

# The conditions for occurrence of critical flow regime of water in the open stream

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**Abstract.** For the solution of engineering problems require increasingly accurate estimates of the hydraulic characteristics of the water streams. To date, it is impossible to consider sufficiently complete theoretical and experimental justification of the main provisions of the theory of turbulence, hydraulic resistance, channel processes. The composition of tasks related to flows in wide channels, turbulence problems are of scientific and practical interest. Various interpretations of the determination of the critical Froude number in wide open water flows based on observations and theoretical transformations are considered. The conditions for the emergence of a critical regime of water flow in an open wide channel are analyzed in order to estimate the critical Froude number and critical depth. Estimates of the critical Froude number for laboratory and field conditions are given. The estimations allow us to consider the proposed approach acceptable for determining the conditions of occurrence of the critical flow regime. The General, physical interpretation of conditions of occurrence of the critical regime of water flow on the basis of phenomenological approach is specified. The results take into account the values of the components of the total specific energy of the section. This shows the estimated calculation. The results obtained theoretically make it possible to compare the above interpretations and determine their applicability, and the results of the analysis can be useful for the estimated calculations of flows in channels and river flows in rigid, undeformable boundaries and with minor channel deformations.

## 1 Introduction

The operation of hydraulic structures in the specified modes and conditions of use, maintenance and operation, that is, the reliability and safety of their work is provided by reliable calculations in the design of structures. The accuracy of calculations of the key characteristics of water flows is necessary in solving engineering problems related to hydraulic engineering, forecasting and regulation of channel processes, prevention of accidents at various hydraulic structures, pressure and non-pressure, including canals and rivers, and the development of measures to eliminate environmental crisis situations.

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Until now, it is impossible to consider sufficiently complete theoretical and experimental substantiation of the main provisions of the theory of turbulence, hydraulic resistance, channel processes, as well as determining the basic parameters, for example, the potential and kinetic energy of the water flow in the pressure and non-pressure channels. In determining the energy of water flow in open channels, unlike pressure flows, often do not take into account the differences and some features of the flow.

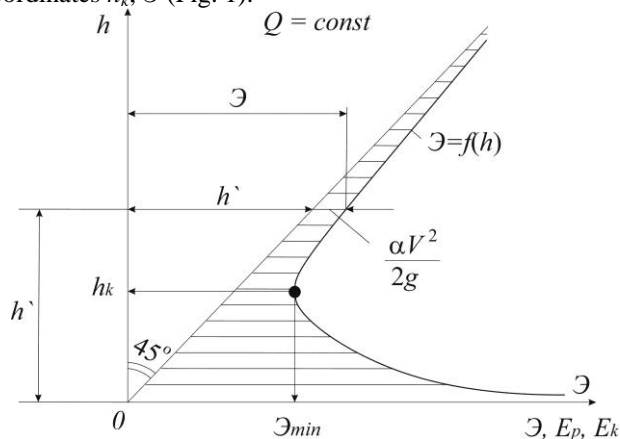
According to the study of the critical regime of open flows, works are known [1-11]. In most cases, hydraulic calculations are carried out using well-known, traditional formulas, often without asking about their applicability to the conditions of water flow in a particular watercourse.

The composition of problems associated with the flow of pressure and non-pressure pipes, wide channels, turbulence problems, axisymmetric flows in smooth and rough pipes are of scientific and practical interest.

## 2 Methods and results

The critical mode is considered to be the boundary state of the flow during its transition from a turbulent state to a calm or from a calm state to a turbulent one.

The critical state of the water flow in the pressure and non-pressure flows formally (without any physical explanation) is usually associated with a minimum specific energy of the cross section at a given flow rate. It is characterized, in addition, by a number of conditions [2, 12-14]: the water flow rate is maximum at a given specific energy; the velocity head is equal to half the average depth of the flow in the channel of a small slope; the Froude number is 1; the velocity of the flow in the channel of small slope with uniform velocity distribution is equal to the velocity of propagation of small gravitational waves created by local disturbance in the basin of small depth. The movement of fluids in critical or close to critical state is unstable. This is due to the fact that a small change in the specific energy of the cross section corresponds to a significant change in depth in the vicinity of the point with coordinates  $h_k; \mathcal{E}$  (Fig. 1).



**Fig. 1.** Energy dependence of the cross section  $\mathcal{E}$  from depth  $h$ :  $E_p$  – potential energy of section,  $E_k$  – kinetic energy of section,  $h_k$  – critical depth.

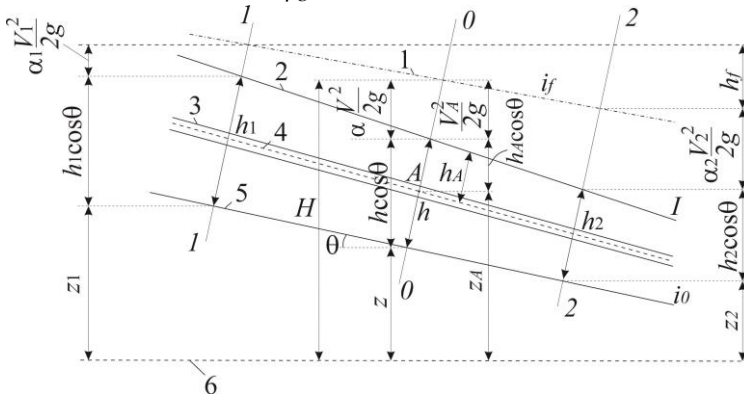
With this deviation, the depth of the flow changes towards a smaller or larger value of the conjugate depths corresponding to the specific energy of the cross section after the change [2, 5, 8, 15-20]. In this case, the water surface is either a bumpy, unstable, wave surface, or there is a hydraulic jump. When designing channels, if the depth of the flow for

most of their length is close to the critical depth, the slope and shape of the channel must be changed to ensure the stability of the structure.

The specific energy of the cross section is determined by selecting the reference plane 0-0, coinciding with the bottom ( $z = 0$ ) and recorded as (Fig. 2):

$$\Theta = h \cos \theta + \frac{\alpha V^2}{2g} = \frac{p}{\rho g} + \frac{\alpha V^2}{2g}, \quad (1)$$

where  $h$  – depth,  $V$  – average speed,  $\frac{p}{\rho g}$  – pressure head.



**Fig. 2.** Schematic cross-section of unconfined channel, indicating depth, energy when water moves slowly: 1 – energy line, 2 – free water surface, 3 – current tube, 4 – line current, 5 – the bottom of the channel, 6 – the reference plane.

The term  $\frac{p}{\rho g}$  in the expression (1) is usually interpreted as the specific energy of the pressure and using the equation of hydrostatics is equal to the depth of the flow  $h$ . For a channel with a low slope and at  $\alpha=1$ , the expression (1) is written as:

$$\Theta = h + \frac{\alpha V^2}{2g} \quad (2)$$

In the usual interpretation of the critical regime it is considered [1], that at depths  $h < h_k$  the flows are turbulent and kinetic energy prevails in them; at  $h > h_k$  the flows are considered calm with the predominance of potential energy. If the basis of the physical interpretation of calm and turbulent regimes to accept these conditions, then the critical regime is logical to associate with the equality of potential and kinetic energy. Under

$\frac{p}{\rho g} = h$  this condition, we write in the form:

$$h_k = \frac{\alpha V_k^2}{2g}, \quad (3)$$

where we find that the critical Froude number corresponding to such an interpretation of the critical regime

$$Fr_k = 2 \quad (4)$$

When integrating (2) with  $\partial\mathcal{D}/\partial h=0$  and  $\omega/B=h_{cp}$  we obtain for a rectangular cross-section of an open flow with a small slope and at  $\alpha=1$  the criterion for the critical state of the flow

$$\frac{\partial\mathcal{D}}{\partial h} = 1 - \frac{V^2 B}{g\omega} = 1 - \frac{V^2}{gh_{cp}};$$

so

$$\frac{V^2}{2g} = \frac{h_{cp}}{2}, \tag{5}$$

which is formulated as follows: in the critical state of the flow, the velocity head is equal to half the average depth [2].

If we assume that the specific potential energy of the cross section is  $h/2$ , the critical condition of equality of the potential and kinetic energy of the cross section is written as:

$$\frac{h_k}{2} = \frac{V_k^2}{2g}, \tag{6}$$

which leads to a critical Froude number  $Fr_k = 1$ .

The condition of equality of the potential and kinetic energy of the cross section, although formal, is physically more transparent than the condition of the minimum specific energy of the cross section (see below), which does not have any physical interpretation.

Let us consider the potential energy of the open flow section in more detail. The potential energy of the mass of liquid passing through the volume element  $dz$  at a height  $z$  is equal to the work on the delivery to this height  $z$  from the plane 0-0 of the amount (weight) of the liquid  $\rho g u \cdot dz \cdot 1s$ , which passes through  $dz$  in the longitudinal direction per unit time.

$$dE_p = \rho g u \cdot dz \cdot z \tag{7}$$

Using the power profile of the velocity:

$$u = u_{\max} \left( \frac{z}{h} \right)^n, \tag{8}$$

where the exponent  $n$  depending on the coefficient of hydraulic resistance  $\lambda$  в открытых широких каналах [9]:

$$n = 1.25 \sqrt{\lambda} \tag{9}$$

The potential energy of the liquid passing through this section per unit time can be found as follows:

$$E_p = \int_0^h \rho g u z \cdot dz = \rho g \int_0^h u_{\max} \left( \frac{z}{h} \right)^n z dz = \frac{\rho g u_{\max} h^2}{n+2} \tag{10}$$

Then the specific potential energy of the flow passing through the cross section per unit time will be equal to:

$$E_{p1} = \frac{\rho g u_{\max} h^2 (n+1) \cdot 1}{(n+2) \rho g u_{\max} h \cdot 1} = h \frac{n+1}{n+2} \tag{11}$$

The calculation results  $E_{p1}$  for different  $n$  are given in table 1.

**Table 1.** The dependence of the exponent  $n$  in the formula (8) on the potential energy  $E_{p1}$ .

$n$	0	0.1	0.14	0.2
$E_{n1}$	$0.5h$	$0.524h$	$0.533h$	$0.545h$

With this method of determining the potential energy of the cross section, its difference from  $h/2$  at  $n > 0$  is small, but still noticeable.

In determining the minimum specific energy of the cross section are determined by the conditions under which this minimum is achieved for a given flow rate  $Q$  [8]. In this case, for a rectangular cross-section of the channel:

$$h_k = \sqrt[3]{\frac{\alpha Q^2}{g B^2}} = \sqrt[3]{\frac{\alpha q^2}{g}} \tag{12}$$

The critical Froude number  $\frac{\alpha V_k^2}{g h_k} = Fr_k$  is in this case (at  $\frac{p}{\rho g} = h$ ) equal to 1.

A similar analysis, performed under the condition that the potential energy of the cross section is close to  $h/2$ , gives a slightly greater value of the critical depth:

$$h_k = \sqrt[3]{2} \sqrt[3]{\frac{\alpha q^2}{g}} \tag{13}$$

and the critical Froude number  $Fr_k = \frac{1}{2}$ .

A common physical interpretation of the transition of the water flow in an open flow from the quiet mode to the turbulent mode is such a flow velocity that exceeds the velocity of propagation of small wave perturbations  $C$ , usually determined by the Laplace formula:

$$C = \sqrt{gh} \tag{14}$$

Then the condition corresponding to the critical flow regime is written as:

$$C = V_k = \sqrt{g h_k} \tag{15}$$

Equation (15) can easily be cast to the form:

$$\frac{V_k^2}{g h_k} = 1$$

The coincidence of the critical Froude number obtained on the basis of the "wave" interpretation with the critical Froude number obtained from the condition of the minimum specific energy of the cross section, it would seem, allows us to assume that both the

critical mode and the corresponding Froude number  $Fr_k = 1$  are determined physically reasonable and unambiguously.

We discuss the last physical interpretation of the critical regime in more detail. Note that in this analysis, the velocity  $C$  was determined by the Laplace formula, which is a special case of a more General expression for the wave velocity [21]:

$$C = \sqrt{\frac{g\lambda_\epsilon}{2\pi} \operatorname{th} \frac{2\pi H}{\lambda_\epsilon}}, \quad (16)$$

where  $\lambda_\epsilon$  is the wavelength of the disturbance on the flow surface.

Comparison of the formulas (15) and (16) showed that the Laplace formula corresponds quite accurately to the dependence (16) only at  $\frac{\lambda_\epsilon}{h} > 6\pi$  ( $6\pi = 18.84$ ). Thus, the use of the dependence (14) in the analysis of critical regimes is possible only for wave disturbances whose length is almost 20 times or more than the depth of the flow. When  $\frac{\lambda_\epsilon}{h} < \pi$ , the value  $\operatorname{th} \frac{2\pi}{\frac{\lambda_\epsilon}{h}}$  becomes close to 1, and the dependence (16) is simplified to the form:

$$C = \sqrt{gH} \sqrt{\frac{1}{2\pi} \frac{\lambda_\epsilon}{h}} \quad (17)$$

Using the dependence (17), it is true for wave perturbations having a length commensurate with the depth of the flow, the condition  $V_k=C$  gives:

$$Fr_k = \frac{V_k^2}{gh_k} = \frac{1}{2\pi} \frac{\lambda_\epsilon}{h_k}, \quad (18)$$

and  $Fr_k$  is equal to 1 only if the value  $\frac{\lambda_\epsilon}{h_k} = 2\pi$  (which is beyond the range of waves of small length).

The calculations according to (18) show that the critical Froude number is significantly less than 1 under wave disturbances, the length of which is less than  $2\pi h$ .

Thus, the "wave" interpretation of the critical regime does not give unambiguous results on the critical Froude number.

The only difference between turbulent and calm flows is the bumpiness of the free surface, which is associated with the manifestation of turbulent pulsations of velocity and pressure on the surface. This physical feature can be used as a more General physical interpretation of the critical regime.

It is obvious that small-scale disturbances on the flow surface can be suppressed by surface tension forces. Analysis performed using Laplace's formula for pressure caused by surface tension at free surface curvature:

$$p_{\mathcal{L}} = \frac{2\sigma}{r}, \quad (19)$$

where  $\sigma$  is the surface tension.

Perturbations with a radius of curvature of the free surface  $r < 1.5$  cm will be suppressed by surface tension. Photos of the free surface of turbulent flows, made with low exposure, reveal the stochastic nature of the tuberosity, which is mixed along the flow at a rate close to the flow rate (Figure 3). Adopting the standard of turbulent pressure pulsations in the bottom flow area

$$p' = 3.5\rho u_*^2 \tag{20}$$

and considering that this perturbation will "manifest" on the surface of the flow by stochastic tuberosity in height  $h'$ , we write

$$p' = \rho gh' = 3.5\rho u_*^2, \tag{21}$$

where

$$h' = \frac{3.5u_*^2}{g} \tag{22}$$



**Fig. 3.** Free surface of turbulent flow.

Using the known relationship between the dynamic and average flow rate  $u_*^2 = \frac{\lambda}{8} V^2$ , the expression (22) is written as

$$\frac{V^2}{gh} = \frac{8}{3.5} \frac{1}{\lambda} \frac{h'}{h} \tag{23}$$

The expression (23) shows that the height of the tuberosity on the free surface of the flow increases with the number of Froude and the coefficient of hydraulic resistance  $\lambda$ .

A numerical estimate of the Froude number at which a distinguishable tuberosity appears on the flow surface for laboratory conditions at  $h \sim 0.1$  m and  $\lambda = 0.02$  gives

$$\left( \frac{V^2}{gh} \right)_k = \frac{8}{3.5} \cdot \frac{1}{0.02} \cdot \frac{10^{-3}}{10^{-1}} = 1.14.$$

For natural conditions (for example, a mountain river,  $h \approx 1$  m;  $\approx 10^{-2}$  m;  $\lambda = 0.025$ ) the critical number of Froude will be

$$\left( \frac{V^2}{gh} \right)_k = \frac{8}{3.5} \cdot \frac{1}{0.025} \cdot \frac{10^{-2}}{10^{-1}} = 0.91.$$

The given estimates of the critical Froude number, performed on the basis of the proposed phenomenological approach, give numerical values of the critical Froude number close to 1.0, which allows us to consider the proposed approach physically and formally acceptable for determining the critical flow regime.

Various interpretations of the appearance of the critical regime of water flow in an open stream are considered:

- Physical interpretation.
- Energy interpretation
- "Wave" interpretation.

In the first interpretation, the generally accepted critical Froude number is equal to one, provided that the specific kinetic and potential energy of the cross section is equal, in this case, the specific potential energy of the cross section is  $h_k/2$ . However, this interpretation is the most transparent for understanding.

The second interpretation shows that the potential energy of the  $\mathcal{E}_n$  at a different coefficient of hydraulic resistance is slightly different from  $h/2$ , but in a rectangular channel the minimum specific energy of the cross section corresponds to the critical number of Froude equal to 1.

In the third, "wave" interpretation of the evaluation of the critical Froude number, based on the Laplace formula for determining the wave velocity in a wide channel, it is easy to obtain the traditional  $Fr_k = 1$ , but only for long waves. For short waves  $Fr_k$  is less than one, which shows an example of calculation. Visual picture of the differences between turbulent and tranquil flows, this interpretation is clearly, it can be used in the generalization of the physical interpretation of the critical flow regime.

Based on the phenomenological approach, three interpretations of the conditions of formation of the critical regime in an open stream are considered, estimates of the critical Froude number and the critical depth for different conditions are made. Evaluation calculations for laboratory and natural conditions show the acceptability of the Analyzed traditional formulas used in hydraulic, hydrological calculations, for their applicability.

The concept of critical mode is used in solving many hydraulic problems of calculation of water facilities. The concepts of critical depth and critical Froude number are used. Traditionally, these concepts according to B. A. Bakhmetev are associated with the concept of minimum specific energy.

This article describes other possible more accurate determination of the critical globemark, which is taken into account in the analysis of the forms of the free surface flow. The comparison of RCR calculated on the basis of energy, wave and pulsation approach allowed to establish the influence of the coefficient of hydraulic resistance on the critical number of Froude and to obtain the corresponding calculated dependences.

As a result of the analysis, a critical Froude number close to one is obtained.

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