Optimization calculation of well function $W(u, r/B)$ based on BP neural network

Zhang Yuxia

School of Water Conservancy and Hydroelectric, Hebei University of Engineering, handan, Hebei, 056000, China

Abstract. In order to obtain the calculation result of the well function $W(u, r/B)$ in the groundwater more quickly and the obtained result is more accurate than the approximation obtained by the conventional interpolation. Based on the selection of BP neural network structure and the selection of parameters, the corresponding BP neural network model was established to study the well function $W(u, r/B)$. The results show that the partial data of well function are trained by BP neural network, and the prediction result is faster and more accurate than the approximate solution obtained by the equal logarithmic distance trapezoidal segmentation method. It can be seen that using BP neural network to learn and train, the network model obtained is used for solving and calculation, which is a convenient, fast and accurate calculation method.

1 Introduction

As a kind of valuable resource, it is of great significance for the development of society to study the movement law of groundwater, make correct evaluation and make reasonable development and utilization [1]. The groundwater well function is an important part of groundwater research. The well function of unsteady motion is the exponential integral function [2], which has no analytic solution and can only be solved approximately by numerical calculation method [3]. The standard curve of well function $W(u, r/B)$ is a curve group. Each r/B has a standard curve, which is generally solved by look-up table method. When there is no corresponding reference value in the table, interpolation method is usually chosen to solve, with slow calculation speed and large human error, and the result is not accurate enough. In addition, there is also approximate well function programming [4] to solve the calculation. In this paper, BP neural network is used to solve and calculate the well function $W(u, r/B)$. Through the learning and training the relationship between $u, r/B$ and $W(u, r/B)$, we obtain the neural network model, which makes the solution faster and more accurate than the approximate solution obtained by equal logarithmic interval trapezoidal segmentation method.

2 Principle of BP neural network

As one kind of the artificial neural networks, BP neural Network (back-propagation Network) was proposed by the group of scientists headed by Rumelhart and McClelland in 1985, which was the most widely applied, most intuitive and most easily understood neural

Network model [5]. BP neural network can learn and store a large number of input-output pattern mapping relationships without revealing the mathematical equation describing such mapping relationship in advance, and it has a strong nonlinear mapping ability [6]. BP neural network is a kind of multi-layer neural network with three or more layers, which not only has an input layer and an output layer, but also has one or more hidden layers [7]. The neural units between each layer are fully connected, while the neurons in each layer are not connected [8-9]. Its learning rule is to use the gradient descent method to continuously adjust the weights and thresholds of the network through back propagation [10], so as to minimize the sum of squared errors between the actual output of the neural network and the expected output [11].

3 BP neural network model

3.1 Training data generation and processing

The training data is selected from the value of the well function of the overcurrent system when the adjacent aquifers are elastically released [12], u and r/B are input vectors, and $W(u, r/B)$ is the output vector. Where u ranges from 0.000001 to 8; r/B ranges from 0.001 to 9. When the training data is relatively small, the error between the predicted value and the numerical solution obtained after learning and training by BP neural network will be relatively large, which cannot reach the expected accuracy target. Therefore, the interval of u and r/B is chosen as 10^n , $0.5 \cdot 10^n$, and n is the corresponding order of magnitude. Table 1 shows some training data:

* e-mail: 2437987184@qq.com

Table 1. Training data

$\frac{r/B}{u}$	0.001	0.0015	0.002	...	0.009	0.0095	0.01	0.015	...	8.5	9
0.000001	13.0031	12.7460	12.4417		9.6532	9.5451	9.4425	8.6319		0.0002	0.0001
0.000002	12.4240	12.2825	12.1013		9.6532	9.5450	9.4425	8.6319		0.0002	0.0001
0.000003	12.0581	11.9606	11.8322		9.6530	9.5450	9.4425	8.6319		0.0002	0.0001
...											
0.000008	11.1279	11.0898	11.0377		9.6292	9.5286	9.4314	8.6318		0.0002	0.0001
0.000009	11.0135	10.9795	10.9330		9.6184	9.5204	9.4251	8.6316		0.0002	0.0001
0.00001	10.9109	10.8803	10.8382		9.6059	9.5106	9.4176	8.6313		0.0002	0.0001
0.00002	10.2301	10.2147	10.1932		9.4383	9.3673	9.2961	8.6153		0.0002	0.0001
...											
7	0.0001	0.0001	0.0001		0.0001	0.0001	0.0001	0.0001		0.0000	0.0000
8	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000	0.0000		0.0000	0.0000

Due to the large gap between the maximum and minimum values of the data, the logarithm was taken before the data normalization processing [13]. The following formula was adopted:

$$y = \frac{y_{\max} - y_{\min}}{x_{\max} - x_{\min}} (x - x_{\min}) + y_{\min}$$

Where: x is the original data; $x_{\max}$ and $x_{\min}$ are the maximum and minimum values of the original data; y is the transformed target data; $y_{\max}$ and $y_{\min}$ are the maximum and minimum values of the target data

3.2 Selection of BP neural network structure and determination of parameters

In this BP neural network, the number of nodes in the input layer is 2, and the number of nodes in the output layer is 1. Since the selection of the number of nodes in the hidden layer is very complicated, the change of the number of nodes can affect the relationship between input and output. It has a relationship with the number of input and output nodes and the problem requirements, and there is no definite analytical solution. According to the formula for reference in Shang Gang etc. [14] the article preliminary selected range of 2 ~ 12. Firstly, a single layer of hidden layer is selected for training, but the error of the solution result is relatively large, so consider choosing double hidden layer for training. To carry out the training, after multiple trainings, the double-layer hidden layer can meet the accuracy requirements, so take four layers neural, node number in both of the two hidden layer is 12. The topology structure of the four-layer feed forward network is shown in Figure 1.

Figure 1. Schematic diagram of four-layer network topology

The BP neural network uses the function newff to create a feed forward neural network. The transfer function of the hidden layer is tangent S-type tansig, and the transfer function of the output layer is purelin. The training function trainlm based on the LM algorithm and the steepest descent method learngdm of the momentum are selected. The error allowable value of network training is 1e-12, the minimum training gradient is 1e-10, the number of training is 20000, and the number of display training is 50.

4 Comparison of training results

The data in literature [4] was selected as the validation data, and the predicted value and prediction error of the verification data by BP neural network were shown in Figure 2 and Figure 3.

Figure 2. Neural network predicts output

Figure 3. Neural network prediction error

The predicted value of the verification data is compared with the numerical solution error result and the error result in the literature [4] as shown in Table 2.

Table 2. Comparison table of error results

u	r/B	W(u, r/B)			relative error (%)	
		predictive value	numerical solution	approximation	predictive value and numerical solution	approximation and numerical solution
0.000001	0.001	13.0030	13.0030	13.0518	0.00	0.37
0.005	0.009	4.7222	4.7222	4.7328	0.00	0.22
0.000001	0.055	6.0388	6.0388	6.0597	0.00	0.34
0.009	0.4	2.2269	2.2269	2.2304	0.00	0.15
0.000001	9.0	0.0001	0.0001	0.0001	0.00	0.00
8.0	0.001	0.0000	0.0000	0.0000	0.00	0.00
8.0	9.0	0.0000	0.0000	0.0000	0.00	0.00
0.1	0.035	1.8207	1.8207	1.8247	0.00	0.22
0.05	0.15	2.3776	2.3776	2.3825	0.00	0.22
1.0	0.4	0.2135	0.2135	0.2144	0.00	0.42
0.7	0.08	0.3732	0.3732	0.3745	0.00	0.35

From the comparison of the above error results, it can be seen that the error between the predicted value and the numerical solution of the validation data and the approximate value and the numerical solution in literature [4] is small.

5 Conclusion

The optimization calculation of well function W (u, r/B) by BP neural network solves the problem that the calculation speed of solving by interpolation method is slow and the result is not accurate when there is no corresponding numerical solution in the solution table. The error between the predicted value and the numerical solution solved by the BP neural network and the error results of the approximate solution and the numerical solution obtained by the equal logarithmic distance trapezoidal segmentation method showed that the prediction accuracy of the BP neural network was more accurate. Therefore, it is convenient and quick to solve the well function after learning and training with BP

neural network, and the calculation can be more convenient and accurate in practical applications in the future.

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