

Modes of magnetic field generation in models of a $\alpha\Omega$ -dynamo with a power type α -generator

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Abstract. In the dynamic model $\alpha\Omega$ -dimensions are simulated reversions of the magnetic field with a varying intensity of the α -generator. The change of the α -generator intensity as a result of synchronization of higher modes of the velocity field and the magnetic field is regulated by a function $Z(t)$ with a power kernel. Dynamo modes are obtained for two types of radial component in the scalar parameterization of the α -effect. The results were analyzed depending on the change in the exponent of the kernel of the function $Z(t)$ and the type of the power kernel, also a comparative analysis with the results of the study [9], where the exponential kernel of the function $Z(t)$ was used.

1 Introduction

The magnetic field is one of the main factors in the existence of life, since it creates a screen from radiation. Therefore, studies of models that track the evolution of the magnetic field as a whole over a long period of time are relevant.

The authors, using the large-scale dynamo model developed by [1] for several years, have been investigating the possibility of the occurrence of inversions in a magnetic field provided that the velocity field is constant. In the study [2] used the small-mode approximation, which includes the minimum number of modes for which dynamo can work, namely, one hydrodynamic and two magnetic poloidal $B_1(t)$ and toroidal $B_2(t)$. As a result of the computational experiment for the model were obtained inversions both in a magnetic field and in a viscous fluid velocity field, and there are not only cases of field attenuation, but also their undamped mutual generation.

The next stage of the work was the study of the system, into which α - and Ω -generators were algebraically introduced, that are responsible for the turbulent and laminar components of the velocity field of the magnetohydrodynamic system (MHD-system) [3]. The numerical calculations showed that inversions in a magnetic field occur, however, both the velocity field and the magnetic field decay quickly, and the periods of oscillations in the fields under consideration are almost identical. In the time subsequent work, fluctuations were introduced, which made it possible to understand their influence on the nature of inversions [4, 5]. As a result the following modes of dynamo were obtained: quasi-periodic with reversed fault and with missing reversed fault, the magnetic field with damped and without oscillations.

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Since in the study of MHD-systems there were cases of an unlimited increase in the magnetic field that were not related to the divergence of the numerical solution algorithm, the function $Z(t)$ [9] was introduced into the MHD-system for regulating degree of the α -effect influence. The general solution of the function $Z(t)$ is

$$Z(t) = \int_0^t J(t - \tau) \mathbf{B}^2(\tau) d\tau, \quad (1)$$

where $J(t)$ is the kernel of integral function. The choice of the form of the function $Z(t)$ is determined by the presence of, firstly, the solution of the ordinary differential equation with oscillations, and secondly, a fairly simple general algorithm for moving from integro-differential equations to the numerical solution scheme [6]. In study [9], the kernel has the exponential form $J(t) = e^{-bt}$. In to accepted restrictions of the model the following dynamo modes were obtained: the chaos, the magnetic field with damped oscillations, the steady regime of the magnetic field, dynamo burst, the steady-state regime.

In this paper, we pose the problem of determining dynamo-modes for a function $Z(t)$ with a power kernel. The power kernel defined by the following functions

$$J(t) = t^k \cdot e^{-bt}, \quad (2)$$

$$J(t) = t^k \cdot e^{-bt} \cdot \cos(at). \quad (3)$$

Also there are pose the problem to constructing phase planes and studying the dynamics of changes in dynamo modes with increasing degree of the nucleus, and comparison with of the results of work [9].

2 Equations and model parameters, formulation of the problem

We assume in the $\alpha\Omega$ -dynamo model that the velocity field \mathbf{v} and the magnetic field \mathbf{B} are axially symmetric in a spherical shell of viscous incompressible liquid rotating around an axis Oz with a constant angular velocity Ω . We consider that the velocity field of a viscous fluid \mathbf{v} is zero on the inner $r = r_1$ and the outer $r = r_2$ spherical envelope boundaries; the magnetic permeability of the inner and outer core are the same, the medium outside the core ($r > r_2$) is not conductive. We assume that the mean flow $\bar{\mathbf{v}}$ has the character of differential rotation, corresponding to the modes $\mathbf{v}_{k,1,0}^T$ from the linear shell $\{\mathbf{v}_{k_1,1,0}^T, \mathbf{v}_{k_2,2,0}^P, \mathbf{v}_{k_3,3,0}^T, \mathbf{v}_{k_4,4,0}^P, \dots\}$ is invariant under the Coriolis drift. Any such mode generates the rest of the chain [7].

The velocity field of a viscous fluid is approximated by the following combination [7]:

$$\mathbf{v} = u(t) \mathbf{v}_0 = u(t) (\alpha_1 \mathbf{v}_{0,1,0}^T + \alpha_2 \mathbf{v}_{0,2,0}^P + \alpha_3 \mathbf{v}_{0,3,0}^T + \alpha_{11} \mathbf{v}_{1,1,0}^T + \alpha_{13} \mathbf{v}_{1,3,0}^T), \quad (4)$$

where \mathbf{v}_0 is the Poincare mode, $|v_0| = 1$, $u(t)$ is the velocity amplitude, the components of the velocity field are independent of time.

The magnetic field is represented by the minimum number of lower eigenmodes $\mathbf{B}_{0,1,0}^P, \mathbf{B}_{0,2,0}^T, \mathbf{B}_{0,3,0}^P$, sufficient to obtain an oscillating dynamo

$$\mathbf{B} = B_2^T(t) \mathbf{B}_{0,2,0}^T(\mathbf{r}) + B_1^P(t) \mathbf{B}_{0,1,0}^P(\mathbf{r}) + B_3^P(t) \mathbf{B}_{0,3,0}^P(\mathbf{r}), \quad (5)$$

where the components of the magnetic field are considered independent of time and the component $\mathbf{B}_{0,1,0}^P$ is dipole.

The physical parameters of the fluid are assumed to be unchanged, the turbulence in the core is isotropic and we use the scalar parametrization of the α -effect as a function $\alpha(r, \theta) = \alpha(r) \cos\theta$, where

$\max|\alpha(r, \theta)| \sim 1$. Two variants of the radial part of the α -effect are used: $\alpha(r) = -\sin(\pi(r - r_1))$ and $\alpha(r) = r$.

Magnetohydrodynamic equation (MGD equation), containing the equation of Navier-Stokes, the magnetic field \mathbf{B} induction equation, continuity condition of the velocity field \mathbf{v} , the magnetic field \mathbf{B} solenoidal condition, boundary conditions, equation (1) for the function $Z(t)$ and taking into account the α -effect, in the Boussinesq approximation have the following form, respectively:

$$\begin{aligned} \frac{\partial \mathbf{v}}{\partial t} + (Re_m \mathbf{v} \nabla) \mathbf{v} &= P_m \Delta \mathbf{v} - \nabla P - E^{-1} P_m (e_z \times \mathbf{v}) + (1 + \zeta(t)) \cdot \mathbf{f}_{out} + (\nabla \times \mathbf{B}) \times \mathbf{B}, \\ \frac{\partial \mathbf{B}}{\partial t} &= Re_m \nabla \times (\mathbf{v} \times \mathbf{B}) + \Delta \mathbf{B} + (R_\alpha - Z) \nabla \times (\alpha(r, \theta) \mathbf{B}), \\ \nabla \cdot \mathbf{v} &= 0, \\ \nabla \cdot \mathbf{B} &= 0, \\ \mathbf{v}(\mathbf{r}_1) &= \mathbf{v}(\mathbf{r}_2) = \mathbf{0}, \\ Z(t) &= \int_0^t J(t - \tau) \mathbf{B}^2(\tau) d\tau, \end{aligned} \quad (6)$$

where P_m is magnetic Prandtl number, Re_m is magnetic Reynolds number, R_α is range of the α -effect, \mathbf{f}_{out} is mean mass density of the external-force field, the fluctuations of which are ensured by the stochastic process $\zeta(t)$ with zero mean in this case. This process simulates the spontaneously arising and vanishing coherent effect of the discarded higher modes of the velocity field [7].

3 The numerical simulation

We apply the Galerkin method to the system (6) and get the following system's form

$$\begin{aligned} \frac{\partial u}{\partial t} &= -P_m u(t) \sum_k \alpha_k^2 \lambda_k + (1 + \zeta(t)) f_{out} + \sum_{i,j,k} \alpha_i L_{ijk} B_j B_k, \\ \frac{\partial B_i}{\partial t} &= Re_m u(t) \sum_{j,k} \alpha_j W_{ijk} B_k - \mu_i B_i + (R_\alpha - z) \sum_k W_{ik}^\alpha B_k, \end{aligned} \quad (7)$$

where f_{out} is mass density of the external-force field, μ_i is the viscous dissipation parameter, λ_i is eigenvalues of the Poincare mode, parameters L_{ijk} , W_{ijk} , W_{ij}^α are the volume integrals of the fields under consideration.

If the kernel $J(t)$ of the function $Z(t)$ has the form (2), then the numerical scheme (7) is supplemented by the equations

$$\begin{aligned} \frac{\partial z_n}{\partial t} &= n \cdot z_{n-1} - b z_{n-1}, \quad n = 1, 2, \dots \\ \frac{\partial z}{\partial t} &= \sum_k B_k^2 - b z, \end{aligned} \quad (8)$$

with the initial condition

$$Z_n(0) = 0, \quad Z(0) = 0. \quad (9)$$

For a kernel of the form (3), the numerical scheme (7) will include the equations

$$\begin{aligned} \frac{\partial z_{s0}}{\partial t} &= -b z_{s0} + z_0, \\ \frac{\partial z_n}{\partial t} &= n z_{n-1} - b z_n - a z_{sn}, \\ \frac{\partial z_{sn}}{\partial t} &= n z_{s(n-1)} - b z_{sn} + a z_n, \quad n = 1, 2, \dots \end{aligned} \quad (10)$$

with the initial condition

$$Z_n(0) = 0, Z(0) = 0, Z_{ns}(0) = 0, Z_s(0) = 0. \tag{11}$$

The computational experiments with the models were carried out at the initial instant of time $t = 0$ for the following boundary conditions

$$u(0) = 1, B_2^T(0) = 0, B_1^P(0) = 1, B_3^P(0) = 0, Z(0) = 0 \tag{12}$$

and the accepted values of the model parameters: magnetic Reynolds number Re_m varied in the range $(0, 1000]$, α -effect amplitude R_α was considered on the interval $(0, 100]$, average density of external force f_{out} is equal to one and the scale factor b is equal to ten.

For the given model parameters, provided that the radial part of the function $\alpha(r, \theta)$ is defined as $\alpha(r) = -\sin(\pi(r - r_1))$, only two dynamo modes with oscillations were obtained: the stationary mode and the magnetic field attenuated. For the case of the scalar radial part $\alpha(r) = r$, a wider range of types of dynamo modes was obtained, which are interesting for further study.

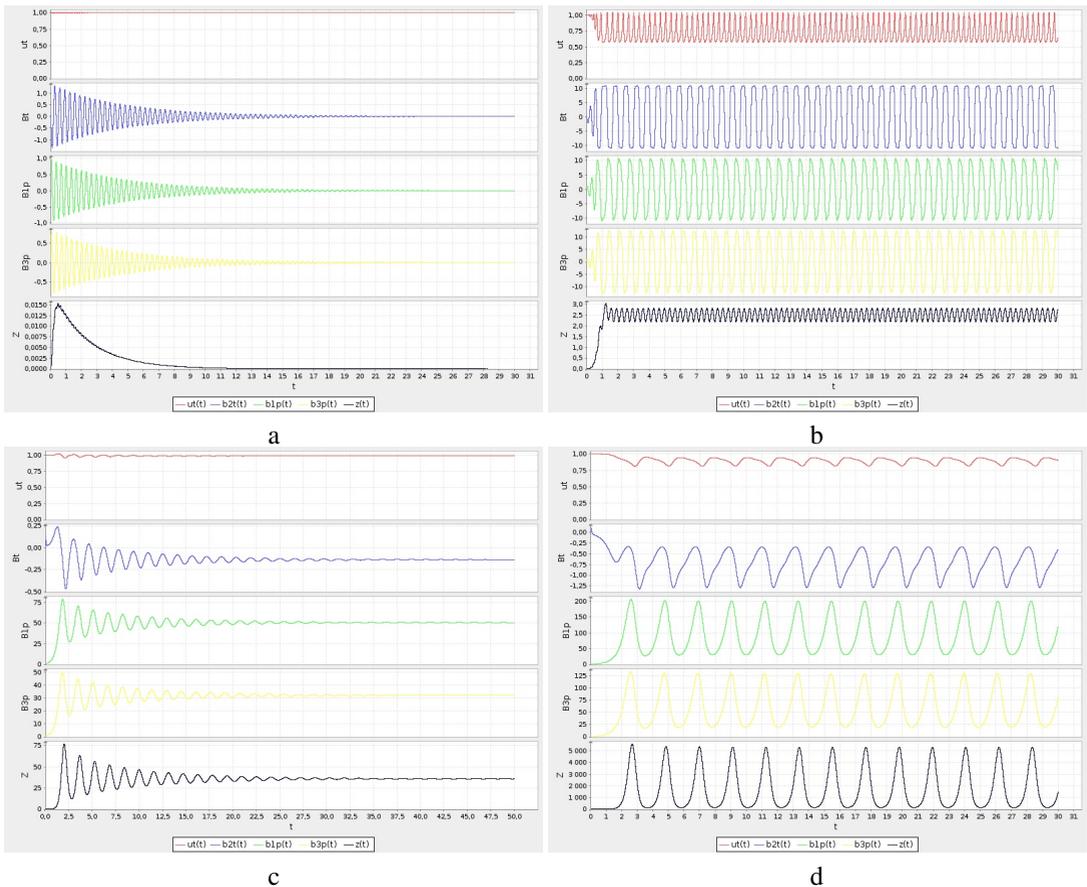


Figure 1. The dynamo regimes for the case of a radial part α -effect as $\alpha(r) = r$ for the kernel $J(t) = t^3 \cdot e^{-bt}$: a) the magnetic field with damped oscillations ($Re_m = 155, R_\alpha = 20$), b) the steady regime of the magnetic field ($Re_m = 150, R_\alpha = 25$), c) output steady-state regime ($Re_m = 35, R_\alpha = 15$), d) vacillation ($Re_m = 40, R_\alpha = 15$).

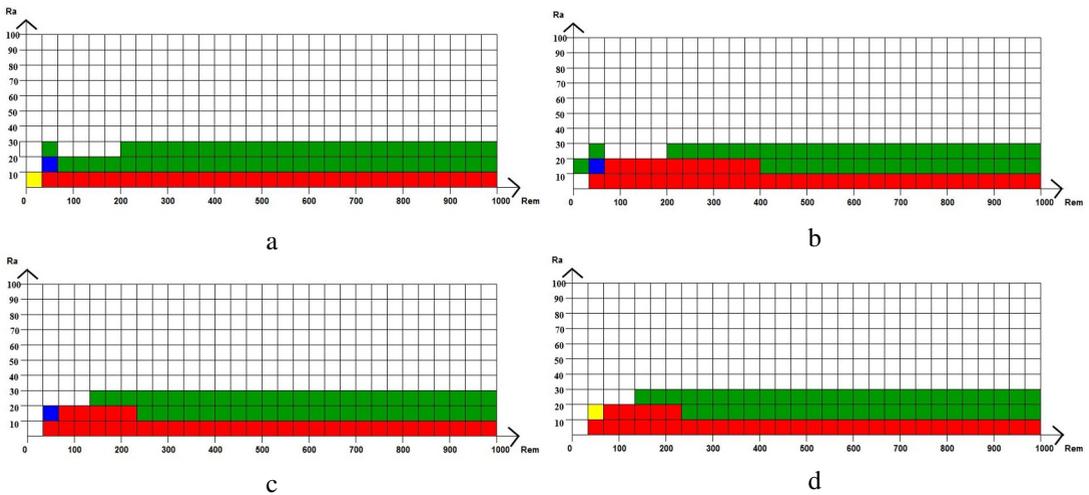


Figure 2. The nature of the generation of the magnetic field as a function of the values of the parameters R_α (turbulent dynamo) and Re_m (large-scale dynamo). Scalar parametrization of the α -effect as a function $\alpha(r) = r$ for kernel a) $J(t) = t \cdot e^{-bt}$, b) $J(t) = t^2 \cdot e^{-bt}$, c) $J(t) = t^3 \cdot e^{-bt}$, d) $J(t) = t^4 \cdot e^{-bt}$. The white area is the generation of a magnetic field without inversions, the red one – is the generation of a field with damped oscillations, the blue – is steady-state regime, the grey – is dynamo burst, the green – is the steady regime, the yellow – is the vacillation.

As a result of the computational experiment for the model with kernel $J(t) = t^k \cdot e^{-bt}$ the following dynamo modes were obtained: lack of inversion, the steady-state regime (Fig.1 c), the magnetic field with damped field (Fig.1 a), the steady regime (Fig.1 b), vacillation (Fig.1 d) and was constructed the distribution of dynamo regimes on the phase plane (Fig.2) for various values of the parameters Re_m and R_α responsible for the Ω - and α -effects.

Note that the kernel exponent $J(t)$ affects the change in amplitudes, but the frequency does not change, the dynamo modes change with the parameter Re_m in the range (0, 400].

A computational experiment for a model with the kernel $J(t) = t^k \cdot e^{-bt} \cdot \cos(at)$ gave similar results (Fig.3 and Fig.4). The modes of magnetic field generation without inversions, with damped oscillations (Fig.3 a), steady-state regime (Fig.3 c), dynamo-burst (Fig.3 d), steady regime (Fig.3 b) and vacillation are obtained. As the degree of kernel increases, the amplitude of oscillations for the obtained dynamo modes increases. Small changes occur when changing the parameter Re_m in the range (0, 150].

4 Conclusions

In the framework of the large-scale model of the $\alpha\Omega$ -dynamo with a power-law varying intensity of the α -generator, it is possible to reproduce various dynamo modes that are observed in real dynamo systems for the case of the radial part of the α -effect given as a function $\alpha(r) = r$.

The model is resistant to changes in the parameters f_{out} , a and b .

Compared with the results obtained for the exponential kernel of the function $Z(t)$ in [9], on the phase plane, the region of appearance of the changing modes narrows in the parameter R_α .

In the considered models, the mode of vacillation was obtained, which was not observed in [9].

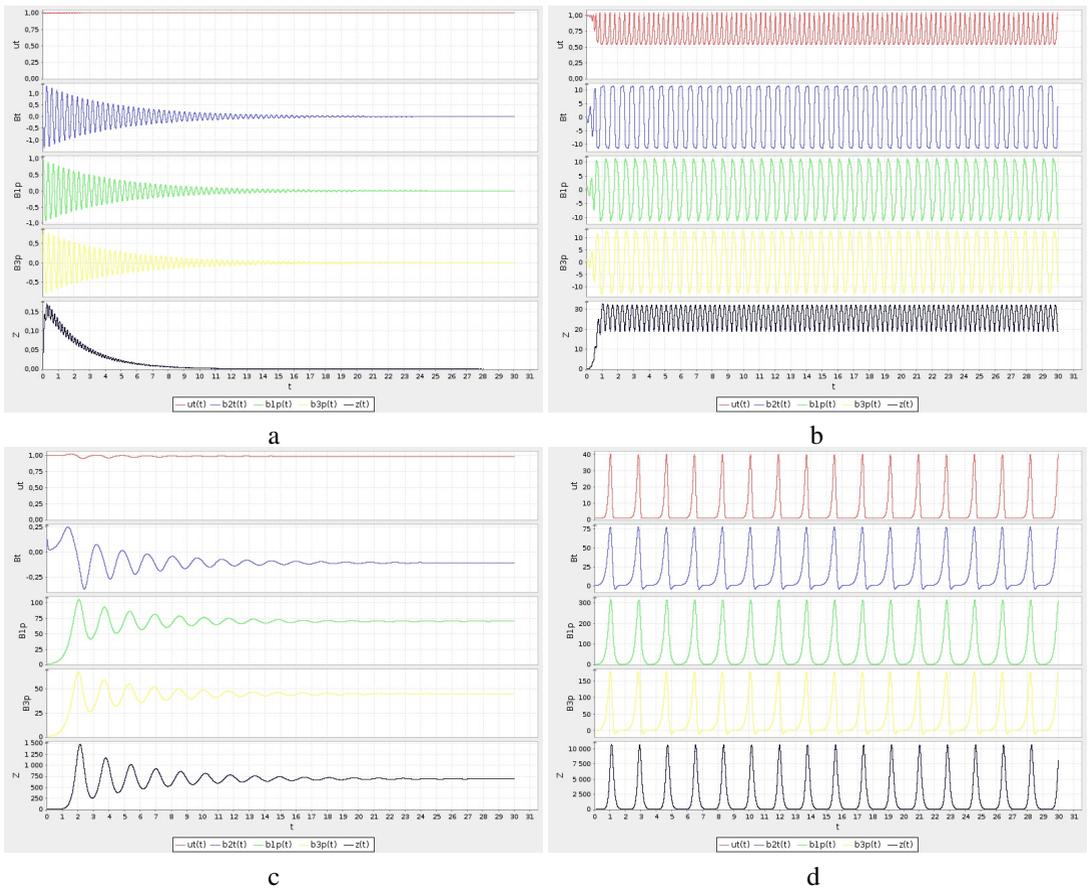


Figure 3. The dynamo regimes for the case of a radial part α -effect as $\alpha(r) = r$ for the kernel $J(t) = t^2 \cdot e^{-bt} \cdot \cos(at)$: a) the magnetic field with damped oscillations ($Re_m = 155$, $R_a = 20$), b) the steady regime of the magnetic field ($Re_m = 160$, $R_a = 25$), c) output steady-state regime ($Re_m = 35$, $R_a = 15$), d) dynamo burst ($Re_m = 0.1$, $R_a = 15$)

In the case of a power-law kernel $J(t) = t^k \cdot e^{-bt}$, the region of damped oscillations increases, and with an increase in the exponent in a small region of the phase plane, the stationary regime changes to vacillation (Fig.2).

For the case of a power-law kernel $J(t) = t^k \cdot e^{-bt} \cdot \cos(at)$, the phase planes with degrees $k = 3$ and $k = 4$ coincide, which means that with increasing degree the number of modes decreases and their location on the phase plane ceases to change (Fig.4 c, d).

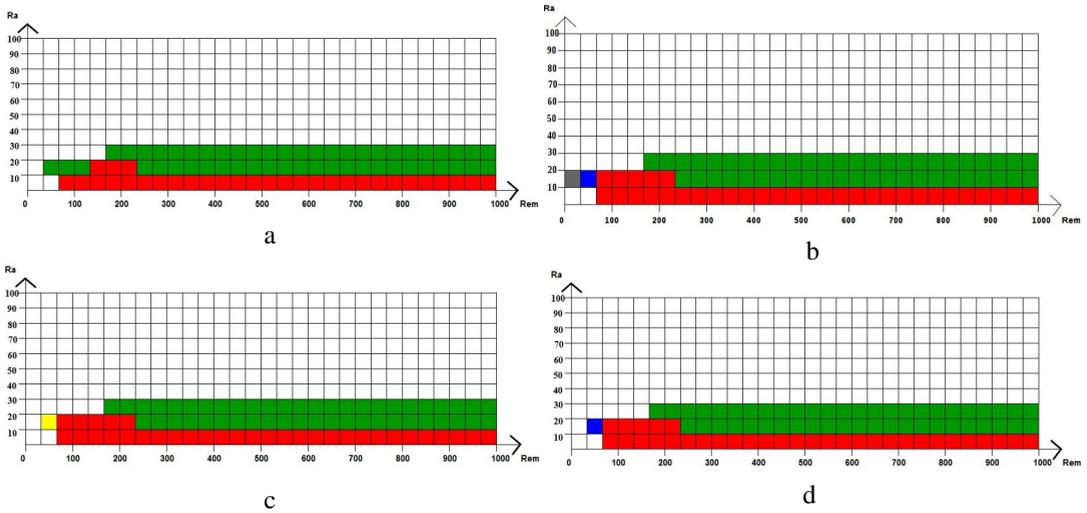


Figure 4. The nature of the generation of the magnetic field as a function of the values of the parameters R_α (turbulent dynamo) and Re_m (large-scale dynamo). Scalar parametrization of the α -effect as a function $\alpha(r) = r$ for kernel a) $J(t) = t \cdot e^{-bt}$, b) $J(t) = t^2 \cdot e^{-bt}$, c) $J(t) = t^3 \cdot e^{-bt}$, d) $J(t) = t^4 \cdot e^{-bt}$. The white area is the generation of a magnetic field without inversions, the red one – is the generation of a field with damped oscillations, the blue – is steady-state regime, the grey – is dynamo burst, the green – is the steady regime, the yellow – is the vacillation.

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