

Stability of counter vortex flows in hydraulic engineering construction

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Abstract. In the framework of linear theory, the stability of counter vortex flows with respect to non-axisymmetric perturbations is investigated numerically. The main flow field calculation results have been obtained as the solutions of the Navier-Stokes equations. The amplification coefficients are calculated, the regions of instability of the flow are defined.

1 Introduction

Counter vortex flows are formed in the interaction of two (or more) coaxially rotating flows, swirled in mutually opposite directions. In technical applications, counter vortex flows are used in combustion chambers, gas turbine engines, heat exchangers, bioreactors, fermenters, cooling towers, and other technical devices [1-3].

In hydrotechnical construction, counter vortex flows are used in vortex spillways to quench the energy of high-speed water flows [4], in counter-vortex aerators [5, 6] to create two-phase water-air flows. A detailed classification of devices using the interaction effect of oppositely swirling flows of liquid and gas is presented in [7].

The hydraulic characteristics of counter vortex flows were studied in [8]. An analytical study of the main flow field based on the Oseen model, is given in [9]. The study of local stability of swirling flows using the Rayleigh number criterion is presented in [10].

This paper is devoted to numerical modeling of hydrodynamics and stability of counter vortex flows using the full system of Navier-Stokes equations to calculate the main flow. The stability of the swirling axisymmetric flows is considered on the assumption of local parallelism using numerical method: the problem of the normal modes developing against the background of the axisymmetric flow determined by the velocity profiles in local cross sections of the flow is solved.

2 Hydrodynamic flow model

We will consider a viscous incompressible flow in an axisymmetric channel with solid impermeable walls. Internal swirling flow is fed to the central part of the channel ($0 \leq r \leq r_0$). External flow, rotating in the opposite direction, is fed into the peripheral part

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of the channel ($r_0 \leq r \leq r_1$). The present analysis is based upon the numerical solution of the full Navier-Stokes equations. In the cylindrical coordinate system r, φ, z the Navier-Stokes equation can be represented in terms of the stream function ψ , the vorticity Ω and azimuthal velocity V_z in form:

$$\frac{1}{r} \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial r} \right) = -\Omega \quad (1)$$

$$\frac{\partial \Omega}{\partial t} + \frac{\partial}{\partial z} (V_z \Omega) + \frac{\partial}{\partial r} (V_r \Omega) = \frac{1}{\text{Re}} \left[\frac{\partial^2 \Omega}{\partial z^2} + \frac{\partial^2 \Omega}{\partial r^2} + \frac{\partial}{\partial r} \left(\frac{\Omega}{r} \right) \right] + G^2 \frac{1}{r} \frac{\partial (V_\varphi)^2}{\partial z} \quad (2)$$

$$\frac{\partial V_\varphi}{\partial t} + \frac{\partial}{\partial z} (V_z V_\varphi) + \frac{1}{r} \frac{\partial}{\partial r} (r V_r V_\varphi) + \frac{V_r V_\varphi}{r} = \frac{1}{\text{Re}} \left[\frac{\partial^2 V_\varphi}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V_\varphi}{\partial r} \right) - \frac{V_\varphi}{r^2} \right] \quad (3)$$

$$V_r = -\frac{1}{r} \frac{\partial \psi}{\partial z}, \quad V_z = \frac{1}{r} \frac{\partial \psi}{\partial r}, \quad \Omega = \frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r} \quad (4)$$

System (1) – (4) is presented in conservative form and contains the two dimensionless parameters: the swirl number $G = W_0 / U_0$ and the Reynolds number $\text{Re} = U_0 \eta_0 / \nu$ (where ν is the kinematic viscosity). Here the radius η_0 is taken as the characteristic length, the axial V_z and the radial V_r velocities are related to the maximum axial velocity U_0 at the channel inlet, and the azimuthal velocity is related to its maximal value W_0 at the channel inlet ($z = 0$).

The flow is considered in the cylindrical domain D ($0 \leq z \leq z_k, 0 \leq r \leq r_k, r_k = r_1 / r_0$). The boundary conditions involve specifying the velocity profiles at the inlet section, no-slip conditions on the rigid surfaces, and symmetry conditions on the axis $r = 0$ and soft boundary conditions should be given at the outlet section $z = z_k$. The set of boundary conditions can be written in form

$$\psi = f(r), \quad V_\varphi = V_{\varphi 0}(r), \quad \frac{\partial \psi}{\partial z} = 0, \quad 0 \leq r \leq r_k, \quad z = 0 \quad (5)$$

$$\psi = \text{const}, \quad V_\varphi = 0, \quad \frac{\partial \psi}{\partial r} = 0, \quad 0 < z \leq z_k, \quad r = r_k \quad (6)$$

$$\psi = 0, \quad V_\varphi = 0, \quad \Omega = 0, \quad 0 \leq z \leq z_k, \quad r = 0 \quad (7)$$

$$\frac{\partial \psi}{\partial z} = \frac{\partial \Omega}{\partial z} = \frac{\partial V_\varphi}{\partial z} = 0, \quad 0 \leq r \leq r_k, \quad z = z_k \quad (8)$$

The function $f(r)$ is defined from the initial distribution of the axial velocity for $r = 0$ according to (4). The initial velocity distribution is defined as:

$$V_{z0}(r) = \begin{cases} 1, & 0 \leq r \leq r_{c1} \\ 1 - k_1(r - r_{c1})^2, & r_{c1} \leq r \leq 1 \\ V_z^* - k_2(r - r_{c2})^2, & 1 \leq r \leq r_{c2} \\ V_z^*, & 1 \leq r \leq r_{c2} \\ V_z^* - k_3(r - r_{c3})^2, & r_{c3} \leq r \leq r_k \end{cases} \quad (9)$$

$$V_{\varphi0}(r) = \begin{cases} s_1 r, & 0 \leq r \leq r_{a1} \\ 1 - s_2(r - r_{a1})^2, & r_{a1} \leq r \leq r_{a2} \\ 1 - s_3(r - r_{a2})^2, & r_{a2} \leq r \leq 1 \\ V_\varphi^* - s_4(r - r_{a3})^2, & 1 \leq r \leq r_{a3} \\ \frac{Ar_k}{r} [1 - \exp(-B(\frac{r^2}{r_k^2})], & r_{a3} \leq r \leq r_{a4} \\ b_0 + b_1 r + b_2 r^2, & r_{a4} \leq r \leq r_k \end{cases} \quad (10)$$

The values of the coefficients in expressions (6) – (7) are equal to $A = 0.554$, $B = 8$, $r_{c1} = 0.94$, $r_{c2} = 1.06$, $r_{c3} = 3.94$, $r_{a1} = 0.9$, $r_{a2} = 0.94$, $r_{a3} = 1.6$, $r_{a4} = 3.94$, $r_k = 4$.

To solve numerically the boundary value problem (1)–(4), (6)–(10) the finite difference method [11] was used with a uniformly spaced grid 81×257 for $z_k = 30$. The time step δt was taken from the range between 0.05 and 0.2. The solution of the boundary-value problem (1)–(4) with boundary conditions (5)–(10) depends on four parameters: the Reynolds number Re , the swirl number G , the initial values of the external flow velocities V_z^* and V_φ^* . The flows were investigated over the following ranges of the parameters: $30 \leq Re \leq 500$, $0 \leq G \leq 3$, $0.1 \leq V_z^* \leq 0.8$, $-1.6 \leq V_\varphi^* \leq 0$. The total number of cases calculated was about 200. The most important properties of the flows are associated with the development of recirculation zones in the near-axis and near-wall sections in the neighbourhood of the tangential swirler. The most characteristic streamline patterns $\psi = \text{const}$ are shown in Fig. 1.

Comparison of the numerical results with the experimental data [12] for turbulent flows in a vortex chamber with oppositely rotating flows is given in [11]. Good agreement between the results was obtained using a turbulent analogue of the Reynolds number, calculated from the turbulent viscosity ν_t using the following expression

$$Re_t = \frac{U_0 r_0}{\nu_t} = \frac{1}{\chi} \sqrt{\frac{8}{\lambda}} \quad (11)$$

where λ is the hydraulic drag coefficient, χ is the universal constant. The coefficient λ varies in the range of 0.011–0.03. The values of the universal constant χ are 0.2, 0.07 for water and air, respectively. In this case, the turbulent Reynolds number is 80–135 for water and 230–385 for air. Therefore, the calculations performed in the selected range of Reynolds numbers allow us to correctly describe the flow hydrodynamics. For all cases considered in [12], the axial velocity distribution on the flow axis, the diameter of the reverse zone and the counter flow rate are in good agreement with experimental data.

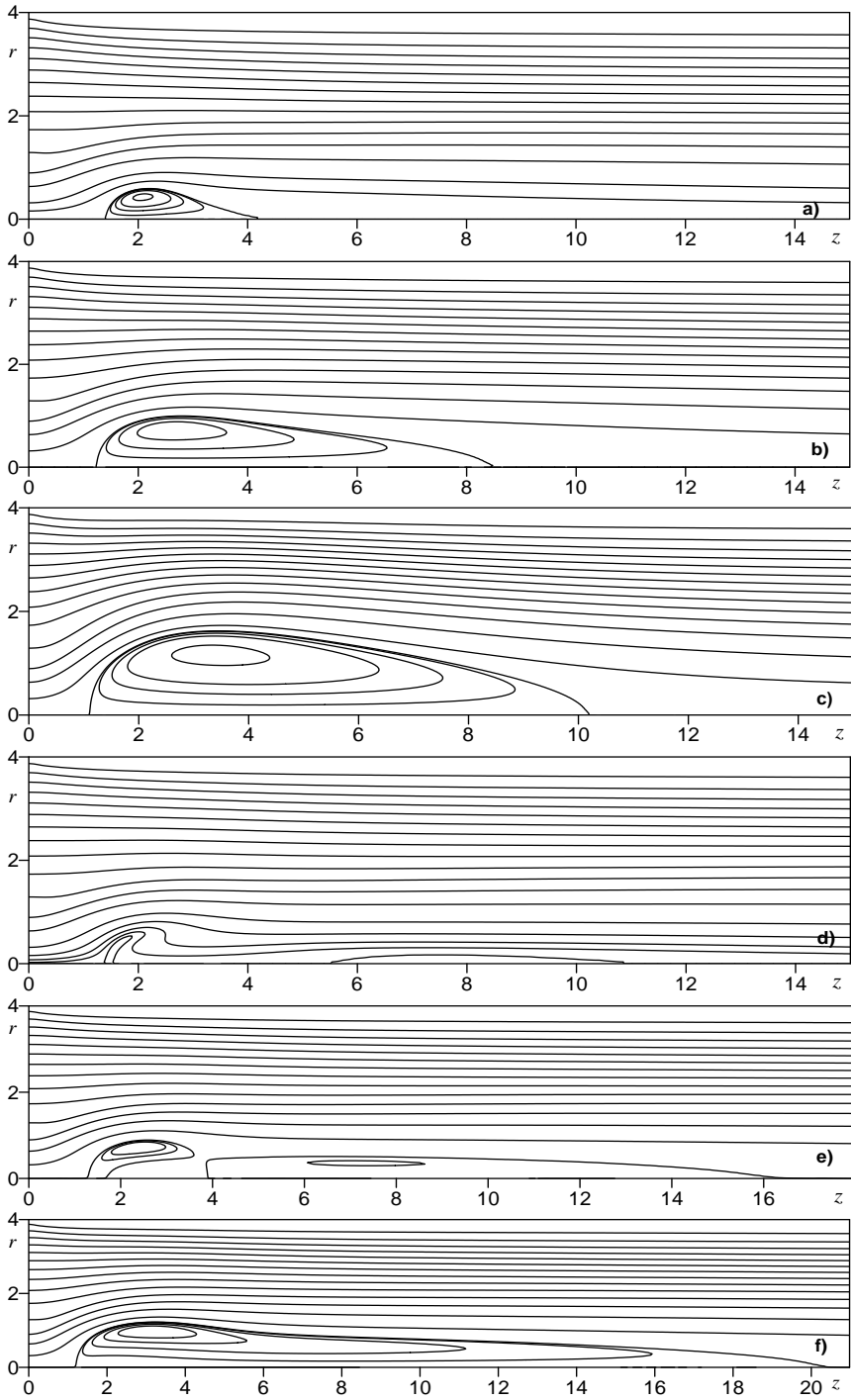


Fig. 1. Streamlines at $Re = 100$; $G = 1.8$; $V_z^* = 0.3$; $V_\phi^* = 0; -0.2; -0.4$ (a-c); $Re = 250$;
 $G = 1.8$; $V_z^* = 0.3$; $V_\phi^* = 0; -0.1; -0.2$ (d-f).

3 Hydrodynamic stability

Let us consider small perturbations of the traveling wave type (normal modes) for the calculated flows, determined by the profiles of the axial $U(r)$ and the azimuthal $W(r)$ velocity components in local flow cross sections

$$\{V'_z, V'_r, V'_\phi, p'\} = \{F, iS, H, P\} \exp[i(\alpha z + n\phi - \alpha ct)] \quad (12)$$

Here p is the pressure, α is the wave number, n is the perturbation mode ($n = 0; \pm 1; \pm 2; \dots$), (the positive values of n correspond to the wave propagation in the direction of swirl, whereas the negative ones correspond to that in the opposite direction), c is the wave speed, i is the imaginary unit. Then, for the complex-valued amplitude functions $F(r)$, $S(r)$, $H(r)$, $P(r)$ we obtain the following system of equations:

$$r^2\gamma F + \alpha r^2 P + r^2 S U' = \frac{1}{i \text{Re}} [r(rF) - (\alpha^2 r^2 + n^2)F] \quad (13)$$

$$r^2\gamma S + 2rHW - r^2 P' = \frac{1}{i \text{Re}} [r(rS)' - (\alpha^2 r^2 + n^2 + 1)S - 2nH] \quad (14)$$

$$r^2\gamma H + r^2 S \left(W' + \frac{W}{r} \right) + rnP = \frac{1}{i \text{Re}} [r(rH)' - (\alpha^2 r^2 + n^2 + 1)H - 2nS] \quad (15)$$

$$\alpha r F + (rS)' + nH = 0 \quad (16)$$

Here $\gamma = \alpha(U - c) + nW/r$, $\text{Re} = U_0 L/\nu$ is the Reynolds number, U_0 is the axial velocity for $r=0$, L is the vortex core radius corresponding to the maximal value of $W(r)$, and the prime indicates the derivative with respect to r .

Assuming that the solution near the axis $r=0$ is regular, we come to the following boundary conditions for (13)–(16):

$$S(0) = H(0) = 0, \quad F(0), P(0) - \text{bounded for } n=0 \quad (17)$$

$$S(0) \pm H(0) = 0, \quad F(0) = P(0) = 0 - \text{where } n = \pm 1 \quad (18)$$

$$S(0) = H(0) = F(0) = P(0) = 0 - \text{where } |n| > 1 \quad (19)$$

$$S(r_k) = H(r_k) = F(r_k) = 0 \quad (20)$$

We consider the perturbations (12) for which $\alpha > 0$ is a real number. In this case, an eigenvalue $c = c_r + ic_i$ specifies the phase velocity of the wave c_r , the oscillation frequency $\omega_r = \alpha c_r$, and the amplification coefficient $\omega_i = \alpha c_i$. When $c_i > 0$ and $c_i < 0$, the amplitudes in (12) grow with time and are damped out, respectively.

The method of calculating the eigenvalues consists of several steps. Near the singular point $r=0$ we construct asymptotic solutions by the Frobenius method; these solutions allow us to transfer the boundary conditions to the point $r=r_a$. From this point we continue the solutions by a Runge-Kutta method with automatic step control and with the

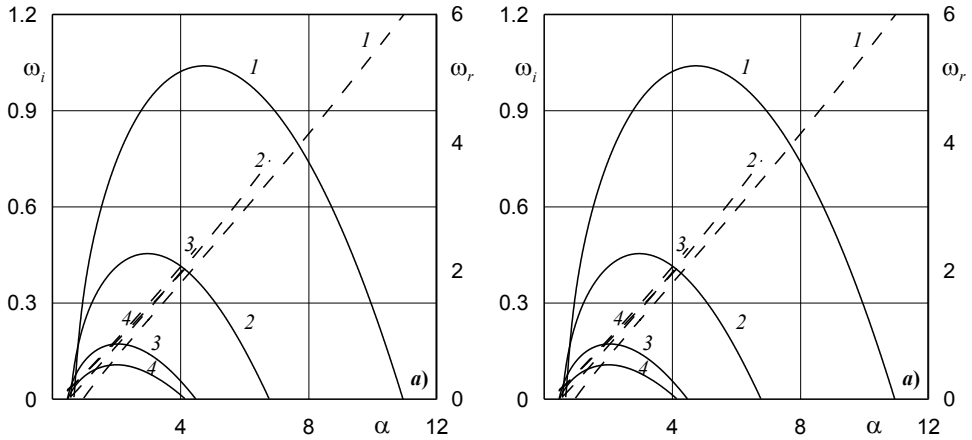


Fig. 2. Amplification coefficients (solid lines) and oscillation frequency (dashed lines) for $Re=100$; $G=1.8$; $V_z^*=0.3$; $V_\phi^*=0$ (a) at $z=0.23; 0.94; 2.5; 5$ (curves 1-4) and $V_\phi^*=-0.2; -0.4$ (b) at $z=0.94; 2.5; 5$ (curves 1-3, 4-6).

Gram-Schmidt orthogonalization. The numerical solutions are matched at the point r_c ($r_a < r_c < r_k$) with Newton's method used for the corresponding characteristic equation. We used this method in [13-15] to analyze the stability of swirling flows of various types.

The calculation examples of problem (13)–(16) with conditions (17)–(20) are presented at Fig. 2. The flow stability was investigated for perturbations (12) with $n=-1$, since, according to [16-17], this mode is the most dangerous for both swirling pipe flow and for free vortices, while in [18] it is shown that it is probably precisely this mode that is observed in the experiments.

4 Conclusions

The mathematical model used to study the hydrodynamics of the counter-vortex flows allows one to fairly accurately describe the structure of swirling flow with the formation of axial recirculation zones.

Considering the local cross sections, we find that as z increases the flow instability first rises and then falls. Thus it is possible to identify a certain region of instability bounded with respect to z and possessing the following properties: for fixed swirl an increase in the Reynolds number amplifies the flow instability, and the region of instability itself grows larger; for fixed Re an increase in swirl leads to only a slight upstream displacement of the boundary of the region of instability; when a reverse flow zone is present, the strongest instability is observed in that zone.

Increasing the swirl of the external flow has a stabilizing effect on the counter vortex flow in all the considered cases.

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