Analysis of HIV Transmission of Commercial Sex Workers and Their Clients with Condom Use Treatment

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Abstract. A model of the HIV/AIDS epidemic among sex workers and their clients is discussed to study the effects of condom use in the prevention of HIV transmission. The model is addressed to determine the existence of equilibrium states, and then analyze the global stability of disease free and endemic equilibrium states. The global stability of equilibria depends on the values of the basic reproduction ratio derived from the next generation matrix of the model. The endemic equilibrium state is globally stable when the ratio exceeds unity. The simulation results are presented to discuss the effect of condom use treatment in preventing the spread of HIV/AIDS among sex workers and their clients. The results show that the effectiveness level in using condoms in sexual intercourse corresponds to the decreasing level of the spread of HIV/AIDS. We use Maple and Matlab software to simulate the impact of condom use.

Keywords: FSWs; clients; endemic; equilibriums; stability.

1 Introduction

HIV/AIDS is a global problem causing mortality worldwide. The largest risk factor of HIV transmission is transmitted through heterosexual contact, 50%, data from WHO [1]. The report of UNAIDS in 2017 [2], about 36.9 million people were living with HIV. The heterosexual contact between female sex workers (FSWs) and their clients without using condoms has a high risk to be contracted HIV infection. WHO has promoted the use of a condom in localization and entertainment areas to prevent the spread of HIV/AIDS among FSWs and their clients. FSWs and their clients have a potential impact on the spread of HIV/AIDS infection in their environment.

In worldwide, the prevalence of HIV/AIDS in prostitutes is higher compared to other populations, even the number of people with HIV/AIDS increases [2]. The increasing of HIV prevalence was supported by FSWs. In the environment society, the FSWs population acquires HIV 13.5 times more than all women, and it has been considered as a key population that contributes highly to the spread of HIV/AIDS.

Modelling in mathematics able to understand the dynamics of the HIV/AIDS epidemic and investigate parameters determining the endemic level of HIV/AIDS infection. Mathematical modelling is an important way to analyze accurately and efficiently the HIV epidemic of FSWs. A number of studies have been done to investigate HIV/AIDS infection via heterosexual or homosexual intercourse.

Mathematical models have been built to study the effect of social and behavior of the community in HIV transmission [3–5]. A model of HIV/AIDS infection in

the homogenous community was studied to understand the global dynamics of the model. The analysis to know the effect of HIV prevention on other epidemics was done by [6].

The impact of genetic on the spread of HIV infection was investigated to identify the threshold parameter [7]. In Kaur [8], Kaur *et.al* studied the role of sex workers in contributing to HIV transmission of clients' regular partners. In this study, they explored the effect of female sex-workers associated to the spread of HIV/AIDS infection in regular partners of their clients.

Omondi [9] studied a mathematical model capturing the dynamics of HIV transmission by considering sexual transmission, and analyzing the effect of pre-exposure prophylaxis (PrEP) use to prevent HIV transmission. They derived the parameter threshold, as the basic reproduction ratio to analyze the global stability of equilibrium states of the HIV/AIDS epidemic.

Another mathematical modeling for the HIV/AIDS epidemic was studied by [10–11] describing the dynamics of HIV/AIDS infection in prostitution and drug abuse.

Here, a model of HIV transmission is developed to describe the dynamics of HIV infection among FSWs and their clients incorporating the effect of condom use. The analysis of the model is studied to examine the disease persistently, and numerical results are presented to study the dynamics of HIV/AIDS infection of FSWs and their clients. The model is proposed to describe the implication of condom use for protecting HIV transmission in digital information using Maple and Matlab software to simulate the dynamic of FSWs and their clients.

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2 Mathematical formula

The total population among FSWs and their clients are classified into two groups namely a group of female sex workers and male clients. Female sex workers are divided into HIV-susceptible individuals (S_f), infectious individuals (I_f), and AIDS acquired individuals (A_f), while their clients are also divided into three subpopulations, namely, HIV-susceptible subpopulation (S_c), infectious individuals (I_c), and AIDS acquired individuals (A_c). The model of HIV/AIDS transmission diagram is given in Fig. 1

Fig. 1. The model of HIV/AIDS transmission diagram

The mathematical model can be presented in the following equation system,

$$\begin{aligned} \frac{dS_f}{dt} &= \lambda_1 - \beta_c(1-\varepsilon)S_fI_c - \mu S_f, \\ \frac{dI_f}{dt} &= \beta_c(1-\varepsilon)S_fI_c - (\mu + \delta_1 + \alpha_1)I_f, \\ \frac{dA_f}{dt} &= \alpha_1I_f - (\mu + \delta_2)A_f, \\ \frac{dS_c}{dt} &= \lambda_2 - \beta_f(1-\varepsilon)S_cI_f - \mu S_c, \\ \frac{dI_c}{dt} &= \beta_f(1-\varepsilon)S_cI_f - (\mu + \delta_1 + \alpha_2)I_c, \\ \frac{dA_c}{dt} &= \alpha_2I_c - (\mu + \delta_2)A_c. \end{aligned} \quad (1)$$

Populations of susceptible female sex workers (FSWs) and their clients are recruited at the constant rate λ_1 and λ_2 , respectively. These individuals die out naturally with a rate μ . Due to the condom use, the infection rate of FSWs population is $\beta_c(1-\varepsilon)I_c$, and the infection rate of the clients population reduces to $\beta_f(1-\varepsilon)I_f$. The populations of FSWs and clients living with HIV infection dies out at the rate δ_1 , while AIDS acquired population die out at the rate δ_2 . HIV-infected populations both FSWs and their clients remove

to AIDS class at rates α_1, α_2 , respectively. The model (1) is analyzed to examine the dynamics behavior and biological understanding.

3 Model Analysis

The existence of endemic state is proved and the stability of equilibria for the system are addressed in the next subsection.

3.1 Endemic state

The next generation matrix for the system (1) can be presented by,

$$G = \begin{pmatrix} 0 & \frac{\lambda_1(1-\varepsilon)\beta_c}{\mu(\mu+\alpha_2)} \\ \frac{\lambda_2(1-\varepsilon)\beta_f}{\mu(\mu+\alpha_1)} & 0 \end{pmatrix} \quad (2)$$

The element G_{ij} shows the average number of new infections in class i affected by an infectious individual of k class for the infection period. The basic reproduction ratio is the radius spectral of matrix G , it is given by

$$\mathfrak{R}_0 = \frac{\beta_c(1-\varepsilon)^2\lambda_1\beta_f\lambda_2}{\mu^2(\mu+\alpha_2)(\mu+\alpha_1)} \quad (3)$$

The basic reproduction ratio shows the number of the next infection affected by one infected individual when the infected individual contacts among the susceptible population. This ratio is determined by parameters the infection rates both female sex workers and their clients when they become infectious. Disease Free State for the system (1) can be written as follows.

$$E_0 = (S_f^0, I_f^0, A_f^0, S_c^0, I_c^0, A_c^0) = \left(\frac{\lambda_1}{\mu}, 0, 0, \frac{\lambda_2}{\mu}, 0, 0 \right)$$

While endemic equilibrium state is given by

$$E^* = (S_f^*, I_f^*, A_f^*, S_c^*, I_c^*, A_c^*), \text{ where}$$

$$S_f^* = \frac{((1-\varepsilon)\beta_f\lambda_1 + \mu(\mu+\alpha_1))(\mu+\alpha_2)}{(1-\varepsilon)\beta_c((1-\varepsilon)\beta_f\lambda_1 + \mu(\mu+\alpha_1))}$$

$$I_f = \frac{\mu^2(\mu+\alpha_1)(\mathfrak{R}_0-1)}{(1-\varepsilon)((1-\varepsilon)\beta_c\lambda_2 + \mu(1+\alpha_2))},$$

$$A_f = \frac{\alpha_1\mu^2(\mu+\alpha_2)(\mathfrak{R}_0-1)}{(1-\varepsilon)((1-\varepsilon)\beta_c\lambda_2 + \mu(\mu+\alpha_2))},$$

$$S_c = \frac{(1-\varepsilon)((1-\varepsilon)\beta_c\lambda_2 + \mu(\mu+\alpha_2))}{(1-\varepsilon)\beta_f((1-\varepsilon)\beta_f\lambda_1 + \mu(\mu+\alpha_1))},$$

$$I_c = \frac{\mu^2(\mu+\alpha_1)(\mathfrak{R}_0-1)}{(1-\varepsilon)((1-\varepsilon)\beta_f\lambda_1 + \mu(\mu+\alpha_1))},$$

$$A_c = \frac{\mu^2(\mu+\alpha_1)(\mathfrak{R}_0-1)}{(1-\varepsilon)\beta_c((1-\varepsilon)\beta_f\lambda_1 + \mu(\mu+\alpha_1))}.$$

The endemic state exists when $\mathfrak{R}_0 > 1$.

3.2 The stability analysis of equilibria

The global stability of equilibria is analyzed applying a Lyapunov function. It is not difficult to see that all solutions of the system (1) are positive and bounded in a compact set

Theorem 3.1: The uninfected state point E_0 of the system (1) is globally asymptotically stable (GAS) if $\mathfrak{R}_0 < 1$.

Proof: We construct a Lyapunov function as,

$$V(t) = (1 - \varepsilon) \frac{\lambda_1}{\mu} \beta_f I_f + (\mu + \alpha_1) I_c.$$

Differentiating the function $V(t)$ to time t results,

$$\frac{dV}{dt} = (1 - \varepsilon) \beta_f \frac{\lambda_2}{\mu} \left((1 - \varepsilon) \beta_c S_f I_c - (\mu + \alpha_1) I_f \right) + (\mu + \alpha_1) \left((1 - \varepsilon) \beta_f S_c I_f - (\mu + \alpha_1) I_c \right) \quad (4)$$

Due to $S_f \leq S_f^0 = \frac{\lambda_1}{\mu}$, $S_c \leq S_c^0 = \frac{\lambda_2}{\mu}$ and by manipulating calculation, the equation (4) can be written as

$$\frac{dV}{dt} \leq \left(\frac{(1 - \varepsilon)^2 \beta_f \beta_c \lambda_1 \lambda_2}{\mu^2} - (\mu + \alpha_1)(\mu + \alpha_2) \right) I_c = (\mu + \alpha_1)(\mu + \alpha_2) (\mathfrak{R}_0 - 1) I_c \quad (5)$$

It is seen that $\frac{dV}{dt} = 0$, when $I_f = I_c = 0$. Plugging

$I_f = I_c = 0$ into the model (1), it is obtained $A_f, A_c \rightarrow 0$,

$S_f \rightarrow \frac{\lambda_1}{\mu}$, and $S_c \rightarrow \frac{\lambda_2}{\mu}$ for $t \rightarrow \infty$. Based on

LaSalle's principle, it shows that the uninfected state is called globally asymptotically stable (GAS).

Theorem 3.2: The endemic state E^* of the system (1) is globally asymptotically stable (GAS) if $\mathfrak{R}_0 > 1$.

Proof: We use a Lyapunov function,

$$V(t) = (S_f - S_f^* \ln S_f) + a_1 (I_f - I_f^* \ln I_f) + a_2 (A_f - A_f^* \ln A_f) + a_3 (S_c - S_c^* \ln S_c) + a_4 (I_c - I_c^* \ln I_c) + a_5 (A_c - A_c^* \ln A_c).$$

Differentiating $V(t)$ with respect to time t ,

$$\begin{aligned} \frac{dV}{dt} &= \left(1 - \frac{S_f^*}{S_f} \right) \frac{dS_f}{dt} + a_1 \left(1 - \frac{I_f^*}{I_f} \right) \frac{dI_f}{dt} \\ &\quad + a_2 \left(1 - \frac{S_c^*}{S_c} \right) \frac{dS_c}{dt} + a_3 \left(1 - \frac{I_c^*}{I_c} \right) \frac{dI_c}{dt} \\ &= \left(1 - \frac{S_f^*}{S_f} \right) (\lambda_1 (1 - \varepsilon) \beta_c S_f I_c - \mu S_f) \end{aligned}$$

$$\begin{aligned} &+ a_1 \left(1 - \frac{I_f^*}{I_f} \right) ((1 - \varepsilon) \beta_c S_f I_c - (\mu + \alpha_1) I_f) \\ &+ a_2 \left(1 - \frac{S_c^*}{S_c} \right) (\lambda_2 - (1 - \varepsilon) \beta_f S_c I_f - \mu S_c) \\ &+ a_3 \left(1 - \frac{I_c^*}{I_c} \right) ((1 - \varepsilon) \beta_f S_c I_f - (\mu + \alpha_2) I_c) \quad (6) \end{aligned}$$

$$\begin{aligned} &= \lambda_1 - (1 - \varepsilon) \beta_c S_f I_c - \mu S_f - \lambda_1 \frac{S_f^*}{S_f} + (1 - \varepsilon) \beta_f S_f^* + \mu S_f^* \\ &+ a_1 \left((1 - \varepsilon) \beta_c S_f I_c - A_1 I_f - (1 - \varepsilon) \beta_c S_f I_c \frac{I_f^*}{I_f} + A_1 I_f^* \right) \\ &+ a_2 \left(\lambda_2 - (1 - \varepsilon) \beta_f S_c I_f - \mu S_c - \lambda_2 \frac{S_c^*}{S_c} + (1 - \varepsilon) \beta_f S_c^* I_f \right) \\ &+ a_2 \mu S_c^* + a_3 \left((1 - \varepsilon) \beta_f S_c I_f - A_3 I_c + (1 - \varepsilon) \beta_f I_f \frac{I_c^*}{I_c} \right) \\ &+ a_3 A_3 I_c^* \quad (7) \end{aligned}$$

For simple calculation, it is introduced new symbols

$$x = \frac{S_f}{S_f^*}, y = \frac{I_f}{I_f^*}, z = \frac{A_f}{A_f^*}, x = \frac{S_c}{S_c^*}, y = \frac{I_c}{I_c^*}, z = \frac{A_c}{A_c^*}, \text{ and}$$

$M = \lambda_1 + \mu S_f^* + a_1 A_1 I_f^* + a_2 \lambda_2 + a_2 \mu S_c^* + a_3 A_3 I_c^*$. From the system (1), it is obtained the relationships for endemic equilibrium as follows,

$$\begin{aligned} \lambda_1 &= (1 - \varepsilon) \beta_c S_f^* I_c^* + \mu S_f^*, \\ A_1 I_f^* &= (1 - \varepsilon) \beta_c S_f^* I_c^*, \\ A_2 A_f^* &= \alpha_1 I_f^*, \\ \lambda_2 &= (1 - \varepsilon) \beta_f S_c^* I_f^* + \mu S_c^*, \\ A_3 I_c^* &= (1 - \varepsilon) \beta_f S_c^* I_f^*, \\ A_4 A_c^* &= \alpha_2 I_c^*, \end{aligned} \quad (8)$$

where $A_1 = \mu + \alpha_1$, $A_2 = \mu + \delta_1$, $A_3 = \mu + \alpha_2$,

$A_4 = \mu + \delta_2$. The equation (7) can be written as,

$$\begin{aligned} \frac{dV}{dt} &= M - \mu S_f^* x - \lambda_1 \frac{1}{x} + (1 - \varepsilon) \beta_c S_f^* x y (1 - a_1) \\ &\quad + ((1 - \varepsilon) \beta_c S_f^* I_c^* - a_3 A_3 I_c^*) y \\ &\quad + (a_2 (1 - \varepsilon) \beta_f S_c^* I_f^* - a_1 A_1 I_f^*) y \\ &\quad + (a_3 - a_2) (1 - \varepsilon) \beta_f S_c^* S_f^* y \\ &\quad - a_2 \mu S_c^* u - a_2 \lambda_2 \frac{1}{u} - a_1 (1 - \varepsilon) \beta_c S_f^* I_c^* \frac{xv}{y} \\ &\quad - a_3 (1 - \varepsilon) \beta_f S_c^* I_f^* \frac{wy}{v}. \quad (9) \end{aligned}$$

Coefficients a_1, a_2 and a_3 are chosen by using coefficients of y, v, xv, uy are equal to zero, it is obtained the equations of the relationship $a_1 = 1$, $a_3 = a_2, a_1 A_1 = a_2 (1 - \varepsilon) \beta_f S_c^*$. The equation (9) becomes

$$\frac{dV}{dt} = M - \mu S_f^* x - \lambda_1 \frac{1}{x} - a_2 \mu S_c^* u - a_2 \lambda_2 \frac{1}{u} - a_1 (1 - \varepsilon) \beta_c S_f^* I_c^* \frac{xv}{y} - a_3 (1 - \varepsilon) \beta_f S_c^* I_f^* \frac{uy}{v}. \quad (10)$$

Terms in the equation (10) contain variables of a set

$$A = \left\{ x, u, \frac{1}{x}, \frac{1}{u}, \frac{xv}{y}, \frac{uy}{v} \right\}. \text{ From the arithmetic means}$$

and geometric means, then these variables can be

grouped into three sub set $\left\{ x, \frac{1}{x} \right\}, \left\{ u, \frac{1}{u} \right\}$, and

$\left\{ \frac{1}{x}, \frac{1}{u}, \frac{xv}{y}, \frac{uy}{v} \right\}$. The equation (10) can be written as

$$\frac{dV}{dt} = b_1 \left(2 - x - \frac{1}{x} \right) + b_2 \left(2 - u - \frac{1}{u} \right) + b_3 \left(4 - \frac{1}{x} - \frac{1}{u} - \frac{xv}{y} - \frac{uy}{v} \right), \quad (11)$$

Coefficients of b_1, b_2 and b_3 are determined by equating terms in the equation (9), and it is obtained the relationship as follows,

$$2b_1 + 2b_2 + 4b_3 = M,$$

$$b_1 = \mu S_f^*,$$

$$b_1 + b_3 = \lambda_1, \quad (12)$$

$$b_2 + b_3 = \lambda_2 = a_2 \mu S_c^*,$$

$$b_3 = a_1 (1 - \varepsilon) \beta_c S_f^* I_c^* = a_3 (1 - \varepsilon) \beta_f S_c^* I_f^*$$

The equation (12) is consistent related to equation (8),

$$\text{thus it is chosen } a_1 = 1, a_2 = \frac{(1 - \varepsilon) \beta_f S_c^*}{\mu + \alpha_1} = \frac{(\mu + \alpha_2) I_c^*}{(\mu + \alpha_1) I_f^*},$$

and $a_2 = a_3$, from the equation (12), it is obtained

$$b_2 = \lambda_2 - (1 - \varepsilon) \beta_c S_f^* I_c^*, b_3 = (1 - \varepsilon) \beta_c S_f^* I_c^*.$$

Thus the equation (11) becomes,

$$\frac{dV}{dt} = \mu S_f^* \left(2 - x - \frac{1}{x} \right) + \left(\lambda_2 - (1 - \varepsilon) \beta_c S_f^* I_c^* \right) \left(2 - u - \frac{1}{u} \right) + (1 - \varepsilon) \beta_c S_f^* I_c^* \left(4 - \frac{1}{x} - \frac{1}{u} - \frac{xv}{y} - \frac{uy}{v} \right) \leq 0. \quad (13)$$

Using the inequality of the arithmetic mean and

geometric mean, it holds $\frac{dV}{dt} \leq 0$, for $x, y, u, v > 0$, and

$$\frac{dV}{dt} = 0, \text{ when } x = y = u = v = 1, \text{ thus the largest}$$

invariant set of the system (1) on the set

$$\left\{ (x, y, u, v) \mid \frac{dV}{dt} = 0 \right\} \text{ is the singleton solution } (1, 1, 1, 1).$$

Using LaSalle invariant principle, the endemic

equilibrium point E^* is called globally asymptotically stable.

4 Numerical Results

Simulation results of the evolution of healthy and infected populations both FSWs and their clients in the long term are presented in this section. Parameter values for simulations are referred in [9-11] as follows,

$$\lambda_1 = 200, \lambda_2 = 100, \beta_f = \beta_c = 0.0004, \quad \mu = 0.0416,$$

$$\alpha_1 = \alpha_2 = 0.116, \delta_1 = 0.1995, \delta_2 = 0.1474, \text{ with initial}$$

$$S_c(0) = 1000, I_c(0) = 700, A_c(0) = 200,$$

$$S_f(0) = 700, I_f(0) = 500, A_f(0) = 200. \text{ In simulations}$$

as presented the next figures, present the effectiveness of the condom use in reducing the spread of in the community among female sex workers and their clients.

Fig. 2. The evolution of healthy FSWs population in different efficacy of condom use treatment.

Fig. 3. The evolution of infected FSWs population in different efficacy of condom use treatment.

Fig. 4. The evolution of healthy costumer population in different efficacy of condom use treatment.

Fig. 5. The evolution of infected costumer population in different efficacy.

In Fig. 2 and Fig. 4, the effect of condoms utilization for sexual intercourse among female sex workers and their clients shows that the number of healthy FSWs and their client populations increase drastically when the effective treatment of condom use increases. Conversely, in Fig. 2 and Fig. 4, the effect of condom utilization for sexual intercourse among female sex workers and their clients shows that the number of healthy FSWs and their clients' populations increases drastically when the effective treatment of condom use increases. Conversely, as in Fig. 3 and Fig. 5, it can be seen that the number of infected FSWs and their customer individuals decrease rapidly when the treatment effectiveness with condom utilization is declined. The number of infected individuals among FSWs and their clients decreases getting fast although the treatment efficacy decreases with constant efficacy values.

5 Conclusion

The policy of completely closing the activities of sex workers in localization does not seem possible. This creates a new problem in the community environment related to the transmission of HIV/AIDS. The most likely thing for sex workers to do is to avoid unprotected sex. Therefore, education is needed for sex workers to be willing to use condoms during sexual intercourse. It should be got the consideration of the government and non-government organizations.

Prevention efforts can be conducted through condom use in sexual intercourse among FSWs with

their clients. Therefore, it is needed awareness of FSWs and their clients, when they have sexual intercourse. Successful in overcoming the HIV/AIDS epidemic in the community among sex workers and their customers will depend on the level of awareness and compliance to use condoms in sexual intercourse. Parameters supporting the model for numerical simulation is difficult to be obtained from experimental data. To overcome the problem, it is needed digital information to analyze data and make the computer program of Matlab and Maple software for numerical simulation.

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