

Monthly model of hot water consumption

Agnieszka Chmielewska^{1,*}

¹Wrocław University of Science and Technology, Faculty of Environmental Engineering,
Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland

Abstract. The article presents a statistical analysis of the monthly consumption of hot water in apartment buildings. On this basis, a monthly model of hot water consumption in multi-family buildings was proposed. This model consists of two sub models: model of average daily consumption of domestic hot water and a model describing changes in consumption of hot water in individual months of the year. To construct the model of average daily consumption of hot water, data on the consumption of hot water was used out of 16 multi-family buildings from the period of two years. The data was supplemented with information on the housing structure in the building. The study of changes in hot water consumption in the following months was based on data obtained from 30 buildings. Statistical analysis showed a strong correlation between the monthly consumption of hot water, the outside temperature and holiday period.

1 Introduction

Modelling the consumption of hot water is complicated. The consumption of hot water in residential buildings depends on sociological and technical factors, which are often impossible to determine. In residential buildings the consumption of hot water depends on: the size of flats, sanitary equipment for apartments, the method of accounting for water consumption, the size and demographic structure of the family, the age of residents, the way of spending free time, lifestyle and lifestyle, the attractiveness of TV programs, etc.. Intake of domestic hot water in residential building is a random volume and is characterised by large variability, both within a period of 24 hours and on particular days of the week. Due to the noticeable need for performing detailed simulations of operation of systems preparing domestic hot water, there is a need for developing profiles of warm water consumption at various time stages. Knowing the monthly water consumption profile seems to be necessary, as it is used for various types of power simulations or selection of solar collectors. Unfortunately, a model that will allow to measure the amount of monthly collection of domestic hot water does not exist in Polish publications, thus this amount is often assumed at a fixed level during the entire year. Whereas in foreign publications, it is possible to find numerous elaborations describing the monthly distribution of domestic hot water consumption. Georg et al. (2015) [1], for instance, presented changes of seasonal consumption of domestic hot water in their elaboration, that indicate a clear drop in the use of domestic hot water in summer. Such changes are explained by the authors, who state that

* Corresponding author: agnieszka.chmielewska@pwr.edu.pl

this is due to higher outdoor temperatures in summer, which affects bathing preferences of users and temperature of water from the mains, used for bathing. Ahmed, Pylsy, Kurnitski (2015) [2] noted, based on measurements of 182 Finnish flats, that unit consumption of domestic hot water in summer amounts to 40 l per day and 46 dm³ per day, in winter. Studies conducted by Mayer (2000) [3] based on hourly measurements from 120 flats in South Africa (including 30 traditional houses, 30 downtown houses and 30 apartments), indicated that the unit hourly consumption of domestic hot water depends on the building type, population density and month of the year. In order to conduct analyses, each group contained 10 facilities with low, average and high population density. In measurements taken by the authors, a reverse trend can be observed as compared to the previous studies (high temperatures in January and February, low temperatures in July and August), which is associated with the climate in South Africa. In such countries as Germany [4], Italy [5], Denmark [6], or Great Britain [7], measurements regarding the demand for domestic power for preparing hot water, are taken monthly. It should be noted that the intake of domestic hot water in the presented studies is closely associated with the usable area of multi-family buildings and the number of days in a given month. In Great Britain [7], the authors additionally introduced reduction indicators of hot water consumption in particular months, besides monthly intake of domestic hot water, taking into account lower intake of hot water in summer.

2 Research method

2.1 Measurement data

Measurements of hot water consumption were made in 42 multi-family buildings located in Wrocław and Zawidów. There was a total of 1376 apartments. Each of the units (the apartments) has been equipped with a residential water meter for measuring the cold and hot water consumption. The model of daily consumption of domestic hot water was made based on the measurement data from 16 multi-family buildings (773 apartments) located in Wrocław. In addition, the analyses were supported by the data obtained from the estate manager. Comprehensive data about the apartments' surface and the annual consumption of hot water came from 626 apartments. The study covered the period from January 2012 to December 2013. Analysis of change in the deviation of daily consumption of domestic hot water in subsequent months, as compared to the average annual value was made on the basis of data from 30 multi-family buildings. The data was gathered every months. After rejecting incomplete and incorrect data, measurements from 585 apartments were used for the analysis. The study covered the period from January 2012 to December 2013.

2.2 Analysis of the measured data

At the beginning, the consumption of domestic hot water in months in each apartment was calculated. For each of the apartments the monthly consumption of the domestic hot water was calculated ($V_{M,DHWi}$), as a difference between the measured values at the beginning and at end of the month. Then, on the basis of equation 1, the deviation of the average daily consumption of hot water in the month in relation to the average daily consumption for each apartment was calculated.

$$S_{M_i} = \frac{\frac{V_{M,DHW_i}}{l_{d,m_i}}}{\frac{1}{12} \sum_{i=1}^{12} \left(\frac{V_{M,DHW_i}}{l_{d,M_i}} \right)} \quad (1)$$

The values calculated according to equation 1 were subjected to statistical analysis. It consisted in examining the significance of differences between the average daily consumption of hot water in the months for the whole year. These analyses showed the desirability of developing a model of hot water consumption in each month of the year.

The publication contains an analysis of the variability of monthly use of domestic hot water within a year. Fig. 1 presents monthly variability and changes of the deviation of domestic hot water consumption in particular buildings, as compared to the average annual value. The diagram was based on medians to eliminate the impact of the deviating values on the results of conclusions. The results presented were compared with data from publications. A model developed in a British standard, i.e. SAP 2012 [7] presents the general trend of monthly changes of actual domestic hot water consumption. However, it does not take into account the non-variable consumption of domestic hot water in the period from January to March, and reduced use of domestic hot water in August. The above discrepancies may be caused by varying school calendar arrangement or the climate itself that affect preferences of users of domestic hot water. In the case of analyses conducted with data regarding consumption of domestic hot water in a Canadian community, i.e. Solar City [1], significantly better representation of the collected data is visible but differences in the holiday season and in February are noticeable.

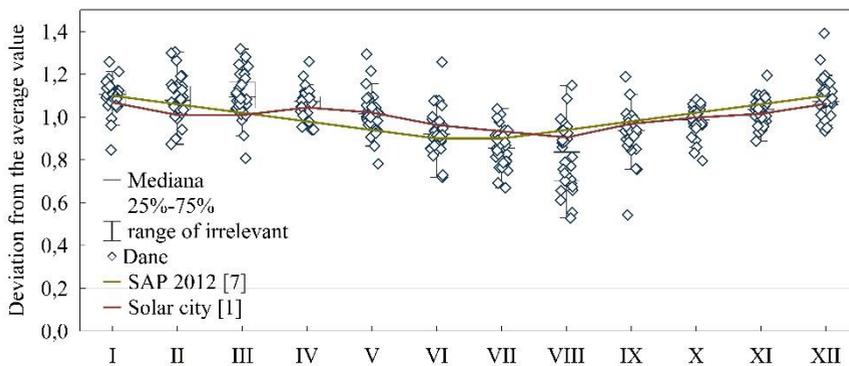


Fig. 1. Change in the deviation of daily consumption of domestic hot water in subsequent months as compared to the average annual value.

Next, the measurements were subjected to a statistical analysis, consisting of verifying whether differences in the amount of daily consumption of domestic hot water in particular months are statistically important. To assess the varied intake of water, the Kruskal-Wallis test was applied which is based on observation ranks. Because the calculated values of p test probability is 0.00, the hypothesis about equal consumption of domestic hot water in particular months can be rejected. It is also stated that there are at least two months when this value varies. Then, a test of multiple comparisons of average ranks for all samples was conducted, based on which it may be stated that there are statistical differences between January, February and March, and months from July to October, among others.

Fig. 2 and Fig. 3 present the deviation in the monthly use of domestic hot water from the average annual value, calculated on the basis of measurement of monthly consumption separately for buildings located in Wrocław and Zawidów. In both of the analysed locations, water consumption reduces in the summer period but this reduction is significantly more noticeable in Wrocław. This may be associated with a bigger number of people going to large cities during the summer holiday, or students leaving Wrocław for the same reason.

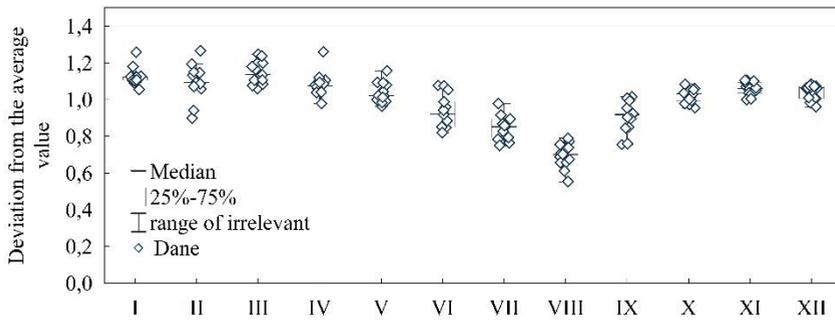


Fig. 2. A change in the deviation of daily consumption of domestic hot water in subsequent months as compared to the average annual value in Wrocław.

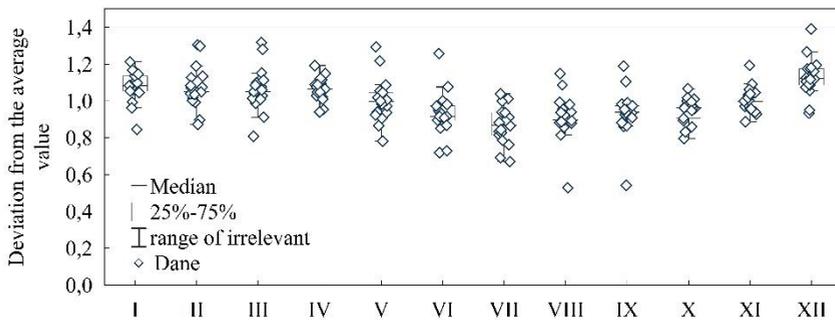


Fig. 3. A change in the deviation of daily consumption of domestic hot water in subsequent months as compared to the average annual value in Zawidów.

3 Monthly model of domestic hot water consumption

Two alternative methods were proposed for developing a model for describing the monthly consumption of domestic hot water. The difference between them consists of choosing independent variables being the basis of measurement and different approach to the method of analysing measurement data. First model ($s_{Mi}^{v.1}$) was developed using the method of multiple regression. The model enables to estimate changes in the deviation of monthly consumption of domestic hot water from the average annual value. Variables for this model were chosen using Spearman's rank correlation coefficients. In the second model ($s_{Mi}^{v.2}$) it was assumed that water consumption is related only to a given month in a year. This assumption enabled to use data regarding domestic hot water consumption from particular flats to expand the database regarding consumption of water in buildings. On this basis, a model was developed describing the changes in the deviation of monthly consumption of domestic hot water from the average annual value. The bootstrap method was used for expanding the database. Initially, the database contained data regarding consumption of domestic hot water in subsequent months for 583 flats located in 30 buildings. The data enabled to draw 1000 new configurations of buildings with various number of flats. The proposed model of monthly domestic hot water consumption was presented in equation 2. It was based on deviation of the average daily consumption of domestic hot water in particular months (s_{Mi}) from the average annual value (V_{srD}) and number of days in a given month ($l_{d,Mi}$).

$$V_{Mi} = s_{Mi} \cdot V_{srD} \cdot l_{d,Mi} \quad (2)$$

3.1 Average daily consumption of the domestic hot water in multi-family buildings

In the [8], it was evidenced that when indicating the average daily domestic hot water consumption in multi-family buildings, it is worth basing one's elaborations on the residential structure, as this enables to obtain significantly better quality of estimates. The relation described in [8] and [9] was presented in equation 3. It was based on surface of flats with a certain number of rooms.

$$V_{sRD} = 0,0021 \cdot A_{M1} + 0,0018 \cdot A_{M2} + 0,0017 \cdot A_{M3} + 0,011 \cdot A_{M4} + 0,011 \text{ [m}^3/\text{doba]} \quad (3)$$

where:

A_{M1} – area one-room apartments [m²], A_{M2} – area two-room apartments [m²], A_{M3} – area tree-room apartments [m²], A_{M4} – area four-room apartments [m²].

Because information about the area of flats may be difficult to obtain, in this publication, the author proposed simplification of the model of average daily domestic hot water consumption. The proposed model based on the number of flats with a certain number of rooms was presented in equation 4.

$$V_{sRD} = 0,063 \cdot n_{M1} + 0,084 \cdot n_{M2} + 0,106 \cdot (n_{M3} + n_{M4}) + 0,015 \text{ [m}^3/\text{doba]} \quad (4)$$

where:

n_{M1} – number of one-room apartments [m²], n_{M2} – number of two-room apartments [m²], n_{M3} – number of tree-room apartments [m²], n_{M4} – number of four-room apartments [m²].

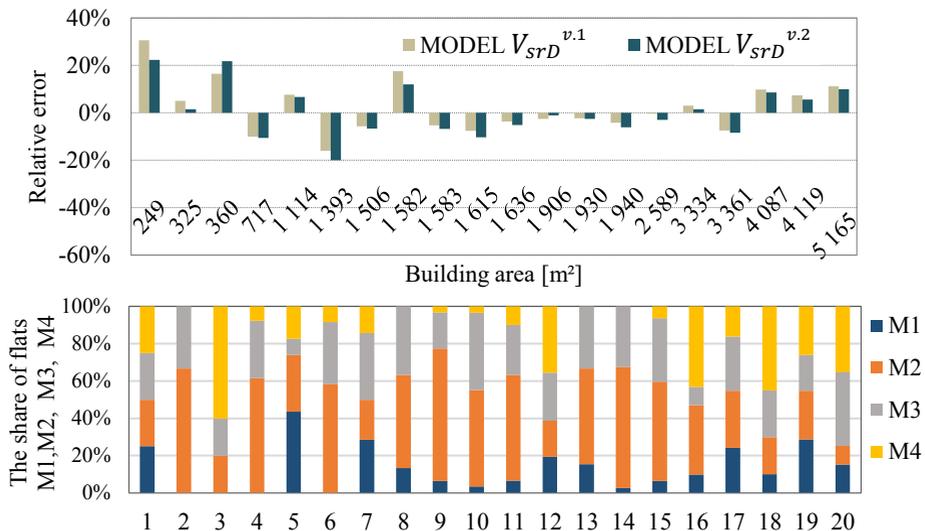


Fig. 4. Relative error of fitting the models to the actual measurement data from buildings with different housing structure.

Fig. 4. presents the relative errors of estimating daily consumption of domestic hot water indicated according to equation 5 as well as the structure of flats and area of buildings. For verifying the model, data obtained by means of the bootstrap method was used, which enabled to draw buildings with non-typical structure from measurement of 626 flats (database A). Both proposed models describe the daily consumption of domestic hot water relatively well. The biggest error can be observed in building 1 characterised by equal arrangement of flats with varying size. The smallest error in the case of both models occurs in buildings 12,

13, 15 and 16. Average unconditional error in estimating in all buildings chosen at random in model $V_{srD}^{v.1}$ amounts to 8.65%, while in model $V_{srD}^{v.2}$ 8.5%.

$$\delta_{V_{srD}} = \frac{V_{srD} - V_{srD,rzecz}}{V_{srD,rzecz}} \quad (5)$$

The conducted analyses indicated that using both of the proposed models, it is possible to indicate daily domestic hot water consumption correctly. The following model was adopted for further analyses and as the final one: $V_{srD}^{v.2}$ based on the number of n-room flats. It is easier to use as it doesn't require information about the area of n-room flats, but merely the number of such flats in a building.

3.2 Model deviation of the average daily consumption of domestic hot water in particular months from the average annual value

In the construction of calculation models, it is a big problem to determine the number of independent variables (explanatory) that should be included in the model. After analysing the measurement data and literature, it was decided that the explanatory variables to be considered in the model are: share of days-off from school in a month, share of Sundays and public holidays in a month, share of holiday days in a month, share of Fridays and Sundays in a month and outside temperature. To test the link between these variables and the consumption of hot water, Sperman's rank correlation was used. The analyses showed a strong correlation between the deviation of the average monthly consumption of hot water from the annual average value and the outside temperature and the share of holiday days. These analyses also showed that the remaining variables do not affect the monthly consumption of hot water. The temperature of the outside air assumed for modelling was transformed according to the equation 6, where $T_{zewn,srM}$ is the average monthly temperature during the year, and T_{Mi} is the average monthly temperature in a particular month.

$$T_{zewn.skor.} = 1 + 0,2 \cdot \frac{T_{zewn,srM} - T_{Mi}}{T_{zewn,srM}} \quad (6)$$

The share of individual days in the month was calculated using the equation 7, where l_w is the number of holiday days and $l_{d,Mi}$ is the number of all days in the month.

$$u = \frac{l_w}{l_{d,Mi}} \quad (7)$$

The multiple regression analysis performed using Statistica package estimated the model parameters. The final model ($s_{Mi}^{v.1}$) is described by the equation 8. The coefficient of determination of the model (R^2) is 0.64.

$$s_{Mi}^{v.1} = 0,363 \cdot T_{zewn.skor.} - 0,184 \cdot u + 0,671 \quad (8)$$

In the second proposed model ($s_{Mi}^{v.2}$) to improve the quality of parameter estimation the bootstrap method was used. The bootstrap method allowed to create 1000 new buildings. The created database consists of four groups of 250 buildings, characterized by the number of apartments. The created base consists of four groups of 250 buildings with the number of apartments. In the first group there were buildings with the number of flats from 4 to 10, in the second group there were buildings with the number of flats from 10 to 30, in the third group there were buildings with the number of flats from 30 to 50, and in the fourth group there were buildings with a number apartments from 50 to 70. In each group, there was a visible relationship between the size of the building and the scatter of the data received. This means that in larger buildings we can describe the monthly distribution of hot water consumption with greater accuracy. After analysing the obtained values of medians and averages in particular groups, it was found that their values differ by a maximum of 1%. All

four groups of buildings were used to build the model. On their basis, the average deviation of hot water consumption in individual months was calculated. The structure of the model is presented in table 1.

Table 1. Model ($s_{Mi}^{v.2}$) of the monthly deviation of DHW consumption from the annual average value.

Month	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Model $s_{Mi}^{v.2}$	1.10	1.09	1.11	1.07	1.01	0.92	0.88	0.77	0.93	1.00	1.04	1.07
Trust -95%	1.10	1.09	1.11	1.07	1.01	0.91	0.87	0.77	0.93	1.00	1.04	1.07
Trust +95%	1.10	1.09	1.11	1.07	1.01	0.92	0.88	0.78	0.94	1.01	1.05	1.07

3.3 Verification of developed models

Next, a comparison was made between both proposed models and a profile characterized by a constant distribution of DHW consumption in all months. Fig. 5 shows the absolute error of fitting the $s_{Mi}^{v.1}$ model and the $s_{Mi}^{v.2}$ model in individual months to the actual measurement data. The measurement data were from 6 multi-family buildings. The analyses covered the period from January 2012 to December 2013.

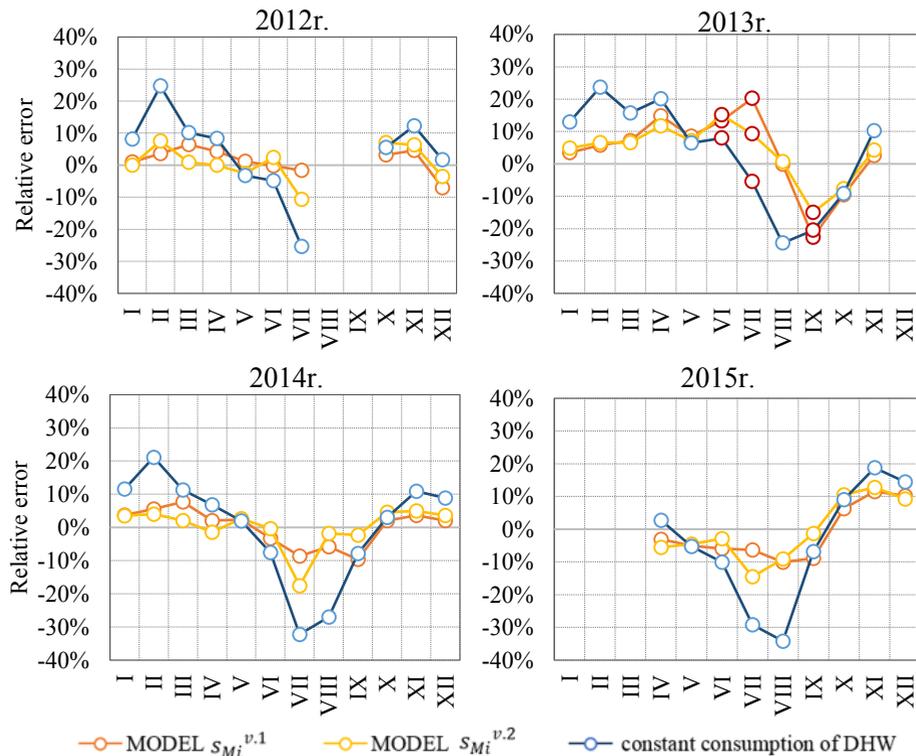


Fig. 5. Absolute error of deviation of the average daily consumption of hot water from the average annual value based on the model $s_{Mi}^{v.1}$, $s_{Mi}^{v.2}$ and constant consumption of hot water.

In 2013, in June, July and September, there were failures in the boiler room, which disturbed the standard distribution of DHW consumption. These months have been excluded from the analysis. The graphs in 2012, 2014 and 2015 show a trend indicating the largest mismatch of all models for measurement data in the summer period, in July and August. Relative error in this period based on the $s_{Mi}^{v.1}$ model is not greater than 10% (August 2015).

In the $s_{Mi}^{v.2}$ model the biggest error at 18% is in July 2014. The constant profile of hot water consumption has an error of 34% in August 2015. Analyses showed that the $s_{Mi}^{v.1}$

model best describes the distribution of water consumption during the holiday season. On the other hand, the $s_{Mi}^{v.2}$ model gives better results for the months like March, April and May. The adoption of a fixed profile of hot water consumption is the worst in describing the distribution of hot water consumption during the year.

$$\sigma_{s_{mi}} = s_{mi} - s_{mi,rzecz} \quad (9)$$

Table 2 presents the analysis of the mean absolute error determined based on the equation 9. The analysis of both developed models were carried out in 2012–2015. The smaller average error of estimation from the period of four years showed the $s_{Mi}^{v.1}$. This error is 5.3%. The $s_{Mi}^{v.2}$ model error is only 0.1% larger than the $s_{Mi}^{v.1}$ model and its average error is 5.4%. The model $s_{Mi}^{v.2}$ is only 0.1% less than the $s_{Mi}^{v.1}$ model and its average error is 5.4%. Unfortunately, to use the $s_{Mi}^{v.1}$ model, you need to know the average monthly outdoor temperature and the school calendar. The second model ($s_{Mi}^{v.2}$) is easier to use because the additional information is not required.

Table 2. The mean error for the models $s_{Mi}^{v.1}$ and $s_{Mi}^{v.2}$.

Error in year:	2012	2013	2014	2015	average value of the error
Model $s_{Mi}^{v.1}$	3%	7%	3%	6%	5.3%
Model $s_{Mi}^{v.2}$	4%	6%	4%	8%	5.4%

4 Conclusions

In this study, the authors demonstrated the ability to describe the consumption of hot water based on the number of flats with a given number of rooms in the flat. The study also showed that when calculating the monthly consumption of hot water, it is necessary to apply the reduction coefficients in the summer and the growth coefficients in winter. It has also been shown that the use of a simple $s_{Mi}^{v.2}$ model based on correction factors in individual months gives similar results as in the case of using the $s_{Mi}^{v.1}$ model. However, the use of the $s_{Mi}^{v.1}$ model is more troublesome as it requires knowledge of the calendar of holiday days and outside temperature.

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