Economic Dispatch of Micro-grid Based on Sequential Quadratic Programming—Model and Formulation

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Abstract. This paper proposes an economic dispatch strategy for micro-grid based on the sequential quadratic programming (SQP) method. In this paper, in the off-grid operating state, according to the load, the kinds of micro sources and the operating cost, the power output minimizing the operating cost is calculated. When the system is connected to the grid, considering the tiered price, generation cost of CHP (Combined heat and power) based on micro-turbine, and generation cost of fuel cell, the paper calculates micro source power output based on the minimum operating cost. The characteristics of the proposed model and its benefits are investigated through several case studies.

1 Introduction

Compared with the traditional power system, micropower sources in micro-grid generally include intermittent DG and micro gas turbine. With the development and application of renewable energy, micro-grid construction is imperative. How to ensure the safe, stable and economic operation of micro-grid is a hot issue of current research [1-2].

This paper introduces an economic dispatch strategy of micro-grid. The main contribution of this paper is the formulation of the economic dispatch problem based on sequential quadratic programming. The rest of this paper is organized as follows: Section 2 gives the mathematical model of the economic dispatch problem. In Section 3, realization of sequential quadratic programming is presented. Section 4 gives the conclusions.

2 Mathematical Model

2.1 Establishment of the Mathematical Model of Micro-grid Economic Dispatch

The objective function of micro-grid economic dispatch can be formulated as [3]:

$$\min z = \sum_{i=1}^{N}\left[C_i \times F_i(P_i) + OM_i(P_i)\right] + C_{buy} - C_{sell} \quad (1)$$

$$OM = \sum OM_i(P_i) = \sum_{i=1}^{N} K_{OM_i} P_i \quad (2)$$

where, $F_i(P_i)$ is the fuel consumption function of unit i, P_i is its output power. C_i is the fuel price, $OM_i(P_i)$ is the operation and maintenance cost of unit i. For different types of distributed generation, the value of coefficient K_{OM_i} is not the same. C_{buy} and C_{sell} are the electricity purchase expense of the micro-grid system from the external grid or the electricity sales income to the external grid in the grid-connected state, and $C_{buy} = C_{sell} = 0$ in the off-grid operation state.

The constraints are

$$\begin{cases} \sum_{i=1}^{N} P_i - P_L - P_{\text{loss}} = 0 \\ P_i^{\min} \le P_i \le P_i^{\max} \\ f_{\min} \le \dfrac{\sum_{i=1}^{N} P_i - P_{\text{loss}} - P_L}{K_L} \le f_{\max} \end{cases} \quad (3)$$

where, $\sum_{i=1}^{N} P_i$ is the total power generation. P_L is the load size. $P_i^{\min}$ and $P_i^{\max}$ are the minimum and maximum output of the unit i. $f_{\min}$ and $f_{\max}$ are the minimum and maximum value of the frequency, respectively. K_L is the scaling factor between power difference and frequency difference.

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2.2 Micro-grid system structure

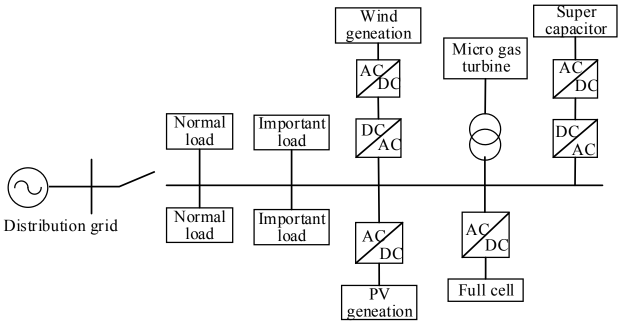

Figure 1. System structure

The system shown in Figure 1 is built based on Yantai micro-grid, and the distribution network with a voltage level of 380V is adopted as the point of common coupling (PCC). In the system, micro source includes photovoltaic power generation, fuel cell power generation, wind power generation, and micro-gas turbine power generation. Its capacity configuration is as follows: photovoltaic power 5 kW, wind power 20 kW, micro gas turbine 130 kW and fuel cell 15 kW.

3 Sequential quadratic programming

3.1 Idea of Sequential Quadratic Programming

The optimal search direction of sequential quadratic programming [4-5] (SQP) is determined by the variable scale method, and the scale matrix is used as the symmetric positive definite iteration matrix in quasi-newton method. In each iteration, the objective function value is minimized, and the positive definite direction matrix is maintained so that the Hessian matrix can be approximated. Therefore, SQP has the feature of global convergence and maintains local superlinear convergence, which is a very effective method and is recognized as one of the most effective algorithms for solving nonlinear constrained optimization problems. This algorithm can obtain the optimal solution of the original problem by converting the original problem into a series of quadratic programming subproblems, and obtain the quadratic approximation of Lagrange function, so as to provide the approximation degree of quadratic programming subproblem, and can also calculate the optimization problem with strong nonlinear.

For a constraint problem

$$\min f(x) \tag{4}$$

With constraints:

$$G_i(x) = 0 \qquad i = 1, 2, \cdots, m_e \tag{5}$$

$$G_i(x) \le 0 \qquad i = m_e + 1, \cdots, m \tag{6}$$

where, $x = [x_1, x_2, \cdots, x_n]$ is the design parameter vector, $f(x)$ is the objective function, $G(x) = [g_1(x), g_2(x), \cdots, g_m(x)]$ is the function vector, m_e is the boundary value of equality constraint and inequality constraint, both $f(x)$ and g(x) are nonlinear functions. The algorithm solves the Quadratic Programming (QP) subproblems based on (7) the Quadratic approximation of Lagrange function:

$$L(x, \lambda) = f(x) + \sum_{i=1}^{m} \lambda_i g_i(x) \tag{7}$$

where, λ_i is the Lagrange factor.

By linearizing the nonlinear constraints, the QP subproblem can be obtained, its objective function is

$$\min \frac{1}{2} d^T H_k d + \nabla f(x_k)^T d \tag{8}$$

The constraint conditions are:

$$\nabla g_i(x)^T d + g_i(x) = 0, i = 1, \cdots, m_e \tag{9}$$

$$\nabla g_i(x)^T d + g_i(x) \le 0, i = m_e + 1, \cdots m \tag{10}$$

where, d is the search direction of the whole variable, the symbol ∇ represents the gradient, and the matrix H_k is the positive definite quasi-Newtonian approximation of Hessian matrix of Lagrange function, which is calculated by BFGS method. Eq. (8) can be solved by any QP algorithm, and the new iterative equation can be formed as follows:

$$x_{k+1} = x_k + \alpha_k d_k \tag{11}$$

where, d_k represents a vector that x_k refers to x_{k+1}. The scalar step parameter α_i is determined by appropriate linear search process so that an index function is worth enough reduction.

3.2 Realization of SQP

3.2.1. Update of Hessian matrix

$$H_{k+1} = H_k + \frac{q_k q_k^T}{q_k^T s_k} - \frac{H_k^T H_k}{s_k^T H_k s_k} \tag{12}$$

where,

$$q^k = \nabla f(x_{k+1}) + \sum_{i=1}^{m} \lambda_i \nabla g_i(x_{k+1}) - \left| \nabla f(x_k) + \sum_{i=1}^{m} \lambda_i \nabla g_i(x_k) \right| \tag{13}$$

$$s_k = x_{k+1} - x_k \tag{14}$$

In each iteration, the BFGS method is used to calculate the positive definite quasi-newton approximation H of Hessian matrix of Lagrangian function. The Hessian matrix remains positive as long as $q_k^T s_k$ is positive and H is initialized to a positive definite matrix.

3.2.2. Solve quadratic programming subproblems

Each major iteration of the SQP method needs to solve a sub-problem as shown below, and its objective function is

$$\min_{d \in R^n} q(d) = \frac{1}{2} d^T H d + c^T d \tag{15}$$

The constraint conditions are

$$A_i d = b_i, i = 1, \cdots, m_e \tag{16}$$

$$A_i d \le b_i, i = m_e + 1, \cdots, m \tag{17}$$

where, A_i is the *i*th row of $A \in R^{m \times n}$.

The quadratic programming problem shown in Eq. (15) can be expressed as (a) equality-constrained quadratic programming problem and (b) inequality-constrained quadratic programming problem.

(a) the quadratic programming problem with equality constraints can be expressed as follows:

The objective function is

$$\min f(d) = \frac{1}{2} d^T Q d + c^T d \tag{18}$$

The constraint conditions are

$$Ad = b \tag{19}$$

where, $Q \in R^{n \times n}$ is symmetric, $c \in R^n, b \in R^m$ and $A \in R^{m \times n}$. Assume that $rankA = m < n$.

(b) the quadratic programming problem with inequality constraints can be expressed as follows:

The objective function is

$$\min f(d) = \frac{1}{2} d^T Q d + c^T d \tag{20}$$

The constraint conditions are

$$Ad \ge b \tag{21}$$

The basic idea of SQP method is that, at the site of an approximate solution, the original nonlinear programming problem is simplified to a quadratic programming problem, calculate the optimal solution, if any, is believed to be the optimal solution of the original nonlinear programming problem, otherwise, construct a new quadratic programming problem, continue the iteration until the solutions approximate constraints of the original problem.

The solution flow chart is shown in Figure 2.

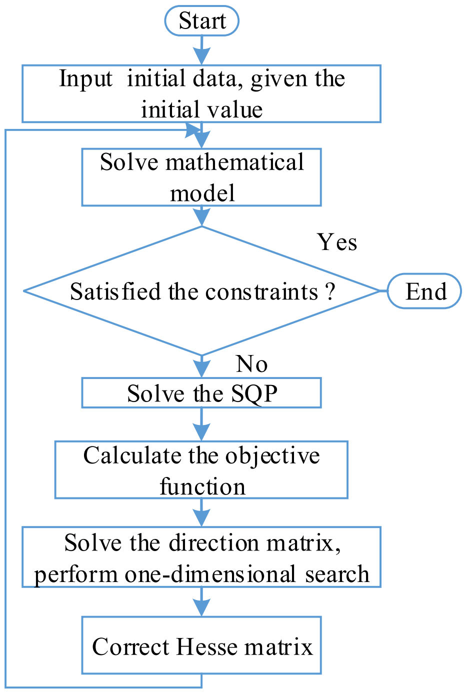

Figure 2. Sequential quadratic programming method flowchart.

For the convenience of expression, the general steps of SQP are given below:

Step 1: give initial value x_0, initial symmetric positive definite matrix H_0, set k=0.

Step 2: solve quadratic programming at x_k, get d_k.

Step 3: set $x_{k+1} = x_k + \alpha_k d_k$, where the step length α_k is obtained by some linear search.

Step 4: modify H_k to get H_{k+1}, so that H_{k+1} remains positively symmetric.

Step 5: set k=k+1 and return step 2.

4 Conclusions

A novel economic dispatch strategy and solution methodology for micro-grid are presented in this paper. We establish a non-linear objective function to achieve economic dispatch with constraints. This paper proposes to use the sequential quadratic programming (SQP) method for micro-power active power output optimization. The SQP method proposed in this paper is applied to the economic dispatch of the micro grid system.

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References

1. Liu, H.T., Ji, Y., Zhuang, H.D., Wu, H.B. (2015) Multi-objective dynamic economic dispatch of

https://doi.org/10.1051/e3sconf/201913601010

microgrid systems including vehicle-to-grid. Energies, 8:4476-4495.

2. Xu, X., Jia, H.J., Chiang, H.D. (2015) Dynamic modeling and interaction of hybrid natural gas and electricity supply system in micro-grid. IEEE Transactions on Power Systems, 30: 1212-1221.
3. Wei, B., Wang, Z.W., Li, L.I. (2007) Analysis of economic efficiency for cold heat, and electricity triple co-generation system with miniature gas turbine. Thermal Power Generation, 36: 1-5.
4. Jin, P., Ai, X., Xu, J.J. (2012) An economic operation model for isolated microgrid based on sequence operation theory. Proceedings of the CSEE, 32: 52-58.
5. Chen, X., Wang, X.M., Zhou, J. (2012) UAV flight performance optimization based on sequential quadratic programming method. Computer Simulation, 29: 99-102.