Vibration model as a system of coupled oscillators in a direct current electric motor

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Abstract. This article describes a vibration model connected by individual nodes of a DC motor. The purpose of the article is a mathematical model allowing to predict the work of machines with a depreciation resource and search for ways to increase the information content of assessing the functioning of the rolling stock condition without dismantling them. Currently, the operational reliability of the DC motor is reduced after the development of the service life. Study and evaluation of the vibration effect on the brush-collector node. The main components of the DC motor (bearings, electrical brushes) are unrecoverable. Therefore, any interference with their normal operation leads to premature wear of the entire DC motor. Existing methods for life-extending maintain the operational reliability of a DC motor equal or close to the passport data for several years. This model allows vibration diagnostics without dismantling the DC motor and stopping the rolling stock. The mathematical model describes the oscillations of each node of the DC motor in the direction of movement of the rolling stock, showing the excess of the permissible value. After mathematical processing of the results, a machine reliability model under study is obtained (usually in the form of polynomials) as a function of the needed parameters – the influencing factors. Due to this model, pre-detected defects can save on repairs and maintenance in the future. This model is the basis of software for online diagnostics. The proposed model allows providing an objective use of repair and restoration compositions depending on the current technical condition, which will lead to an increase in the service life of the mechanism and savings in operating costs. On the one hand, extending the operating life of a DC motor should produce an economic effect, and on the other hand, technical measures to maintain operational reliability cause an increase in material costs.

1 Introduction

The model of vibrations can be represented as a system of connected nodes of which a direct current motor (DC motor) consists, we will call it oscillators. Due to the individual motor elements' masses, stiffness and damping factors for these elements, such as mechanical transverse excitation of the rotating armature, collector, fan, bearings, vibrations of brush holder, poles and column (for example, due to the movement of the object with DC motor) (Fig. 1), in modeling, we can observe the emergence of complex structures that can be described as a superposition of harmonic oscillations called normal oscillations [1-10].

2 Methods

Assuming that the displacements during vibrations are directed only vertical, we will have a one-dimensional system of five “particles” with masses $m_i (i=1, \dots, 5)$ connected by springs. We assume that the mass of springs is zero, the stiffness coefficients (or elasticities, or force constants) are equal to k_i , the damping coefficients are c_i .

Fig. 1. Model of motor vibration.

Let us denote x_i as i mass displacement from the balance position along the displacement axis. The boundary point x_5 (the base to which the column is attached) will be assumed fixed, which is expressed by the boundary condition $x_5=0$. And the displacement of

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$$\left\{ \begin{aligned} m_1 \cdot d_t^2 x_1 &= \frac{k_1 k_2 k_3 k_4}{k_1 k_2 + k_2 k_3 + k_2 k_4 + k_4 k_1} (x_1 - x_2 - x_3) - \frac{c_1 c_2 c_3 c_4}{c_1 c_2 + c_2 c_3 + c_2 c_4 + c_4 c_1} (\partial_t x_1 - \partial_t x_2 - \partial_t x_3) + F_1; \\ m_2 \cdot d_t^2 x_2 &= \frac{k_4 k_6}{k_4 + k_6} (x_1 - x_2) - \frac{c_4 c_6}{c_4 + c_6} (\partial_t x_2 - \partial_t x_3) + \frac{k_1 k_2 k_3 k_4}{k_1 k_2 + k_2 k_3 + k_2 k_4 + k_4 k_1} (x_1 - x_2 - x_3) - \\ &\quad - \frac{c_1 c_2 c_3 c_4}{c_1 c_2 + c_2 c_3 + c_2 c_4 + c_4 c_1} (\partial_t x_1 - \partial_t x_2 - \partial_t x_3); \\ m_3 \cdot d_t^2 x_3 &= \frac{k_3 k_5}{k_3 + k_5} (x_1 - x_3) - \frac{c_3 c_5}{c_3 + c_5} (\partial_t x_1 - \partial_t x_3) + \frac{k_4 k_6}{k_4 + k_6} (x_1 - x_2) - \frac{c_4 c_6}{c_4 + c_6} (\partial_t x_2 - \partial_t x_3) + \\ &\quad + \frac{k_1 k_2 k_3 k_4}{k_1 k_2 + k_2 k_3 + k_2 k_4 + k_4 k_1} (x_1 - x_2 - x_3) - \frac{c_1 c_2 c_3 c_4}{c_1 c_2 + c_2 c_3 + c_2 c_4 + c_4 c_1} (\partial_t x_1 - \partial_t x_2 - \partial_t x_3); \\ m_4 \cdot d_t^2 x_4 &= -k_9 (x_4 - x_1) - c_9 (\partial_t x_4 - \partial_t x_1) + \frac{k_3 k_4}{k_3 + k_4} (x_4 - x_2 - x_3) + \frac{c_3 c_4}{c_3 + c_4} (\partial_t x_4 - \partial_t x_2 - \partial_t x_3); \\ m_5 \cdot d_t^2 x_5 &= k_9 (x_4 - x_5) + c_9 (\partial_t x_4 - \partial_t x_5) - \frac{k_7 k_8}{k_7 + k_8} x_4 + \frac{c_7 c_8}{c_7 + c_8} \partial_t x_4 + F_2 \end{aligned} \right. \quad (4)$$

the column, for example, when an object moves on an unequal surface, will be set by affecting the external force F_2 of one or another kind. The force F_1 describes the transverse mechanical excitations of the shaft, armature, collector, bearings and fan in rotating. [11]

The irregularity of the collector should not exceed 0.05 mm; a sinusoidal profile negatively affects the brush-collector node. The future model should include a model of the irregularities of the form.

$$Z_4(t) = A \sin(\omega t) \leq 0.05 \sin(\omega t) \quad (1)$$

The oscillations behavior of a multi-mass system with elastic constraints will be described by classical mechanics equations. Newton's laws must be used:

$$F_1 = -F_2, F = ma = m dv(t)/dt \quad (2)$$

Hooke Law is

$$F = -kZ(t) \quad (3)$$

Since the force acting on the i mass is determined only by compression and tension of the springs connecting with it, the motion equations for each of the masses will have the form (4). Where $k_c = 0$, the system (1) divides into independent equations and the motion of each mass does not depend on its nodes.

The motion equation (1) describes the longitudinal oscillations, i.e. movement along the system directed from left to right. We can also show that similar equations describe the transverse oscillations of N coupled point masses.

The values of the displacements $x_1 = x_2 = x_3 = x_4 = x_5 = 0$ correspond to the system balance.

Upper frequency is equal to

$$\omega_1 = \sqrt{(k + 2k_c)/m} \quad (5)$$

In this mode, both particles oscillate in antiphase (the displacements are directed in opposite directions).

Movement on the lower frequency

$$\omega_2 = (k/m)^{1/2} \quad (6)$$

corresponds to the oscillations of both particles in phase. Noting that in (2), the motion does not depend on k_c .

The general motion of such a system is a superposition of both normal oscillations. [12]

1. If ω_1 and ω_2 are not connected by a simple relation, the resulting displacement is a complex function in time.

2. If the connection is weak, then $\omega_1 \approx \omega_2$ and then the beating are observed in the oscillations of $v_f(t)$ and $v_i(t)$.

In this case, the elements of the nodes quickly oscillate with an angular frequency

$$(\omega_1 + \omega_2)/2 \quad (7)$$

and amplitude varying sinusoidally with a beating frequency

$$(\omega_1 - \omega_2)/2 \quad (8)$$

The frequency of normal system vibrations consisting of n elements with masses m is determined by the formula:

$$\omega_n = 2 \sqrt{\frac{k}{m}} \sin \frac{n\pi}{2(N+1)} \quad (9)$$

where

$n = 1, \dots, N$ is the oscillation number.

To solve the equation system (1), it is necessary to specify the initial positions of the masses and their initial

$$\begin{cases}
 d_t x_1 = v_1 \\
 m_1 \cdot v_1 = \frac{k_I k_2 k_3 k_4}{k_I k_2 + k_2 k_3 + k_2 k_4 + k_4 k_I} (x_1 - x_2 - x_3) - \frac{c_I c_2 c_3 c_4}{c_I c_2 + c_2 c_3 + c_2 c_4 + c_4 c_I} (v_1 - v_2 - v_3) + F_1; \\
 d_t x_2 = v_2 \\
 m_2 \cdot v_2 = \frac{k_4 k_6}{k_4 + k_6} (x_1 - x_2) - \frac{c_4 c_6}{c_4 + c_6} (v_2 - v_3) + \frac{k_I k_2 k_3 k_4}{k_I k_2 + k_2 k_3 + k_2 k_4 + k_4 k_I} (x_1 - x_2 - x_3) - \\
 - \frac{c_I c_2 c_3 c_4}{c_I c_2 + c_2 c_3 + c_2 c_4 + c_4 c_I} (v_1 - v_2 - v_3); \\
 d_t x_3 = v_3 \\
 m_3 \cdot v_3 = \frac{k_3 k_5}{k_3 + k_5} (x_1 - x_3) - \frac{c_3 c_5}{c_3 + c_5} (v_1 - v_3) + \frac{k_4 k_6}{k_4 + k_6} (x_1 - x_2) - \frac{c_4 c_6}{c_4 + c_6} (v_2 - v_3) + \\
 + \frac{k_I k_2 k_3 k_4}{k_I k_2 + k_2 k_3 + k_2 k_4 + k_4 k_I} (x_1 - x_2 - x_3) - \frac{c_I c_2 c_3 c_4}{c_I c_2 + c_2 c_3 + c_2 c_4 + c_4 c_I} (v_1 - v_2 - v_3); \\
 d_t x_4 = v_4 \\
 m_4 \cdot Z_4(t) \cdot v_4 = -k_9 (x_4 - x_1) - c_9 (v_4 - v_1) + \frac{k_3 k_4}{k_3 + k_4} (x_4 - x_2 - x_3) + \frac{c_3 c_4}{c_3 + c_4} (v_4 - v_2 - v_3); \\
 d_t x_5 = v_5 \\
 m_5 \cdot v_5 = k_9 (x_4 - x_5) + c_9 (v_4 - v_5) - \frac{k_7 k_8}{k_7 + k_8} x_4 + \frac{c_7 c_8}{c_7 + c_8} \partial_t x_4 + F_2
 \end{cases} \quad (13)$$

displacement rates, i.e. formulate the initial conditions of the Cauchy problem:

$$\begin{aligned}
 &x_1(0), x_2(0), x_3(0), x_4(0), x_5(0) \\
 &v_1(0), v_2(0), v_3(0), v_4(0), v_5(0)
 \end{aligned} \quad (10)$$

where $v_i(0) = \partial_t x_i|_{t=0}$, and also determine the forces impacting the system. The force acting on the motor armature, in the simplest case, can be represented as harmonic:

$$F_1 = F_{1,0} \cos(\omega t + \varphi) \quad (11)$$

As for the impact on the motor column, for example, when the whole system moves on an unequal surface, then, in general, since the distribution of inequality is random, the amplitude and frequency of external influence on the column described by F_2 force are random variables. The impact force can be specified, for example, as a random Gaussian noise:

$$F_2 = F_{2,0}(t) \cos[\omega(t) + \varphi] \quad (12)$$

where the time-amplitude and frequency will be generated using random number sensors.

Thus, by equalities (4-12), we completely determined the model describing the associated vibrational oscillations of individual motor components, and hence the motor as a whole. In the general case, we cannot analytically solve the constructed model, including a system of non-uniform second-order differential equations, describing the oscillations of a coupled

system of elements, taking into account damping and external influences, one of which is random. Individual parts of the solution, for example, in the case of free oscillations in the absence of external influences, can be studied analytically, of course. But the solution to the problem we have set requires the use of statistical approaches to studying the vibration characteristics of such a complex system as a motor. And now the motor model is fully defined, we can begin consideration of the vibration simulation algorithm [13]

Due to irregularities on the side of the collector, the beating frequency lies in a wide range from 750 to 850 Hz, from 150 to 500 Hz – developing bearing defects, from 200 to 450 Hz – defects in the motor stator and at the poles.

3 Simulation algorithm results

In accordance with the adopted laws and the calculated kinematic equivalent circuit, we can record the balance of mechanical forces for each element separately.

For simulating the dynamic behavior of five coupled masses, a system of five second-order differential equations is first described as a system of first-order differential equations using the natural notation $dx_i = v_i$ (13).

The Euler-Cromer algorithm is used in calculating the displacements and velocities of the associated masses m_i ($i=1, \dots, 5$). For each pair of system equations (5) it will look as:

$$\begin{aligned} v_i^{n+1} &= v_i^n + a_i^{n+1} \Delta t, \\ x_i^{n+1} &= x_i^n + v_i^{n+1} \Delta t, \\ t^n &= t^0 + n \Delta t, i=1, \dots, 5 \end{aligned} \quad (14)$$

where superscripts correspond to the step number in t , a_i^{n+1} is the right-hand side of the even equation of system (5) for the i mass divided by this mass with superscripts n for x and v .

Since the Cauchy problem was initially posed and we identified second-order differential equations, we must also set initial conditions:

$$x_i|_{t=0} = x_i^0, \quad v_i|_{t=0} = v_i^0; \quad i=1, \dots, 5 \quad (15)$$

The Euler-Cromer Method has the second-order of approximation apropos the integration step Δt , i.e. $O(\Delta t^2)$.

According to the information, the system friction is neglected, indicating a significant difference in the coercive power (from the collector) and free power (in the mass of nodes). The element frictional forces are included in the model. But the determining factor is the extrinsic influence of the collector and the mechanical properties of the elements. [14-16]

Algorithm (13) is easily implemented by software (Fig. 2).

Fig. 2. Mathematical software model description.

As a result of numerous experiments, we established the correlation presence between the operating time of the collector node y and the beating of the collector x :

$$(y - \bar{y}) = r \frac{\sigma_y}{\sigma_x} (x - \bar{x}) \quad (16)$$

where $\bar{y}, \bar{x}$ - are arithmetic average values of the parameters of the beating and the operating time of the collector node; σ_x, σ_y are standard parameter deviations of the beating and the operating time of the collector node; r is the correlation coefficient.

The specified parameters and deviations are determined by the formulas:

$$\bar{x} = A_x + h_x \frac{\sum m_x \Delta x}{N} \quad (17)$$

$$\bar{y} = A_y + h_y \frac{\sum m_y \Delta y}{N} \quad (18)$$

$$\sigma_x = \left[\frac{\sum m_x \Delta x^2}{N} - \left(\frac{\sum m_x \Delta x}{N} \right)^2 \right] h_x \quad (19)$$

$$\sigma_y = \left[\frac{\sum m_y \Delta y^2}{N} - \left(\frac{\sum m_y \Delta y}{N} \right)^2 \right] h_y \quad (20)$$

where $\Delta x, \Delta y$ are the reduced beating values of the collector and its operating time; A_x, A_y are beating and operating time values, having the highest failure rate; h_x, h_y are values of time intervals for measuring parameters of beating and operating time.

The values of $\Delta x, \Delta y$ are calculated by the formulas:

$$\Delta x = (x - A_x)/h_x, \quad \Delta y = (y - A_y)/h_y \quad (21)$$

The correlation coefficient r between the parameters x and y is determined from the expression

$$r = \frac{\sum m_{xy} \Delta x \Delta y - \frac{\sum m_x \Delta x \sum m_y \Delta y}{N}}{N \sigma_x \sigma_y} \quad (22)$$

As a result of calculations using formulas (18) – (22), we obtain a different type of expression (19):

$$Y = B + kx \quad (23)$$

where B and k are the constants for a particular type of reservoir.

If we substitute the maximum allowable value of the beating in expression (23), then we obtain the average time of failure of the collector node.

Fig. 3. Diagram of the vibration spectrum with the specified bearing defects (red) and brush-collector node (blue).

We can imagine from this diagram (Fig. 3) the expansion in a Fourier series on the interval $(-T; T)$ in the range $[0; 1000]$ has the form:

$$f(x) = \frac{a_0}{2} + \sum a_n \cos\left(\frac{\pi \cdot n \cdot x}{T}\right) + b_n \sin\left(\frac{\pi \cdot n \cdot x}{T}\right) \quad (24)$$

Table 1. Model of motor diagnostics.

Motor characteristics	Assigned	Measure type	Frequency	Perfect motor	Diagnosed motor	Yes/No	Defect type
Power	10	kW	0	0	0	NO	No
Number of collector plates	81	ea.	2.5	0.33829	0.2247273	NO	No
Number of brushes	8	ea.	5	0.43725	0.0082658	NO	No
Number of pairs of poles	4	ea.	7.5	0.37799	0.0165316	NO	No
Power-line frequency	50	Hz.	10	2.466	0.7337296	YES	Working mode
Working motor coefficient	0.5	no.	12.5	0.37799	0.5549885	YES	Working mode
Permissible wear coefficient	0.75	no.	15	5.369	0.0302612	NO	No
Motor speed of rotation	900	U/min	17.5	4.633	0.0055339	NO	No
Voltage	220	V.	20	0.93433	0.0137296	NO	No
Number of bearings	2	ea.	22.5	0.31815	0	NO	No
Armature frequency	30		25	0.31815	0.0192635	NO	No
			127.5	0.15879	0.154035	NO	No
			130	0.05983	0.319621	NO	No
			132.5	0.19906	1.213	YES	Bearing
			135	0.25832	1.132	YES	Bearing
			137.5	0	0.231001	NO	No
			140	0	0.296521	NO	No
			142.5	0.05983	0.635357	YES	Bearing
			145	0.03969	0.858692	YES	Bearing
			147.5	0.05983	0.369706	NO	No
			150	0	0.219451	NO	No

Finally, we have from the obtained data:

$$f(x)=317.225000000000+\sum 634.45 \cdot \cos (0.0008 \cdot \pi \cdot n \cdot x)(25)$$

Using the STATISTICA software package, the curve, when applied to a diagram, proves that it is distributed according to *anormal (exponential) law*.

4 Results

Having developed a method for calculating vibrodiagnostics using the formulas, it is possible to create a model for an operated DC motor, which can evaluate the operational life of the components and the entire DC motor.

According to the DC motor data (table 1, columns 1-3), we can determine the operating parameters. Column 4 has the frequency from 0 to 1000 Hz, which indicates the spectrum of the DC motor. Columns 5 and 6 show the spectra of an ideal (operable) and diagnosed (operated) DC motor. Then “yes/no” indicates whether there are a defect and the type of it and the degree of its development (color indicator). These data can be obtained at the diagnostics.

An additional table 2 is compiled for a more detailed analysis of the defect. It shows the wear degree of the node, its operational state and the residual resource of the nodes.

Table 2 consists of two main parts: the frequencies at which defects appear (they are separate for each DC motor); the stages of defect development are determined according to acceptable ranges (up to 0.7 – a normal stage, from 0.7 to 0.8 – acceptable wear, 0.9 – pre-emergency stage, and 1 – emergency mode). These stages are presented in Figure 4.

Fig. 4. Degree of development of the defect in the nodes of the electric motor.

5 Conclusion

Thus, this technique can prevent breakdowns and accidents on the rolling stock, save on expensive equipment and spare parts. The method of

Table 2. Assessment of the wear degree of the motor components.

Defectrate					Defect developmental stages				
Bearing defect	Brush defect	Brush wear	Collect or beating	Collector platerip	To brush defect (split, fracture)	To bearing defect	To collector plate defect	Brush wear per holder	To adhesion of col. plate and armature
100	400	400	2875	843.75	0	0	0	0	0
101.25	400.31	401.25	2870	843.437	0.0062	0.01666	0.00174	0.00623	0.00296
102.5	400.62	402.5	2865	843.125	0.0124	0.03333	0.00349	0.01242	0.00593
103.75	400.93	403.75	2860	842.812	0.0187	0.05	0.00524	0.01857	0.00889
105	401.25	405	2855	842.5	0.0249	0.06666	0.00700	0.02469	0.01186
106.25	401.56	406.25	2850	842.187	0.0311	0.08333	0.00877	0.03076	0.01484

vibrodiagnostics calculating allows going from traditional scheduled preventive repair to the repair according to the condition of the operating DC motor on the rolling stock without stopping and dismounting.

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