Modelling of turbidity distribution along channels

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Abstract. The purpose of the article is to develop the required and sufficient conditions under which numerical methods can be used for engineering calculations and for scientific research of hydrodynamic processes in solving practical problems related to surveying of pollutants diffusion in water flows. The conducted studies consisted in the finding out conditions under which mathematical modelling using hydrodynamic equations allows to solve engineering problems of channel hydrodynamics and, in particular, to numerically simulate the transport of suspended particles in channels. A number of additional nature of numerical models were studied in addition to approximation and stability, such as averaging over probability and over time averaging. It was noted that only stationary processes could be described by equations if they are obtained from the Reynolds equations, i.e. when using the Reynolds equations, an important class of problems with a pulsating flow under constant boundary conditions is excluded from consideration. And, if the equations are obtained directly from the conservation laws, then all the desired variables have the meaning of actual quantities averaged over the scale. That is even in the case of statistically stationary flows, using such equations, it is possible to solve nonstationary problems on large time scales.

1 Introduction

Turbulence is an inherent property of the flow of liquid media. Turbulent flow regimes are inherent in currents of natural and artificial channels. Therefore, in the mathematical modelling of flows, it is necessary to take into account dissipative processes related to viscosity, thermal conductivity, diffusion of components and the corresponding processes of turbulent heat – mass transfer. Otherwise, inadequate characteristics of hydrodynamic flows can be obtained.

More than a century of research experience shows that the problem of turbulence is extremely complex and, so far, it has not been possible to obtain any simple analytical solutions for describing the processes occurring in turbulent flows. Turbulence has a

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stochastic nature and is a fundamentally three-dimensional unsteady nature and includes a wide and continuous spectrum of spatial and temporal scales [1 - 3].

In some cases, turbulence is a decisive factor determining the speed and nature of the processes, such as mixing and transfer of suspensions. In such cases, it is necessary to take particular care over the introduction of simplifications into the systems of equations since this can lead to the impossibility of applying for modelling real flows and mass transfer and which could significantly change or even do not allow to obtain a picture of flows.

2 Hydrodynamic equations

The solution of the general equations of hydrodynamics is a very complex problem, which is traditionally solved by introducing certain hypotheses. As practice shows [5 - 12], for reservoirs, the horizontal dimensions of which are much greater than the depth, this can be done by introducing a large scale to consider the phenomenon. In this case, depending on the degree of desensitization, three-dimensional equations of baroclinic liquid, two-dimensional Saint-Venant equations, one-dimensional equations, and zero-dimensional (balance) equations could be obtained.

The general equations of hydrodynamics can be written in the following way [13]:

$$\frac{\partial u_i \rho}{\partial t} + \frac{\partial u_i u_j \rho}{\partial x_j} + \frac{\partial \rho}{\partial x_i} = \frac{\partial \tau_{ij}}{\partial x_j} + g_i \rho$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial u_i \rho}{\partial x_i} = 0; \quad \frac{\partial S_r}{\partial t} + \frac{\partial S_r u_j}{\partial x_j} = q_{Sr} \tag{1}$$

$$\rho = f\left(S_r\right); \quad i = 1,2,3 \,,$$

here:
ui - projection of the current velocity vector onto the axis xi,
p - hydrodynamic pressure,
τi - component of shear stress tensor,
ρ - density,
gi - component of the gravitational acceleration vector,
qSr - internal sources of substance for Newtonian fluid.

$$\tau_{ij} = \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \nu \rho \tag{2}$$

where:
ν - kinematic coefficient of viscosity,
Sr - some substance that determines density (temperature, salinity).
In [4], it was shown that with the introduction of scale:

$$M_L = L_n^2 T \tag{3}$$

here Ln - linear scale in plan (provided that Ln >> h, where h - flow depth); $T = L_n / U$, here U – representative velocity.
and considering the case when

$$\frac{\rho_{\max} - \rho_{\min}}{\rho_{\max} + \rho_{\min}} << 1 \tag{4}$$

as well as using the turbulent viscosity hypothesis for moments

$$\tau_i = \nu_T \frac{\partial u_i}{\partial z} \tag{5}$$

here νT – is be find out with either using the von Kármán turbulence model or "k-ε" model [5 - 11], the assumption of small changes in all characteristics along the horizontal coordinate as compared with changes along the vertical one, results in hydrostatic pressure at a given scale and, thus, the following system of equations can be obtained:

$$\begin{cases} \dfrac{\partial u_i}{\partial t} + \dfrac{\partial u_i u_j}{\partial x_j} + \dfrac{\partial u_i w}{\partial z} + g\left(\dfrac{\partial z_\Gamma}{\partial x_i} + \dfrac{1}{\rho}\displaystyle\int_z^{z_\Gamma} \dfrac{\partial \rho}{\partial x_i} \partial z \right) = \dfrac{\partial}{\partial z} \nu_T \dfrac{\partial u_i}{\partial z} + \dfrac{\partial}{\partial z} \mu \dfrac{\partial u_i}{\partial z} \dfrac{1}{\rho} \\ \dfrac{\partial u_i}{\partial x_i} + \dfrac{\partial w}{\partial z} = 0 \\ \dfrac{\partial S_r}{\partial t} + \dfrac{\partial S_r u_j}{\partial x_j} + \dfrac{\partial S_r w}{\partial z} = \dfrac{\partial}{\partial z} D \dfrac{\partial S_r}{\partial z} + q_{Sz} \\ \rho = \rho(S_r) \end{cases} \tag{6}$$

Here D – is vertical diffusion coefficient (similar toν_T), it is usually assumed that D = ανT, Sr - average substance concentration in scale (3).

The same equations can be obtained from the Reynolds equations with the additional assumption of the possibility of neglecting turbulent interactions between liquid jets in the plane. Thus, two different approaches lead to the same form of equations. However, in terms of expected results, these approaches are not equivalent. Indeed, all dependent variables u_i, q_i and h in (6) have different meanings depending on whether these equations are derived from the Reynolds equations or directly from the conservation laws. In the first case, these values averaged first by probability and then on a large scale, in the second, these are actual values, averaged by the same scale.

When considering a statistically stationary flow in the case of an ergodic process, averaging over probability is equivalent to averaging over infinite time. Therefore, if we assume that equations (6) are obtained from the Reynolds equations, then in the considering case they can only be stationary. At the same time, if equations (6) are obtained directly from the conservation laws, then all dependent variables u_i, q_i and h in (6) are the current values averaged by scale. That is, even in the case of statistically stationary flows, using equations (6), it is possible to solve non-stationary problems in time scales over $L_n / U_{C.B}$, where $U_{C.B}$ - disturbance drift rate.

Thus, when using the Reynolds equations, an important class of problems with pulsating flow under constant boundary conditions is excluded from consideration.

All of the above, strictly speaking, is true for free from boundaries in terms of flow. If the flow is considered near a rough vertical wall, it is impossible to consider the hydrodynamic quantities as actual. If we consider them as averaged over probability, then in the case of pull apart flows we will come, in the same way as before, to the same dead-end result (concerning shear stress between jets). Therefore, it is proposed to consider the hydrodynamic quantities in (6) as averaged over the scale MΔ, much bigger than the roughness scale, but much smaller (3). Then τij will have a meaning similar to the Reynolds stress, where the averaging is carried out not by probability but by scale MΔ. These stresses are determined by the roughness peaks. Stresses at the bottom are also determined by the roughness of the bottom.

The modified Prandtl model [12] or a two-parameter model "k-ε" [5] is usually used to close the system of equations (6).

3 One-dimensional equations

Engineering practice of calculations [13-17] shows that for a certain class of flows there is another scale, within which phenomena can be neglected. Let us consider this class. Let there be a reservoir whose geometry satisfies the following ratio L >> B, here L – the length of the water body along the direction of prevailing flow, B - representative transverse size of the water body. In the future, such a water body will be called a waterway. Direction of the predominant flow let us take as the axis of the waterway and consider this axis straight if r >> B, here r - the radius of curvature of the axis of the waterway.

According to [15], one-dimensional equations for describing flows in such waterways can be written in the following form:

$$\oint_{\sigma_{xt}} Q\partial x + \left(QU + P\right)dt = \iint_{W_{xt}} \left(-g\omega i_{TP} + g\omega i + J + F\right)dxdt$$

$$\int_{\sigma_{xt}} \omega dx + Qdt = \iint_{W_{xt}} qdxdt$$

$$P = \int_{b_s} g\frac{\left(z_S - z_b\right)^2}{2}dy \tag{7}$$

$$\omega = \int_{b_s} \left(z_S - z_b\right)dy$$

Here:

y - transverse coordinate; ω - flow cross-sectional area; Q - water flow through the entire flow cross section; $U = Q/\omega$ - cross sectional flow average velocity; friction slope:

$$i_{TP} = \lambda\frac{U|U|}{2gR} \tag{8}$$

$R = \omega/\chi$ (χ - wetted perimeter); i - average bottom slope; J - specific impulse supplied to the area along with lateral flow rate; q - lateral flow rate; F- force associated with deviation from prismatic channel of the waterway:

$$F = g\int_0^{\chi}\left(z_S - z_b\right)\left(\frac{\partial y_b}{\partial x}\sin\gamma - \frac{\partial z_b}{\partial x}\cos\gamma\right)dl \tag{9}$$

y_b - transverse coordinate of the bottom surface; γ - the angle between the tangent to the bottom line in the plane x = const and the axis OY in plane YZ, normal to the plane of the averaged bottom surface.

These equations are widely used in practice. [13 - 20].

4 Consideration of deformations

Equations (7) are written for the conditions of bottoms, banks, could not be deformed, banks, and slopes. For the case of a wide rectangular channel with a smooth changes in the width of the cross sections problems can be solved toking into account deformations. In this case, the equations are as follows:

$$\begin{aligned}
&\frac{\partial q}{\partial t}+\frac{\partial uq}{\partial x}+gh\frac{\partial z_s}{\partial x}=ghJ-\lambda\frac{u^2}{2};\\
&\frac{\partial q}{\partial x}+\frac{\partial z_s}{\partial t}=0;\\
&S_B\frac{\partial z_b}{\partial t}+\frac{\partial Sq}{\partial x}+\frac{\partial Sh}{\partial t}=0;\\
&\frac{\partial Sq}{\partial x}+\frac{\partial Sh}{\partial t}=K\left(S_H-S\right)
\end{aligned} \tag{10}$$

Having a turbidity distribution along the length of the channel (for example, from experimental data) can find numerical values of “K” could be found from:

$$\frac{\partial SH}{\partial t}+\frac{\partial Sq}{\partial x}=K\left(S_H-S\right) \tag{11}$$

Experimental studies [21] showed that, under certain conditions of formation of the original moving bottom, the first member of the equation could be neglected. Moreover, under conditions of erosion with water without sediments, it is in order of magnitude smaller than other members. Then with a constant flow rate:

$$K=\frac{q\dfrac{dS}{dx}}{S_H-S} \tag{12}$$

Data from [4, 22] were processed to identify the degree of influence of suspended solids in streams on the numerical value of "K". According to [4] the settling velocity (hydraulic size) does not have significant effect on the value of $\frac{K}{U_*}$. On the contrary, the presence of suspended solids reduce its values the stronger the more of suspended solids are being transferred by the stream.

In [4], it was shown that for a zone of flow descent from a berm, the value can be determined in the same way. In the conditions of turbidity-free water flow, the maximum value obtained as the result of experimental and field studies data processing is $\left(\frac{K}{U_*}\right)_{\max}=1{,}26$, and in the conditions of the formed equilibrium flow is $\left(\frac{K}{U_*}\right)_{\max}=0{,}81$.

Thus, by dividing flows into two zones: zone of local erosion and zone of common deformations, using the results of data processing, it is possible to calculate parameters of flows in easily deformed sandy channels.

5 Conclusions

Until today, there is no simple analytical solutions to describe processes in turbulent flows. This is due to the stochastic nature of turbulence flows, which is a process that has a three-dimensional nonstationary nature and includes a wide and continuous spectrum of spatial and temporal scales.

Turbulence in solving engineering problems of channel hydrodynamics is the decisive factor determining the speed and nature of the processes such as mixing and transfer of suspensions.

Long-term studies allowed to formulate certain hypotheses regarding the scope of consideration of phenomena, the application of which permitted to obtain three-dimensional equations of baroclinic fluid, two-dimensional Saint-Venant equations, one-dimensional equations and zero-dimensional equations, each of which has its own specific field of application..

Averaging over probability is equivalent to averaging over infinite time while considering of statistically stationary flows in the case of an ergodic process. Therefore, if the hydrodynamic equations are obtained from the Reynolds equations then stationary processes only can be described by these equations, The use of the Reynolds equations leads to the exclusion from consideration of an important class of problems, i.e. problems of pulsating flows under constant boundary conditions. If the equations are obtained directly from the conservation laws, then all the required variables have the meaning of actual quantities, averaged over the consideration scale. That is, even in the case of statistically stationary flows, using such equations, it is possible to solve nonstationary problems on large time scales.

The results of processing the turbidity distribution along the length of the channel conducted to identify the degree of its influence on the deformation processes allows to calculate parameters of flows in easily deformed sandy channels.

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