Self-oscillations at the frequency of subharmonics in nonlinear electric chains and systems

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Abstract. It is known that the occurrence and existence of autoparametric oscillations (AIC) at the subharmonic frequency (GHC) in power lines (power lines) and in power supply systems is extremely undesirable, since they cause ferroresonant overvoltages at different frequencies. At the same time, there is an extensive class of nonlinear electric circuits in which the excitation of the AIC at the frequency of the SGC forms the basis of frequency-converting devices serving as secondary power sources. It is shown that single-phase-three-phase nonlinear systems are, to one degree or another, equivalent circuits of power lines, the main elements of which are: longitudinal compensation capacitors, transverse compensation reactors, and transformers with non-linear characteristics. The regularities of the excitation of the GCC at the frequency (ω / 3) of the power lines were studied, theoretical and experimental studies of the equivalent model of single-phase-three-phase circuits with nonlinear inductance were carried out. For a theoretical analysis of the steady-state mode of SGK at a frequency (ω / 3) with inductive coupling, the frequency-energy approach is used. The conditions of existence and critical parameters of the circuit are determined, and the mechanism of the appearance of the SGC at the frequency (ω / 3) is also studied.

1 Introduction

Electric systems in large quantities contain elements with significant inductance (generators, transformers, reactors, etc.). On the other hand, power lines have capacities relative to the ground and between phases. Often, to control the voltage and increase the stability of parallel operation, additional capacities relative to the ground are included in the line cut (Fig. 1).

Combinations of such inductances and capacitances create a number of complex oscillatory circuits in the circuit of the electrical system. In normal operation, the capacitance and inductance systems of these circuits are shunted by the load or connected directly to the terminals of a powerful source so that free oscillations cannot develop in them.

With various commutations in the system, part of the oscillatory circuits can be determined and energetic oscillations develop in them, leading to significant overvoltages [1-7,10-12]. These overvoltages are resonant in nature and, in principle, can have a very significant duration. Resonant overvoltage is one of the most difficult to analyze types of overvoltages, since in most cases there are steel cores, the magnetization characteristics of which are nonlinear.

Recently, the study of nonlinear oscillations in electrical systems has been developed, since the presence of longitudinal compensation capacitors and chokes with steel cores in the power lines open up wide possibilities for the formation of various oscillatory circuits and highlights resonant overvoltages [1-7,10,11].

2 Subharmonic resonance in power lines (power lines)

In practice, longitudinal capacitive compensation is used to increase line throughput. Capacities are switched on longitudinally at one or several points of the line. Usually capacitors are installed on one of the switching points of the line.

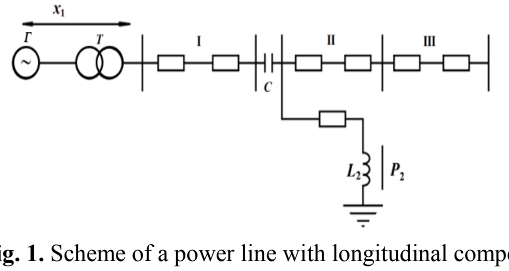

Fig. 1. Scheme of a power line with longitudinal compensation.

Suppose a transmission break occurred in section II so that only section I, compensation capacitance C, and reactor P remained in operation. The equivalent circuit is shown in Fig.2a.

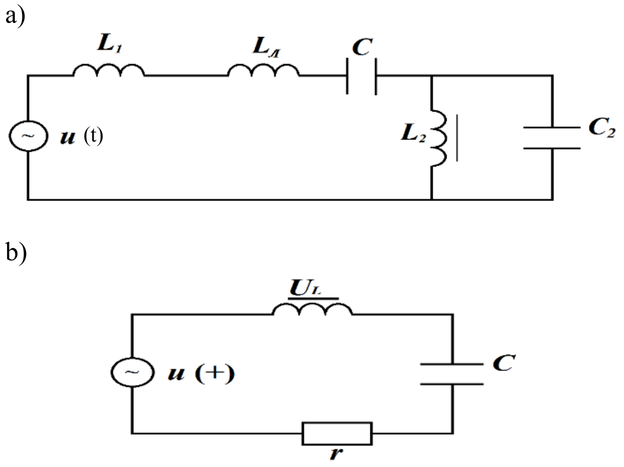

Fig. 2. Equivalent equivalent circuit.

The natural frequency of the circuit, consisting of C and L2, is: $\omega_0 = \frac{\omega}{\sqrt{X_L/X_C}}$;
where ω_0 is the network frequency, $X_L = \omega L_2$ is the inductive reactance of the reactor, $X_C = \frac{1}{\omega C}$ is the capacitive resistance of the longitudinal compensation.
If the reactor power is $Q_P = 0{,}2$, then the lines $\left(\frac{X_1}{X_L} \approx 5\right)$, and the capacitor resistance is $X_C \approx \frac{Z_L}{3}$,, then $\frac{\omega}{\omega_0} = \frac{1}{\sqrt{3}} < \frac{1}{3}$. Therefore, in the oscillatory circuit L, C, subharmonic oscillations (SHO) with a frequency of $\left(\frac{1}{3}\right)$ are possible. The line capacitance is much smaller than the longitudinal compensation capacitance, therefore, for a subharmonic current, it represents a very large resistance and should not have a significant effect on the development of SHO.

The analysis of overvoltages in the case under consideration can well be carried out using a single-frequency equivalent circuit, converting it to the form in Fig. 2b, where the line and source inductance are sequentially combined with the nonlinear reactor inductance in the form of the equivalent inductance L(i), C is the longitudinal compensation capacity and R - active resistance equivalent inductance.

3 Analysis of the steady state SHO mode at a frequency $\left(\frac{\omega}{3}\right)$.

To study the patterns of excitation of subharmonic oscillations of power lines, theoretical and experimental studies of the equivalent model in single-phase-three-phase circuits with non-linear inductance were carried out (Fig. 2b).

The frequency-energy approach was used to analyze the steady-state regime of the SGC at a frequency $\left(\frac{\omega}{3}\right)$ with inductive coupling [8, 9, 10].

Subsequently, the energy equilibrium equations are compiled for the frequency divider and the condition is imposed that the ferromagnetic element converts current energy with a frequency of 3ω into current energy with a frequency of ω.

Consider the stea (t) ate excitation of third-order subharmonic oscilla $\left(\frac{\omega}{3}\right)$ in a single-phase ferroresonant circuit using energy ratios with the aim of quantitative and qualitative assessment of electromagnetic processes.

The core magnetization characteristic can be approximated by a cubic polynomial:

$$\psi = ai - bi^3 \tag{1}$$

We take a certain point as a base point on the dependence $\psi = f(i)$ of inductance. The corresponding flux linkage value will be called basic $\psi_б$, current $i_б$. The dependence curve $\psi = f(i)$ is depicted in relative units. Denoting: $\psi_б = \sqrt{\frac{a^3}{b}}$; $i_б = \frac{\psi_б}{a}$; $i_б^3 = \frac{\psi_б}{b}$; or $i_б = \sqrt[3]{\frac{\psi_б}{b}}$ the magnetization curve in relative units from (1), we obtain the current-voltage characteristic of the ferromagnetic element in relative units, we present in the form: $\psi = i - i^3$.

Given the law of current change:

$$i = I_1\cos(\omega t + \varphi_1) + I_3\cos(3\omega t + \varphi_3) \tag{2}$$

Then:

$$\Psi = i - i^3 = A_1\cos(\omega t + \varphi_1) + B_1\cos(3\omega t + \varphi_3) - $$
$$-A_2 sin(\omega t + \varphi_1) + B_2 sin(3\omega t + \varphi_3), \tag{3}$$

Where:

$$A_1 = I_1 - \frac{3}{4}\cdot I_1^3 - \frac{3}{2}\cdot I_1 \cdot I_3^2 - \frac{3}{4}\cdot I_1^2 \cdot I_3 \cdot cos\cdot(3\varphi_1 - \varphi_3)$$
$$B_1 = I_3 - \frac{3}{4}\cdot I_3^3 - \frac{3}{2}\cdot I_1^2 \cdot I_3 - \frac{1}{4}\cdot I_1^3 \cdot cos(3\varphi_1 - \varphi_3)$$
$$A_2 = \frac{3}{4}\cdot I_1^2 \cdot I_3 \cdot sin(3\varphi_1 - \varphi_3);$$
$$B_2 = \frac{1}{4}\cdot I_1^3 \cdot sin(3\varphi_1 - \varphi_3)$$

Inductance Voltage:

$$U_L = \frac{d\Psi}{dt} = -\omega A_1 sin(\omega t + \varphi_1) - \omega A_2 cos(\omega t + \varphi_1) - $$
$$-3\omega_1 B_1 sin(3\omega t + +\varphi_3) - 3\omega B_2(3\omega t + \varphi_3) \tag{4}$$

Replace currents and voltage with their complex images:

$$\dot{I}_1 = jI_1 \cdot e^{j\varphi_1};\ \dot{I}_3 = jI_3 \cdot e^{j\varphi_3}$$
$$\dot{U}_1 = -\omega \cdot A_1 \cdot e^{j\varphi_1} - j\omega \cdot A_2 \cdot e^{j\varphi_1} = j\omega(-A_2 + jA_1)e^{j\varphi_1}$$
$$(5)$$
$$\dot{U}_3 = -3\omega \cdot B_1 \cdot e^{j\varphi_3} - j3\omega \cdot B_2 \cdot e^{j\varphi_3} = -j3\omega(B_2 + jB_1)e^{j\varphi_3}$$

Integrated capacities entering the inductance:

$$\tilde{S}_1 = \frac{1}{2}\dot{U}_1 = \frac{1}{2}\omega I_1^*(-A_2 + jA_1);$$
$$\tilde{S}_3 = \frac{1}{2}\dot{U}_3 I_3^* = \frac{1}{2}\omega I_3(B_2 - jB_1) \tag{6}$$

or: $P_1 = -\frac{1}{2}\omega I_1 A_2 = -\frac{3}{8}\omega I_1^3 I_3 sin(3\varphi_1 - \varphi_3)$ (7a)

$Q_1 = \frac{1}{2}\omega I_1 A_1 = \frac{1}{2}\omega I_1^2 \Big[1 - \frac{3}{4}I_1^2 - \frac{3}{2}I_3^2 - \frac{3}{4}I_1 I_3 cos(3\varphi_1 - \varphi_3) I_6\Big]$ (7b)

$P_3 = \frac{3}{2}\omega I_3 B_2 = \frac{3}{8}\omega I_1^3 I_3 sin(3\psi_1 - \varphi_3) = -P_1$ (7c)

$Q_3 = \frac{3}{2} I_3 B_1 = \frac{3}{2}\omega I_3 \Big[I_3 - \frac{3}{4}I_3^2 - \frac{3}{2}I_1^2 I_3 - \frac{1}{4}I_1^3 cos(3\psi_1 - \varphi_3)\Big]$ (7d)

Introducing the notation: $3\varphi_1 - \varphi_3 = \chi$ we rewrite equation (7):

$P_1 = -\frac{3}{8}\omega I_1^3 I_3 \sin\chi = -a\sin\chi = -P_3$ (8a)

$Q_1 = \frac{1}{2}\omega I_1^2 \left(1 - \frac{3}{4}I_1^2 - \frac{3}{2}I_3^2\right) - \frac{3}{8}\omega I_1^3 I_3 \cos\chi = b_1 - a\cos\chi$ (8b)

$Q_3 = \frac{3}{2}\omega I_3^2 \left(1 - \frac{3}{4}I_3^2 - \frac{3}{2}I_1^2\right) - \frac{3}{8}\omega I_1^3 I_3 \cos\chi = b_3 - a\cos\chi$ (8c)

It follows from (8a) that $P_1 = -P_3$, this confirms the possibility of using a nonlinear inductance as a frequency divider $\left(\frac{\omega}{3}\right)$. Moreover, the division of the frequency ω by $\left(\frac{\omega}{3}\right)$ will be at $P_1 < 0$; $P_3 > 0$ ($\chi = 0 \div 180^0$), and the multiplication of the frequency ω by 3ω, respectively, for:

$P_1 > 0;\ P_3 < 0 \quad (\chi = 0 \div -180^0)$.

By eliminating χ from (8a) and (8b) we have:

$P_1^2 + (Q_1 - b_1)^2 - a^2 = 0$ (9)

By squaring equation (10) and adding, we get

$K_p^2 + (K_q - b_1')^2 - a'^2 = 0$ (11)

Transforming equation (11) taking into account (9) and (10), we obtain:

$$36\frac{I_3^4}{(1-K_q)^2} + 27\frac{I_1^2 I_3^2}{(1-K_q)^2} + 9\frac{I_1^4}{(1-K_q)^2} - 48\frac{I_3^2}{(1-K_q)} - 24\frac{I_1^2}{(1-K_q)} + 16\left[\frac{K_p^2}{(1-K_q)^2} + 1\right] = 0 \quad (12)$$

Expression (12) is an equation of second-order curves (ellipse), which describes the relationship between the squares of the amplitude of the current of the fundamental harmonic i_3^2 and subharmonic i_1^2, i.e. it expresses a curve whose invariants are equal to:

$\delta = \frac{567}{y} > 0$ S=45; $\Delta = 324\,[-8 + 7(k^2 + 1)]$,

where: $k = \frac{k_p}{1-k_q}$ (13)

It is known that $\Delta S = 324 \cdot 45[-8 + 7(k^2 + 1)] \le 0$. (14)

We have equations of real ellipses with coordinates k=0.378; $x_0 = \frac{8}{21} = 0{,}38$; $y_0 = \frac{16}{21} = 0{,}762$ with an ellipse slope $\alpha = 22^0 30'$.

From Fig.3b, the upper parts of the ellipses physically correspond to the region of existence of subharmonic oscillations at a frequency $\left(\frac{\omega}{3}\right)$.

Inequality (14) allows us to determine the parameters of the frequency divider at which stable frequency-dependent SHO can exist in the frequency converting devices at the frequency $\left(\frac{\omega}{3}\right)$. The direct AS separates the region of stable and unstable (dashed lines) the existence of SHO.

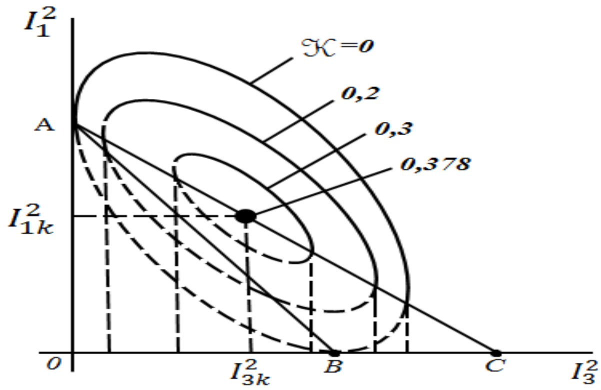

Fig. 3. Graphic representation of the input-output characteristics of the SHO $\left(\frac{\omega}{3}\right)$.

It should be noted that the determinant - Δ is a function of the parameters of the frequency divider circuit, within which the ferromagnetic element can convert the energy of a current of frequency 3ω coming to it from the network into current energy with a frequency of ω. $\Delta \le 0$ corresponds to the steady-state mode of excitation of the SHO, and $\Delta > 0$ corresponds to an imaginary ellipse: physically in this case they cannot exist in the GHS scheme, i.e. Because of the imbalance in the power of conversion and scattering in the system, the SHO breaks down.

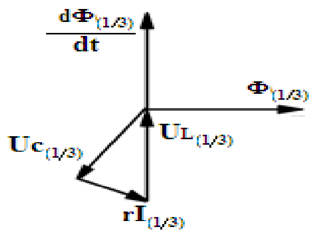

Fig. 4. Vector diagram in the SGK mode $\left(\frac{\omega}{3}\right)$.

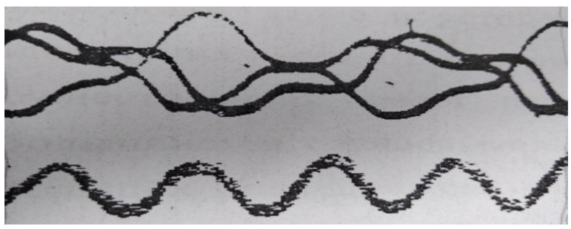

Fig. 5. Oscillograms of the SGK curves $\left(\frac{\omega}{3}\right)$ in three-phase circuits.

For the physical justification of the excitation of subharmonic oscillations according to the results of experimental and theoretical studies [8, 10, 12], the following conclusion can be made:

1. If a purely sinusoidal voltage with a frequency ω is applied to the nonlinear oscillatory circuit, then the SHO can arise in the circuit with a period equal to $\left(\frac{n}{m}\right)\omega$, where n,m are integers and n<m. Most often, oscillations occur with a frequency $\left(\frac{\omega}{m}\right)$, where m is an odd number.

The fact is that the energy for the development of the SHO comes from the network to the beat with fluctuations. It follows that the frequency of the SHO should be in a simple ratio with the frequency of the voltage source.

2. SHO is generated by the circuit itself, since they are not contained in the applied voltage. For the stable existence of the SHO with a certain frequency$\left(\frac{\omega}{m}\right)$, the natural frequency of the circuit should be equal to $\left(\frac{\omega}{m}\right)$. Therefore, in the presence of SHO, the resulting flow acquires such a value that provides the desired natural frequency of oscillations.

3. Since the voltage source does not contain subharmonics, the sum of the subharmonic components of the voltage drop in the coil $\left(U_{L\left(\frac{1}{3}\right)}\right)$, resistance $\left(I_{\left(\frac{1}{3}\right)} \cdot R\right)$ and capacitance $\left(U_{C\left(\frac{1}{3}\right)}\right)$ should be equal to zero, the vector diagram of which is shown in Fig. 4. From the vector diagram it can be seen that this is possible only if the subharmonic component of the current lags behind the subharmonic component of the flow in the coil by an acute angle. Consequently, SHO is excited at relatively lower resistances. Depending on the magnetization curve, subharmonics of odd orders may appear in the circuit under consideration, and subharmonics of even order may appear in systems with magnetization.

4. In contrast to single-phase electro-ferromagnetic circuits in three-phase systems, the higher and lower harmonics create direct, reverse and zero sequence vectors. In general, the phase shifts of the third-order SHO $\frac{\omega}{3}$ between phases in three-phase systems can be represented in three versions: 0^0, 40^0, 80^0; 0^0, 80^0, 160^0; 0^0, 160^0, 320^0, 0^0 (Fig. 5).

5. If phase shifts or phase ratios are measured with respect to the GHS (ω / 3), then the first and third variants of the GHS phase shifts in a three-phase system correspond to a direct phase sequence, and the second variant is the reverse phase sequence. In the general case, phase shifts of n-order subharmonics in an m-phase system can be determined from the expression; $\varphi_k = \frac{2\pi}{mn}(k-1)$, where k is the phase order. Therefore, with symmetric systems, SGCs can create various combinations of asymmetric systems. Depending on the structure and connection diagram of three-phase systems, only higher and lower harmonics can appear in phase, linear and neutral wires.

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