E3S Web of Conferences **97**, 05045 (2019) https://doi.org/10.1051/e3sconf/20199705045
FORM-2019

Mathematical model of vibrating air pump unit

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Abstract. Based on the laws of classical mechanics, in particular, the law of conservation of momentum, the paper describes the developed mathematical model of signal propagation during vibration diagnostics. At the beginning, the problem of signal propagation was investigated, which was reduced to solving the problem of wave propagation. According to the analysis of experimental results investigated, that the attenuated nature of the signals must be taken into account. For this purpose, a mathematical model has been developed, which allows to solve the problem of the propagation of damped signals. Comparative analysis allows to conclude that the constructed model is adequate.

1 Introduction

Currently in the Republic of Uzbekistan about 70% of the land is irrigated using centrifugal pumps, the smooth operation of these pumps is necessary for the successful development of agriculture. In the Action Strategy for the further development of the Republic of Uzbekistan for 2017-2021, special attention is paid to the development of land reclamation and irrigation facilities to increase the level of the national economy". The implementation of this task, aimed at improving the accuracy of diagnosing the state of pumping units, becomes important.

2 Methods

As a rule, when vibrodiagnostics using an accelerometer installed at certain points of the machinery, a signal is recorded in the form of vibration acceleration. This signal, either in the device, which is called the vibrator, or integrated in the computer, is converted into vibration velocity or vibration displacement (Fig. 1). All these three types of signals in vibration diagnostics are considered periodic polyharmonic processes.

The leading direction in vibration diagnostics is the analysis of the spectrum of the vibration signal.

When constructing a mathematical model, we will proceed from the laws of theoretical mechanics, in particular, from the law of conservation of momentum, which leads to the equation of signal propagation in the form of vibration accelerations in integral form [1]. In order to pass from an integral equation to a differential one, suppose that the desired function has second derivatives.

The task of signal propagation is reduced to solving a differential equation of the form:

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$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}, \tag{1}$$

satisfying homogeneous boundary conditions at:

$$u(o,t) = 0;\ u(l,t) = 0; \tag{2}$$

And initial conditions:

$$u(x,o) = \varphi(x);\ \dot{u}(x,o) = \psi(x); \tag{3}$$

To solve the problem using the method of separation of variables. According to this method, equation (1) is represented as:

$$u(x,t) = X(x)Y(t); \tag{4}$$

where: X (x) is a function of variable x only;

Y (t) - function of variable t only.

As a result of substituting equation (4) into equation (1), we obtain the following expression:

$$\frac{X''(x)}{X(x)} = \frac{1}{a^2}\frac{\ddot{Y}(t)}{Y(t)} = -\lambda; \tag{5}$$

where: λ is a constant, which, for the convenience of subsequent calculations, is taken with a minus sign, without assuming anything about its sign.

From the expression (5) we obtain the differential equations for determining.

$X(x)$ and $Y(t)$;

$$X''(x) = \lambda X(x) = 0 \tag{6}$$

$$\ddot{Y}(t) = a^2 \lambda Y(t) = 0 \tag{7}$$

Moreover, the boundary and initial conditions are taken as zero.

It is determined [2] that when the value of λ1 is equal to

$$\lambda_n = \left(\frac{\pi \cdot n}{l}\right)^2; \tag{8}$$

There are nontrivial solutions of the problem (1)

$$X_n(x) = \sin\frac{\pi \cdot n}{l}x; \tag{9}$$

Defined up to an arbitrary factor, which we set equal to one. The same values of λ_n correspond to the solution of equation (7) in the form.

$$y(t) = \frac{a_o}{2} + \sum_{k=1}^{\infty}\left(a_k \cos\left(\frac{2\pi k}{T}t\right) + b_k \sin\left(\frac{2\pi k}{T}t\right)\right), \tag{10}$$

where: flourier coefficient for the k-th harmonic

$$a_k = \frac{2}{T}\int_O^T y(t)\cos\left(\frac{2\pi k}{T}t\right)dt, \qquad b_k = \frac{2}{T}\int_O^T y(t)\sin\left(\frac{2\pi k}{T}t\right)dt,$$

$$y(t) = A_O + \sum_{k=1}^{\infty} A_k \cos\left(\frac{2\pi kt}{T} - \varphi_k\right), A_O = \frac{a_o}{2},$$

- average value.

Equations (10) can also be represented as

$$Y(t) = A_o + \sum_{k=1}^{\infty} A_k \cos\left(\frac{2\pi kt}{T} - \varphi_k\right), \ A_o = \frac{a_o}{2}; \tag{11}$$

where: $A_k = \sqrt{a_k^2 + b_k^2}$ - module of the k-th harmonic of the spectrum;

$\varphi_k = arctg\left(\frac{b_k}{a_k}\right)$ - initial phase of the k-th harmonic of the spectrum.

When k = 1 we have $f_1 = \frac{1}{T}$ (Hz) or circular frequency

$$\omega_1 = \frac{2\pi}{T}.$$

Further, by virtue of linearity and homogeneity, we represent equation (1) as the sum of particular solutions:

$$u(x,t) = \sum_{n=1}^{\infty}\left(A_n \cos\frac{\pi \cdot n}{l}a \cdot t + B_n \sin\frac{\pi \cdot n}{l}a \cdot t\right)\sin\frac{\pi \cdot n}{l}x$$

This solution satisfies this equation and the boundary conditions.

However, due to the decaying nature of signal propagation, in general, the signal propagation equation can be represented as

$$A_o\frac{\partial^u y}{\partial x^u} + A\frac{\partial^2 y}{\partial x^2} + 2B\frac{\partial^2 y}{\partial x \partial t} + C\frac{\partial^2 y}{\partial t^2} + D\frac{\partial y}{\partial x} + Ey = 0 \tag{12}$$

where: $A_o = -\frac{E_o y}{T}$; $\quad A = V_1^2 + V_0^2$; $\quad B = -V_0$; $\quad C = -1$;

$$D = \frac{\partial v_0}{\partial t} + \left(\frac{\partial P}{\partial x}\right)T \quad E = \omega_e^2; \quad V_n^2 = \frac{P}{T}; \tag{13}$$

In particular, from equation (12) one can get equation (1), however, due to the fading nature of signal propagation, the third term of equation (12) cannot be neglected.

In this case, the problem is reduced to solving an ordinary differential equation:

$$\frac{d^2 y_n}{d \cdot t^2} + B_n\frac{d \cdot y_n}{d \cdot t} + C_n y_n = F_n(t) \tag{14}$$

To find a unique solution of the differential equation (14) in partial derivatives, it is necessary to determine the initial and boundary conditions.

Initial and boundary conditions are called the conditions specified at the initial time t. The boundary conditions are specified for different values of spatial variables.

Equation (14) is solved numerically under initial conditions.

 https://doi.org/10.1051/e3sconf/20199705045

$$t = 0; \qquad y_n = y_n(0); \qquad \dot{y}_n = \dot{y}_n(0);$$

To test the adequacy, we compare the results of solving equation (14) with the data of experimental studies and present them in the form of an accelerogram (Fig. 1.)

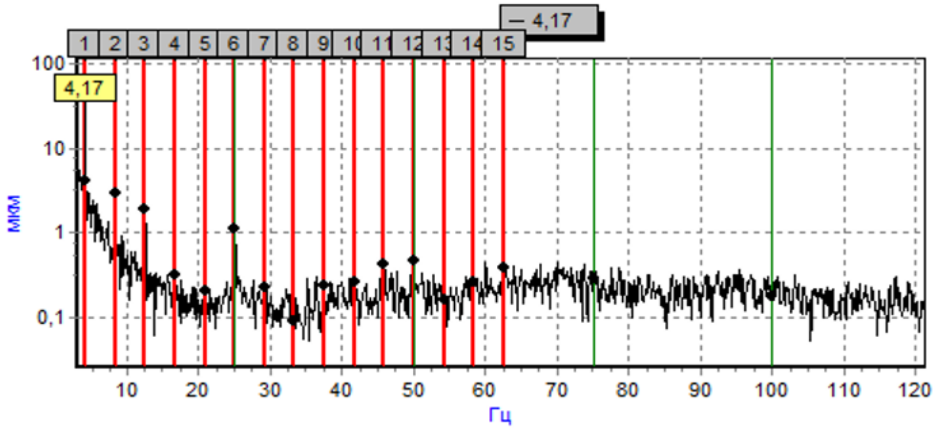

Fig. 1. Accelogram amplitude spectrum of vibration on the camera impeller pumps first Karshi Machine channel.

Using the "Spline interpolation" method, we replace the accelelogram with broken curves (Fig. 2).

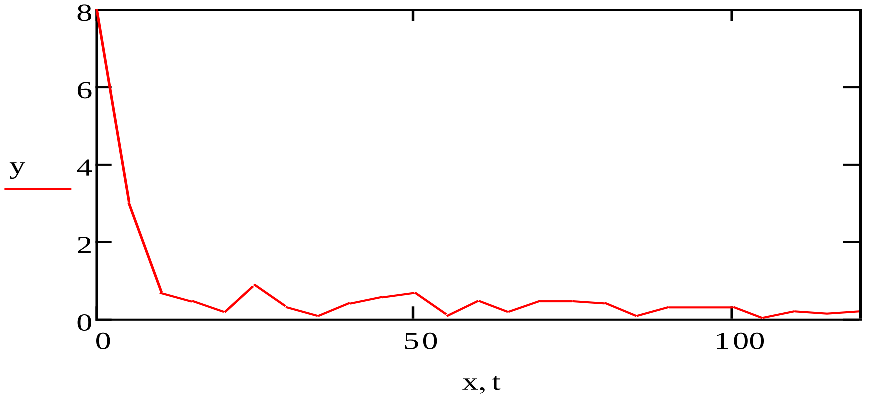

Fig. 2. Experimental result.

Further, according to the well-known method [2], we average the values and as a result we obtain the averaged curve (Fig. 3).

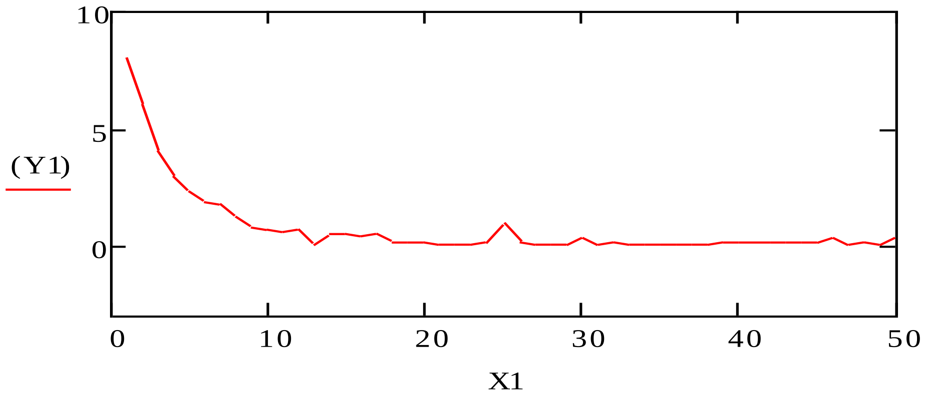

Fig. 3. Approximation of experimental results.

Based on the analysis and processing of experimental data with a given degree of accuracy, it is possible to assert that equation (14) is valid.

In case of replacement by a differential equation of the first order.

$$\frac{d\cdot z}{d\cdot t} = z(t)+\sin(z(t),t) \quad \text{at} \quad x(0)=8$$

The pattern of signal change is shown in Fig. 4.

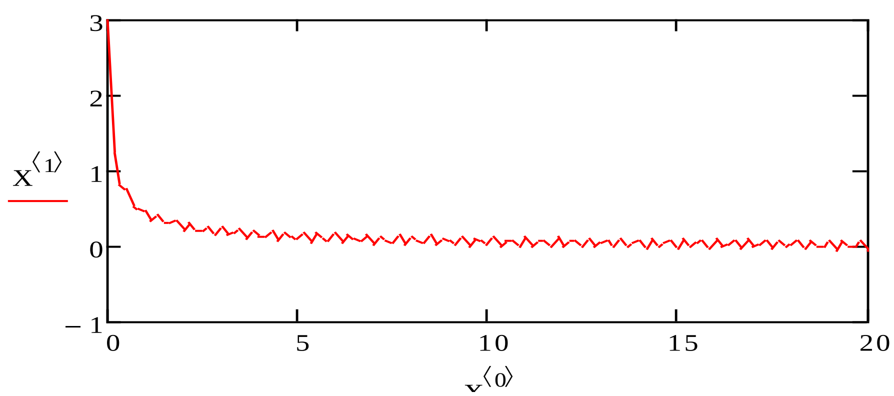

Fig. 4. The pattern of changing signal.

Comparing the experimental and theoretical curves, we can conclude about the adequacy of the mathematical model.

Next, go to the complex values.

For a periodic function of a Fourier series in a complex form, has the form:

$$y(t) = \sum_{k=-\infty}^{\infty} c_k exp\left[\left(\frac{i2\pi k}{T}\right)t\right],$$

where $$c_{л} = \frac{\int_{O}^{T} y(t)exp\left[\left(-\frac{i2\pi k}{T}\right)t\right]dt}{T}, \qquad (15)$$

$$c_k = \frac{(a_k - ib_k)}{2}; \qquad c_{-k} = \frac{(a_k + ib_k)}{2};$$

(coefficients c_k complex conjugate). Relationship (15) is based on the Euler formula

$$e^{-ix} = \cos x - i \sin x; \tag{16}$$

An important characteristic of the vibration signal is the root mean square value (RMS). Given the parseval equality, equations (15) take the following form:

$$\frac{\int_O^T y^2(t)dt}{T} - \left[\frac{\int_O^T y^2(t)dt}{T}\right]^2 = \sum_{k=1}^{\infty} \frac{a_k^2 + b_k^2}{2}, \tag{17}$$

score of this great

$$RMS = \sqrt{\frac{\sum_{k=1}^{L} A^2(k)}{2}}; \tag{18}$$

where: L is the number of lines of the spectrum.

Let the function y (t) be given at times $t_i = y(t_i)$, where: $t_i = i \cdot \Delta t$; Δt – sample rate, $\Delta t = \frac{T}{N}$; *i=1, ... N, N* – number of ordinates in function; *T*- implementation length of the studied function. Further (ti) we will denote as y (i) or ui, i.e. i– e function value

The calculation of the spectrum module is performed by the formula:

$$A_k = (Y(\omega_k) = \sqrt{a_k^2 + b_k^2}; \tag{19}$$

where is the frequency of the k-th harmonic $\omega_k = k \cdot \omega_1,\ f_k = \frac{k}{T};$ the coefficients a_k and b_k are not calculated by the formula, but by a numerical method. So, for example, by the method of rectangles

$$a_k = \frac{2}{N}\sum_{i=1}^{N} y(i)\cos\left(\frac{2\pi}{N}ki\right), b_k = \frac{2}{N}\sum_{i=1}^{N} y(i)\sin\left(\frac{2\pi}{N}ki\right), \frac{a_o}{2} = \frac{\sum_{i=1}^{N} y(i)}{N}, \tag{20}$$

Function at sampling points

$$y(i) = \frac{a_o}{2} + \sum_{k=1}^{N/2} a_k \cos\left(\frac{2\pi}{N}ki\right) + \sum_{k=1}^{N/2-1} b_k \sin\left(\frac{2\pi}{N}ki\right). \tag{21}$$

In complex form, this value is

$$y(i) = \sum_{k=-(N/2-1)}^{N/2} c_k \exp\left(i2\pi ki / N\right),$$

where $c_k = \frac{\sum_{i=1}^{N} y(i)\exp\left(-i2\pi ki / N\right).}{N}$

Considering that $c_k = c_k$

$$y(i) = \sum_{k=0}^{N-1} c_k \exp(i 2\pi k i / N), \tag{22}$$

In the future, the amplitude of the k-th harmonic, calculated by the formula (6), we will denote Y(k).

If the signal is not periodic, then its spectrum is continuous and is determined by the direct Fourier transform:

$$s(\omega) = \int_{O}^{T} y(t) \exp(-j\omega t) dt,$$

The values of the function are determined by the inverse Fourier transform.

At discrete frequencies $f_k = \dfrac{k}{t}$; $s\left(\dfrac{k}{t}\right) = T \cdot c_k$;

where: c_k is the complex Fourier coefficients (the complex coefficient is equal to the area of the rectangle with base f_1 and height s (k / T)).

Based on the Kotelnikov theorem, if the signal has a frequency-limited spectrum ($f \le F_b$), then to restore the signal it is enough to know the spectrum at discrete points $f_{k.}$

Given the discreteness of the representation of the original signal y (i) the desire to represent the spectral density in discrete form, use the discrete Fourier transform (DFT) [3] (with the assumption that the values of y_i)

$$s_k = s(k/T) = \sum_{i=1}^{N} y(i) \exp(i 2\pi k i / N), k = 0,1,\ldots N-1; \tag{23}$$

At $N/2 \le k \ge (N-1)$ $s_k = s_{N-k}$

$$y(i) = \sum_{k=o}^{N-1} s(k) \exp(i 2\pi k i / N)/N, i = 1,\ldots, N; \tag{24}$$

$$c_k = c(k/T) = (1/T) \sum_{i=1}^{N} y(i) \exp(-i 2\pi k i / N), k = 0,1,\ldots N-1; \tag{25}$$

If the cutoff frequency $Fc = N/(2\Delta t)$ is significant, then the number of ordinates N at a constant discretization step must also be large, which increases the amount of DFT calculations (the number of calculations is proportional to N2).

In this regard, in practice the methods of fast Fourier transform (FFT) are widely used (4)

For FFT

$$X(k) = N \cdot C(k); \qquad A(k) = Y(k) = (2/N) \cdot X(k); \tag{26}$$

We introduce such notation:

f_g – sampling frequency (Hz), sample rate $\Delta t = 1/fg$ (sec), sale length T = N/fg; sec

f_1 – first harmonic frequency (Hz), $f_1 = \dfrac{1}{T} = \dfrac{f_g}{N}$; $f_k = k;\cdot f_1$ - frequency of the k-th harmonic (the realization length T determines the accuracy of the spectrum reproduction);

L – the number of lines in the spectrum

$$L = \frac{F_b}{f_1} = \frac{F_b \cdot N}{f_g} \tag{27}$$

where: F_b – high frequency. Based on the Kotelnikov theorem, $f_g > 2F_b$, often take $f_g = 2{,}56$ $F_b(256=2^8)$;

F_H- lower frequency (Hz) if $T > 1/F_H$ that $f_1 < F_H$;

f_p - rotor speed (Hz);

3 Discussion

Consider a practical case of importance:

$f_g = 2{,}56 \cdot F_b$ Hz, $N = 2^p$. Then $K = 1002^{p-8}$: at p=10 (N=1024) L=400, at p=11 (N=2048) L=800, at p=12 (N=4096) L=1600, at p=13 (N=8192) L=3200. As we see, if $f_g > 2 \cdot F_b$, then the number of lines of the spectrum L<N/2.

If additionally $F_b = 5000$ Hz (f_g =12800 Hz), that $T = 2^{p-7}/100$; $f_1 = 100/2^{p-7}$; $f_k = k \cdot 100/2^{p-7}$; relative harmonic number in rotor frequency $m_k = f_k/f_p = k/2^{p-8}$ (f_p=50Hz). At p=11 $m_1=1/8$, $m_2=1/4$, $m_3=3/8$, $m_4=1/2$, $m_5=5/8$, $m_6=3/4$, $m_7=7/8$, $m_8=1$, $m_9=9/8$, . . . , $m_{800}=100$; $m_k=1$ at $f_k=f_p$. If a f_p=50 Hz, that $k=2^{p-8}$, at p=11 k=8.

Comment: number of lines in increments $f_p L_p/2p^{-8}=100$ and does not depend on p. For sampling rate 12,8 кHz, the frequency step for the spectrum is: at N=1024 f_1=12,5 Hz, at N=2048 f_1=6,25, at N=4096 f_1=3,125 Hz, at N=8192f_1=1,5625 Hz.

RMS value is determined as follows [4]:

$$RMS = \left[\int_O^T y^2(t)dt / T\right]^{\frac{1}{2}} = \left[\sum_i y_i^2 / N\right]^{\frac{1}{2}}; \tag{28}$$

Actually, formula (28) is valid in the case, if the constant component a0/2 equals zero. In vibration diagnostics vibration, removed from machinery, Filtered at low and high frequencies and set in the range (FH, Fb) FH>0, so the formula (28) is true for the time function (wave) and coincides in meaning with the formula (19), fair to the spectrum.

In the case when the vibration signal breaks up in time into parts, either when integrating or defining an RMS envelope, you must first remove the constant component

$$RMS = \left[\sum y_i^2 / N - (\sum y_i / N)^2\right]^{\frac{1}{2}}; \tag{29}$$

The number of terms in the formula (29) is equal to L.

In addition to the VHC, in vibration diagnostics is used:

a) peak value - the largest absolute value of the maximum deviations of the oscillating quantity. There is a positive peak value and a negative peak value;

b) span - the difference between the highest and lowest values of the oscillating value.

RMS is the most important indicator, since it takes into account the temporal development of the studied fluctuations, and it directly displays the value associated with the signal energy and, therefore, the destructive power of these oscillations.

4 Conclusions

Comparison of the obtained results with experimental data allows us to conclude that, that the proposed method adequately determines the process under consideration, signal propagation.

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