement scheme and physical-mechanical properties of concrete and reinforcement are supposed to be known. The slip of reinforcement and irregularity of strains of concrete and reinforcement after the formation of cracks are not considered. It is supposed that the number of cycles is not large and fatigue is also neglected.

The trajectory of the inclined crack is approximated by a straight line. Its design direction corresponds to the minimum of external load causing it [5]. This load in its turn can be determined from the energy balance equation with regard to the stress-strain state of an element above the crack.

$$W_s + W_{sc} + W_{sw} + W_{sh} + W_{bt} + W_{bc} = A_q \quad (1)$$

$W_s$ – is the potential energy of the stretched longitudinal reinforcement deformation, $W_{sc}$, $W_{sw}$ – are the potential energy of the compressed longitudinal reinforcement and of the shear reinforcement deformation, $W_{bc}$– is the potential energy of deformation of the compressed concrete above the crack, $W_{bt}$,– is the potential energy of destruction of stretched concrete, $W_{sh}$ – is the potential energy of shift of concrete above the crack, $A_q$ – is the work of the external loads

The beam is considered as a discrete model, consisting of infinitely hard trapezoidal elements, united by ductile connections, which allow mutual rotation of elements. The deformation of a construction takes place in joints, where the internal forces are calculated.

As the results of theoretical analysis of probable field of cracks and the results of experimental study of different authors have shown the intersection of directions of inclined cracks in the support areas of hinged beams loaded by distributed load takes place. Under the action of concentrated forces trajectories of probable cracks converge to the point of application of force in the case of small span of the slice values. That's why the motion parameters of a beam can be obtained from the design of normal sections.

The beam is divided into certain number of segments in length, and in every point the angle of inclination of a crack is determined. At each time step of calculation the differential equation of motion of a beam is solved, and deflections and curvatures are determined at any point of a beam.

In order to obtain deformation the cross-section of a beam has been divided into fibers. For normal sections in bending the hypothesis of flat sections is suitable. In order to obtain the strains in inclined and polyline sections the hypothesis of bilinear sections has been used. For the inclined section the hypothesis is of the form:

$$\begin{cases} \varepsilon_x(y) = \varepsilon_x(h_b) + A \cdot (y - h_b) \cdot \chi, & (0 \le y \le h_b) \\ \varepsilon_x(y) = (y - h_b) \cdot \chi, & (h_b < y \le h) \end{cases}, \tag{2}$$

$A$ is an empirical coefficient, $\chi$ is a curvature. If $A = 1$ the hypothesis of bilinear sections turns to the well-known hypothesis of flat sections

As the compressive stress act above the inclined crack along its upper side, and therefore the compressive stains take place, then in bilinear section the deformations along the X-axis change the jump at a point of fracture of the section $y = h_b$ (see. Fig.2). For the bilinear section:

$$\begin{aligned} &\varepsilon_x(y) = \varepsilon_x(h_b) + A \cdot (y - h_b) \cdot \chi, && (0 \le y \le h_b) \\ &\varepsilon_x(y) = (y - h_b) \cdot \chi, && (h_b < y \le h) \end{aligned}, \tag{3}$$

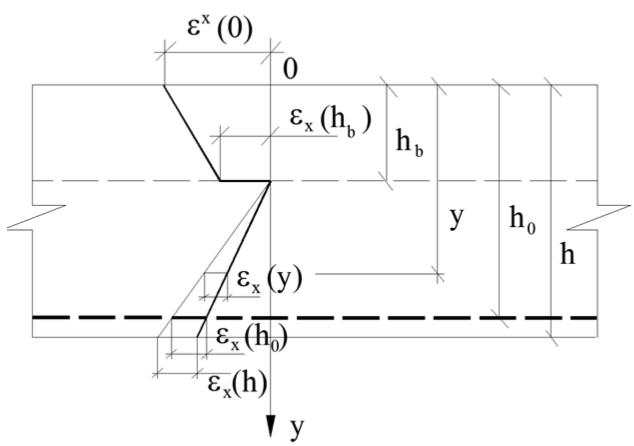


**Fig. 2.** The hypothesis of bilinear sections

The coefficient $A$ of the hypothesis of bilinear sections is adopted to be constant in the process of loading for each section.

The instant position of the neutral axis $y^*$ has been determined as an instant center of gravity of a cross-section of the material with variable elastic modulus. Then the stresses in materials have been determined on a basis of stress-strain curve of materials concrete with regard to the destruction slope of the curve and on a basis of elastoplastic resistance diagram of reinforcement.

The bending moments in normal and inclined sections have been calculated determined with the help of numerical section height integration of stresses.

The failure of constructions under special high intensity loads can occur on the normal and on the inclined sections. That's why it is necessary to develop a criterion, defining the destruction mechanism in the process of loading, particularly after the development of plastic strains in reinforcement.

The failure mechanism of compressed concrete above the crack is determined by the transverse to longitudinal forces $Q_b/N_b$ ratio [6]

$$\begin{cases} Q^{np} = \int_0^{h_b} b \cdot \tau_{xy}(y) \cdot dy \\ N^{np} = \int_0^{h_b} b \cdot \sigma_x(y) \cdot dy \end{cases}, \tag{4}$$

$\tau_{xy}(y)$, $\sigma_x(y)$ – are the tangent and normal stresses of concrete above the crack.

If the point corresponding to the $Q_b/N_b$ ratio lies above the strait line, described by the dependence:

$$\frac{Q^{np}}{N^{np}} = 1.3 \cdot \frac{\theta_{\max}}{\pi}, \tag{5}$$

only the failure due to shear can take place. If this point lies below the strait line , described by the dependence:

$$\frac{Q^{np}}{N^{np}} = 1.1 \cdot \frac{\theta_{\max}}{\pi}, \tag{6}$$

only the failure due to compression can take place. In the interval between these straight lines the destruction due to compression and due to shear are possible.

The longitudinal forces in compressed area of concrete above the crack $N_{b1}$ and $N_b$ in inclined and bilinear section have been calculated with the help of numerical section height integration of normal stresses. The transverse force $Q_{b1}$ can be obtained from the condition of equality to zero of the sum of projections of all forces on the longitudinal axis $\Sigma x=0$ in inclined section.

The transverse force $Q_b$ has been determined form the equation of balance of triangular element above the crack (see Fig3), and thereby the probable failure mechanism is monitored.

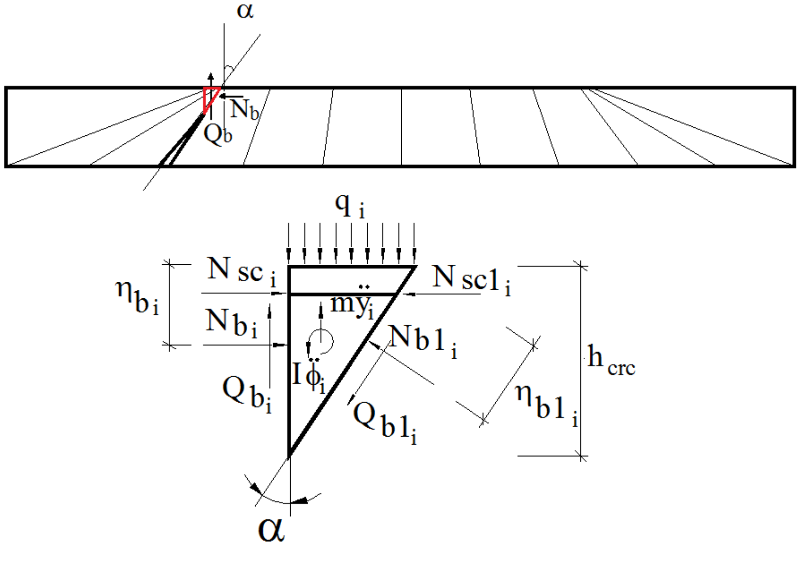


**Fig.3.** The scheme of internal forces in triangular element above the crack

The scheme of the solution is represented in Figure 4

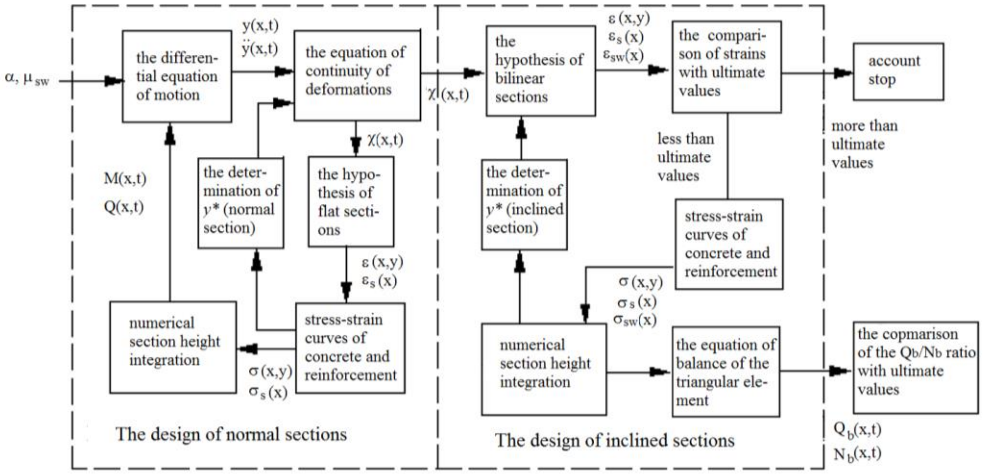


**Fig. 4.** The block sheme of the problem solution

On a basis of proposed model a computer program has been written and the design of hinged beams with different geometrical characteristics and amount of reinforcement under the action of distributed loads and concentrated forces has been carried out. For each section the “moment-curvature” relation has been built. In figure 5 the example of such curve is presented for the value of a crack inclination angle of 30 degrees under static load.

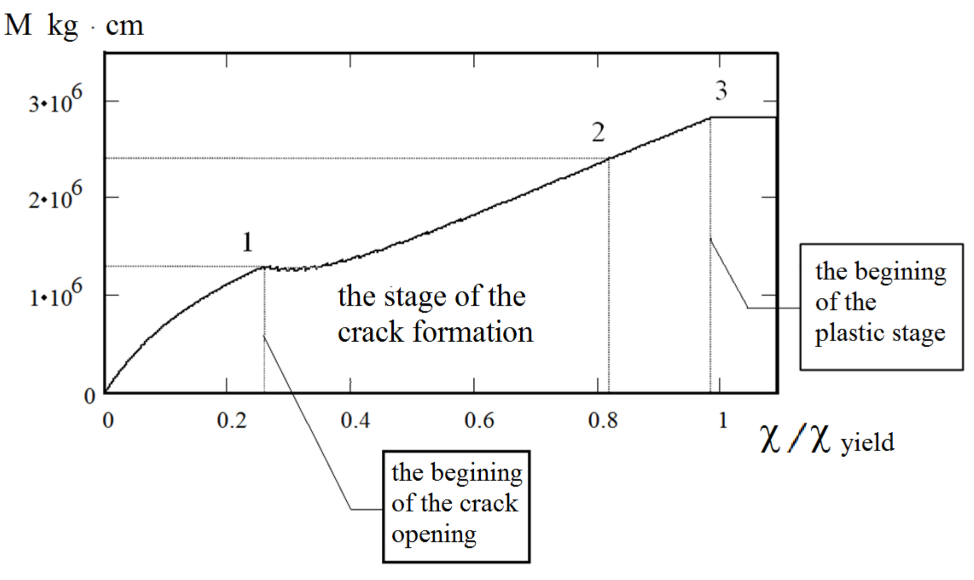


**Fig. 5.** The “moment-curvature” relationship in inclined section (inclination angle 30°) under static load

In order to estimate the probable failure mechanism of a beam the internal forces in the compressed area of concrete have been determined and the function of the $Q_b/N_b$ ratio has been built throughout the loading process, including the plastic stage. It is necessary in order to exclude the fracture of compressed area of concrete due to the shear in plastic stage.

In figure 6 the change in value of the $Q_b/N_b$ ratio from account step is represented during the quasi-static loading process.

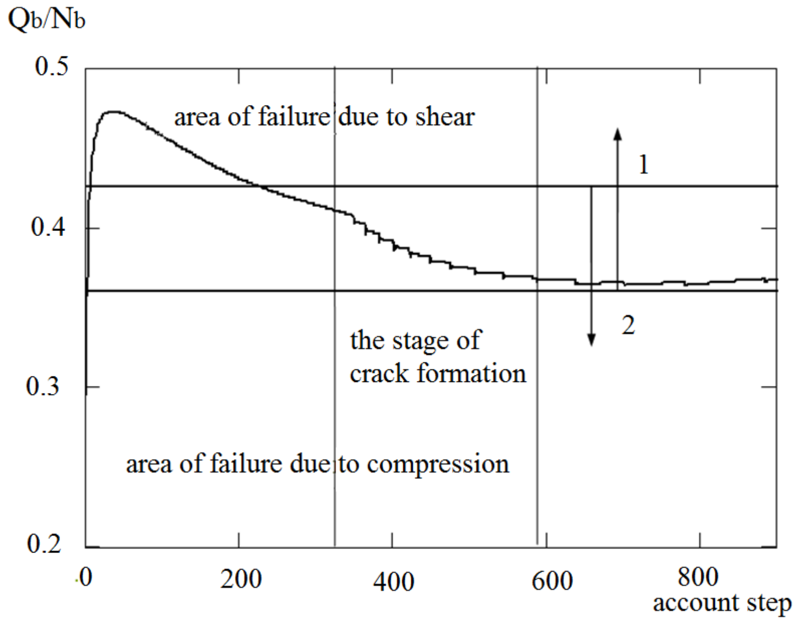


**Fig. 6.** The change of $Q_b/N_b$ ratio during the quasi-static loading process, the crack inclination angle is 30°

As one can see from the graph before the beginning of crack formation, when in the stretched area of concrete plastic strains develop and in the compressed area of concrete strains are close to elastic, the $Q_b/N_b$ ratio decreases smoothly. Then as the crack opens and plastic deformations of compressed concrete increase a sharp decrease of this ratio takes place.

The moment of the beginning of crack opening corresponds to the disabling the first layer of stretched concrete in inclined section (see Fig.5, point 1). The development of the crack lasts during the entire loading period. That's why the completion of the stage of its formation can be determined only conditionally. Approximately this moment corresponds to the stabilization of function of the height of undisturbed concrete area above the crack $h_{crc}$ in the process of loading (see Fig.5, point 2).

After the completion of stage of crack formation until the appearance of plastic strains in stretched reinforcement and then in plastic stage the $Q_b/N_b$ ratio changes very slightly. So if the $Q_b/N_b$ ratio after why the completion of the stage of crack formation does not exceed the value, limiting the area of failure due to compression (see Fig.6, line 2), the failure due to the shear is impossible.

In figure 7 the change in value of the $Q_b/N_b$ ratio from account step is represented during the increase of deflections process under the action of dynamic impact load for the same crack inclination angle.

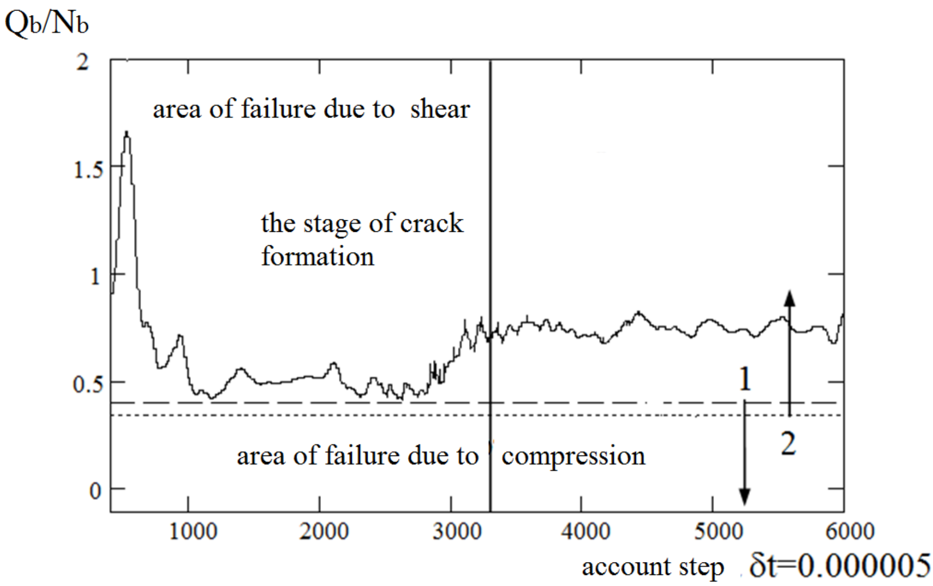


**Fig. 7.** The change of $Q_b/N_b$ ratio during the deflection increase process under the dynamic load $q(t)=q^{max}=const$, the crack inclination angle is 30°

At initial stage of deformation of construction the essential influence of higher forms of oscillations is noticed. The sharp change of value and even of the sign of internal forces is possible. That's why the time dependence of the $Q_b/N_b$ ratio is rather complicated.

During the stage of crack formation and spreading the sharp decrease of the $Q_b/N_b$ ratio takes place as in the case of static load. After the completion of the stage of crack formation the $Q_b/N_b$ ratio fluctuates about a constant value. Therefore one can make a conclusion that the $Q_b/N_b$ ratio is a suitable criterion characterizing the failure mechanism of construction on the inclined section. Using this criterion is possible both under static and under dynamic loads.

To evaluate the influence of the empirical coefficient $A$ of the hypothesis of bilinear sections the dependence of the $Q_b/N_b$ ratio on inclination angle α in hinged beam without shear reinforcement under uniformly distributed load has been built, when the span to height ratio of the beam is equal to 10. The values of the coefficient have been taken $A$ = 1, 2, 3.

As one can see from the graph the effect of the coefficient $A$ value on the $Q_b/N_b$ ratio is in insignificant for the sections close to normal (if $\alpha \leq 20°$). For other sections the shape of the curve preserves but as the $A$ value increases, the maximum of the $Q_b/N_b$ ratio increases and drifts towards larger angle values.

As a result one can make the following conclusions:

1. Under the action of the quasi-static load after the end of crack formation the $Q_b/N_b$ ratio remains constant during the loading process. Under the action of the dynamic load the $Q_b/N_b$ ratio fluctuates about a constant value. The general view of the function is similar for the beams with different parameters and for different laws of load variation in time. This criterion defines the failure mechanism of compressed area of concrete.
2. With the beginning of the plastic stage of reinforcement the internal forces $Q_b$ and $N_b$ remain constant under static load and at any time depending on the dynamic load. On this basis on can conclude about impossibility of failure of compressed area of concrete due to the shear in plastic stage.

3. Under action of dynamic loads risk of failure of a beam due to the shear is higher than under action of static loads.

Hereinafter it is intended to consider the field of cracks directions and of the bearing capacity of beams with regard to the plastic behavior of materials under alternating loads when the system of intersecting cracks is formed.

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