Synthesis of random parameter identification algorithms and estimates of current vessel motion parameters

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Abstract. The paper considers the task of synthesis of algorithms for identifying random parameters of a vessel, such as attached masses, moment of inertia, and estimating the current parameters of the vessel's motion from real-time measurements of onboard sensors. The task of the synthesis of algorithms for identifying random parameters of the vessel and evaluating the characteristics of the vessel's movement is to determine (evaluate) the current parameters (attached masses, moment of inertia) and the characteristics of the vessel's motion (position vector, speed) from the measurements of the vessel's motion, angular position and angular velocity of the vessel rotation).

1 Introduction

Knowing the current random parameters of the vessel allows to assess the characteristics of the movement of the vessel more accurately, to predict the parameters of the movement of the vessel at given points in time and to safely control the vessel when passing narrow straits.

To solve the above tasks and implement these algorithms on board of the vessel, it is necessary to have:

- onboard sensors measuring vessel vector mass center position (e.g. GLONASS or GPS sensors);
- onboard rate measuring sensors and the angular velocity and a linear acceleration of the vessel;
- local computer network with special algorithmic software;

on-board clock system.

2 Formulation of the problem

The controlled movement of the vessel in the horizontal plane is determined by a vector system of nonlinear equations formed as

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$$\frac{d\bar{X}}{dt} = f(\bar{X}, \bar{U}, \bar{P}) + \bar{\varsigma}(t),\ \bar{X}(t_0) = \bar{X}_0 \qquad (1)$$

where $\bar{X}(t)$ – phase coordinate vector, describing the movement of the ship; $\bar{U}(t)$ – vector of control actions (forces and moments from the steering device and longitudinal traction); $\bar{P}$ – vessel parameters vector (mass, moment of inertia, draft, length, width, force and moment coefficients, and a number of others); $\bar{\varsigma}(t)$ – vector random process determined by random influences of waves, wind, streams and other factors.

Phase coordinate vector $\bar{X} = (x, y, v_x, v_y, \psi, \omega_z)$, where x, y – vessel coordinates in a local Cartesian coordinate system (LCCS), v_x, v_y, – coordinates of the vessel's velocity vector in a bounded coordinate system (BCS), ψ – heading angle of the vessel; ω_z – angular velocity of rotation of the vessel around the vertical axis. Vector function $f(\bar{X}, \bar{U}, \bar{P}) = (x_t, y_t, a_x, a_y, \omega_z, \omega_{Zt})$, where x_t,y_t, – coordinates of the vessel's velocity vector in LCCS, a_x,a_y – linear acceleration of the vessel in the horizontal plane, ω_{Zt} – angular acceleration of rotation of the vessel, $\bar{P} = (m_{11}, m_{22}, J_{ZP},)$ – random parameters vector.

The coordinates of the speed, linear and angular accelerations of the vessel depend on the phase coordinates and random parameters and are determined by the expressions

$$x_t = v_x\cos(\psi) - v_y\sin(\psi),\quad y_t = v_x\sin(\psi) + v_y\cos(\psi),$$

$$a_x = (F_{xi}(v_y,\omega_z, m_{22}) + F_{xg}(v_x,v_y, \omega_z) + F_{xr}(v_x,v_y,\omega_z) + T_v(v_x,v_y,n_{rot}))/m_{11}, \qquad (2)$$

$$a_y = (F_{yi}(v_x,\omega_z, m_{11}) + F_{yg}(v_x,v_y, \omega_z) + F_{yr}(v_x,v_y, \omega, \delta))/m_{22},$$

$$\omega_{Zt} = (M_{zi}(v_x,v_y, \omega_z,m_{11},m_{22}) + M_{zr}(v_x,v_y, \omega_z,m_{22}) + M_{zg}(v_x,v_y, \omega_z))/J_{zp}.$$

The vector of control actions is the vector $\bar{U} = (n_{об}, \delta)$ vector, where n_{rot} – rotational speed of the propeller (defining a longitudinal rod), δ – rudder angle (determining the forces and moments acting on the vessel).

Regarding the phase coordinate vector $\bar{X}(t)$ at discrete points in time t_i vector is measured as

$$\bar{Y}(t_i) = \bar{\varphi}(\bar{X}(t_i), \bar{P}) + \Sigma(t_i)\,\xi(t_i), \qquad (3)$$

where $\Sigma(t_i)$ - matrix of standard errors of measurement (RMSE); $\xi(t_i)$ – discrete white noise with mutually independent coordinates and unit dispersions.

Vector function $\bar{\varphi}(\bar{X}(t_i), \bar{P})$ has the form

$$\bar{\varphi}(\bar{X}(t_i), \bar{P}) = (x, y, \psi, \omega_Z, a_x, a_y, \omega_{Zt}), \qquad (4)$$

For selected phase coordinates and control actions, in quiet weather and deep water ($\varsigma(t) = 0$), vector function $f(\bar{X}, \bar{U}, \bar{P})$ in the equation (1), forces and moments in expression (4) are presented in the article [1].

The problem of synthesis algorithms random identification parameters and evaluating the characteristics of movement of the vessel is according the vectors of measurements $\mathbf{Y}(t_0)$,$\mathbf{Y}(t_1)$,.., $\mathbf{Y}(t_i)$ to determine estimates of the vector of random parameters $\hat{P}(t_i) = (\hat{m}_{11}(t_i), \hat{m}_{22}(t_i), \hat{J}_{zp}(t_i))$ and phase coordinates vector $\hat{X}(t_i) = (\hat{x}(t_i), \hat{y}(t_i), \hat{v}_x(t_i), \hat{v}_y(t_i), \hat{\Psi}(t_i), \hat{\omega}_Z(t_i))$ by the criterion of the minimum standard deviation of the assessment.

3 Synthesis task

The exact solution to the synthesis problem posed can be solved by polynomial filtration methods [2], [3], [4]. However, this solution is quite complicated in a computational sense. The synthesis problem with acceptable accuracy in a constructive form (in the form of linear recurrence equations) can be solved using the methods of linear Kalman filters [5] and the correlation theory of filtration [6].

Under the assumption that at time t_{i-1} there are estimates $\hat{\mathbf{X}}(t_{i-1})$, $\hat{\mathbf{P}}(t_{i-1})$ and variance error matrices of these estimates $\mathbf{K_{XX}}(t_{i-1})$, $\mathbf{K_{XP}}(t_{i-1})$, $\mathbf{K_{PP}}(t_{i-1})$ and the vector of control actions is also known as $\mathbf{U}(t_{t-1}) = \mathbf{U_o}(t_{t-1})$, we linearize the vector function $\mathbf{f(X,U,P)}$ regarding ratings

$$\mathbf{f(X,U,P)} \approx \mathbf{f}(\hat{\mathbf{X}}(t_{i-1}),\mathbf{U}_o(t_{i-1}),\hat{\mathbf{P}}(t_{i-1})) + \mathbf{A_X}(\hat{\mathbf{X}}(t_{i-1}),\mathbf{U}_o(t_{i-1}),\hat{\mathbf{P}}(t_{i-1}))(\mathbf{X}(t_{i-1}) - \hat{\mathbf{X}}(t_{i-1})) + \mathbf{A_P}(\hat{\mathbf{X}}(t_{i-1}),\mathbf{U}_o(t_{i-1}),\hat{\mathbf{P}}(t_{i-1}))(\mathbf{P}(t_{i-1}) - \hat{\mathbf{P}}(t_{i-1})). \tag{5}$$

Vector rating $\mathbf{X}(t_i)$ according to measurements up to the moment t_{i-1} (i.e. forecast estimate $\hat{\mathbf{X}}(t_{i-1})$ at the time t_i) might be written as

$$\hat{\mathbf{X}}(t_i|t_{i-1}) = \hat{\mathbf{X}}(t_{i-1}) + \mathbf{f}(\hat{\mathbf{X}}(t_{i-1}),\mathbf{U}_o(t_{i-1}),\hat{\mathbf{P}}(t_{i-1}))\,dt, \tag{6}$$

where dt – time rate of measurements income.

Predicted at the moment of time t_i dispersion matrices might be written in the form

$$\mathbf{K_{XX}}(t_i|\, t_{i-1}) = (\mathbf{I} + \mathbf{A_X}(\cdot)dt)\mathbf{K_{XX}}(t_{i-1})(\mathbf{I} + \mathbf{A_X}(\cdot)dt)^T + (\mathbf{I} + \mathbf{A_X}(\cdot)dt)\mathbf{K_{XP}}(t_{i-1})\mathbf{A_P}^T(\cdot)dt +$$
$$+ \mathbf{A_P}(\cdot)dt\mathbf{K_{XP}}^T(t_{i-1})(\mathbf{I} + \mathbf{A_X}(\cdot)dt)^T + \mathbf{A_P}(\cdot)dt\mathbf{K_{PP}}(t_{i-1})\mathbf{A_P}^T(\cdot)dt,$$
$$\mathbf{K_{XP}}(t_i|\, t_{i-1}) = (\mathbf{I} + \mathbf{A_X}(\cdot)dt)\mathbf{K_{XP}}(t_{i-1}) + \mathbf{A_P}(\cdot)dt\mathbf{K_{PP}}(t_{i-1}), \quad \mathbf{K_{PP}}(t_i|\, t_{i-1}) = \mathbf{K_{PP}}(t_{i-1}), \tag{7}$$

where **I –** unit matrix, index "T" – means matrix transpose.

Vector rating $\mathbf{Y}(t_i)$ according to measurements up to the moment t_{i-1} might be written in the form

$$\hat{\mathbf{Y}}(t_i|t_{i-1}) = \boldsymbol{\varphi}(\hat{\mathbf{X}}(t_i|t_{i-1}), \hat{\mathbf{P}}(t_{i-1})). \tag{8}$$

We linearize the vector function $\boldsymbol{\varphi}(\mathbf{X}(t_i),\mathbf{P})$ regarding ratings $\hat{\mathbf{X}}(t_i|t_{i-1})$, $\hat{\mathbf{P}}(t_{i-1})$

$$\boldsymbol{\varphi}(\mathbf{X}(t_i),\mathbf{P}) \approx \hat{\mathbf{Y}}(t_i|t_{i-1}) + \mathbf{C_X}(\hat{\mathbf{X}}(t_i|t_{i-1}), \hat{\mathbf{P}}(t_{i-1}))(\mathbf{X}(t_{i-1}) - \hat{\mathbf{X}}(t_i|t_{i-1})) +$$
$$\mathbf{C_P}(\hat{\mathbf{X}}(t_i|t_{i-1}), \hat{\mathbf{P}}(t_{i-1}))(\mathbf{P} - \hat{\mathbf{P}}(t_{i-1})). \tag{9}$$

The dispersion matrix of errors in the estimation of the predicted measurement vector is determined by the expression

$$\mathbf{K_{YY}}(t_i|\, t_{i-1}) = \mathbf{C_X}(\cdot)\mathbf{K_{XX}}(t_i|\, t_{i-1})\mathbf{C_X}^T(\cdot) + \mathbf{C_X}(\cdot)\mathbf{K_{XP}}(t_i|\, t_{i-1})\mathbf{C_P}^T(\cdot) +$$
$$\mathbf{C_P}(\cdot)\mathbf{K_{XP}}^T(t_i|\, t_{i-1})\mathbf{C_X}^T(\cdot) + \mathbf{C_P}(\cdot)\mathbf{K_{PP}}(t_i|\, t_{i-1})\mathbf{C_P}^T(\cdot) + \textstyle\sum(t_i)\sum^T(t_i). \tag{10}$$

Expressions for matrix elements $\mathbf{A_X}(\cdot)$, $\mathbf{A_P}(\cdot)$, $\mathbf{C_X}(\cdot)$, $\mathbf{C_P}(\cdot)$, are given in the Application. Linearized recurrence algorithms for identifying random parameters and estimating the motion characteristics of a vessel according to [6] are determined by the equations

$$\hat{\mathbf{X}}(t_i) = \hat{\mathbf{X}}(t_i|t_{i-1}) + \mathbf{D_X}(t_i)(\mathbf{Y}(t_i) - \hat{\mathbf{Y}}(t_i|t_{i-1})), \quad \hat{\mathbf{X}}(t_0) = \mathbf{X}_0,$$

$$\hat{\mathbf{P}}(t_i) = \hat{\mathbf{P}}(t_{i-1}) + \mathbf{D_P}(t_i)(\mathbf{Y}(t_i) - \hat{\mathbf{Y}}(t_i|t_{i-1})), \quad \hat{\mathbf{P}}(t_0) = \mathbf{P_0}, \tag{11}$$

where matrices of weighting coefficients of residuals of measurements $\mathbf{D_X}(t_i)$, $\mathbf{D_P}(t_i)$ are determined by the expressions

$$\mathbf{D_X}(t_i) = (\mathbf{K_{XX}}(t_i|\, t_{i-1})\mathbf{C_X}^T(\cdot) + \mathbf{K_{XP}}(t_i|\, t_{i-1})\mathbf{C_P}^T(\cdot))\mathbf{K_{YY}}^{-1}(t_i|\, t_{i-1})$$

$$\mathbf{D_P}(t_i) = (\mathbf{K_{XP}}^T(t_i|\, t_{i-1})\mathbf{C_X}^T(\cdot) + \mathbf{K_{PP}}(t_i|\, t_{i-1})\mathbf{C_P}^T(\cdot))\mathbf{K_{YY}}^{-1}(t_i|\, t_{i-1}). \tag{12}$$

Dispersion matrix

$$\mathbf{K}(t_i) = \begin{bmatrix} \mathbf{K_{xx}}(t_i), \mathbf{K_{xp}}(t_i) \\ \mathbf{K_{xp}}(t_i), \mathbf{K_{pp}}(t_i) \end{bmatrix}$$

satisfies the recurrence matrix equation

$$\mathbf{K}(t_i) = \mathbf{K}(t_i|\, t_{i-1}) - \mathbf{D}(t_i)\mathbf{C}(t_i)\mathbf{K}(t_i|\, t_{i-1}),\ \mathbf{K}(t_{0i}) = \mathbf{K_0}, \tag{13}$$

где $\mathbf{D}(t_i) = \begin{bmatrix} \mathbf{D_x}(t_i) \\ \mathbf{D_p}(t_i) \end{bmatrix}$, $\mathbf{C}(t_i) = \begin{bmatrix} \mathbf{C_x}(t_i), \mathbf{C_p}(t_i) \end{bmatrix}$..

4 Analysis of the accuracy of synthesized algorithms

The accuracy analysis of the algorithms was carried out by the method of statistical modeling for a vessel of the “Volga-Balt” type. Vessel dynamic characteristics and design parameters m_{11}, m_{22}, J_{zz} were determined according to [7-9]. Steering of the vessel $U_o(t) = [n_{rot}, \delta(t)]$ were determined by the rotational speed of the propeller n_{rot} = 3,5 r/s and the angle of steering to the maximum value $\delta m = 30^o$ from the left side to the right with a periodicity T_{per}=210 s.

Accuracy, for various options of on-board sensors for measuring the characteristics of the vessel’s movement are shown in Table 1.

Table 1. Accuracy, for various variants of sensors for measuring of the motion parameters of a vessel

Accuracy Option	σ_x[m]	σ_ψ[rad]	σ_ω[rad/s]	$\sigma_{\omega t}$[rad/s^2]	σ_a[m/s^2]
#1	1	$3*10^{-4}$	10^{-6}	10^{-7}	10^{-4}
#2	3	$3*10^{-3}$	10^{-5}	10^{-6}	10^{-3}
#3	5	$3*10^{-2}$	10^{-4}	10^{-5}	10^{-2}

Accuracy (mathematical expectation and standard deviation) of estimation of motion characteristics and realized random parameters $\mathbf{P} = [m_{11}, m_{22}, J_{zp}]$ ships in steady state for various accuracy options for measuring instruments are shown in Table 2.

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Table 2. The accuracy of the assessment of the characteristics of the vessel in steady state for various options for the accuracy of measuring instruments

Accuracy Option	M $\hat{x}$ [m] σ $\hat{x}$ [m]	M $\hat{V}$ [m/s] σ $\hat{V}$ [m/s]	M $\hat{\psi}$ [rad] σ $\hat{\psi}$ [rad]	M $\hat{\omega}$ [rad/s] σ $\hat{\omega}$ [rad/s]	M $\hat{m}_{11}$ [tf∗s²/m] σ $\hat{m}_{11}$ [tf∗s²/m]	M $\hat{m}_{22}$ [tf∗s²/m] σ $\hat{m}_{22}$ [tf∗s²/m]	M $\hat{J}$ [tf∗s²/m] σ $\hat{J}$ [tf∗s²/m]
#1	0.001 0.026	$1.8{*}10^{-5}$ 0.0038	$0.063{*}10^{-5}$ $1.129{*}10^{-5}$	$0.039{*}10^{-7}$ $2.294{*}10^{-7}$	-0.0016 0.041	0.005 0.035	15.6 149
#2	-0.019 0.18	-10^{-4} 0.006	$9.3{*}10^{-6}$ $1.07{*}10^{-4}$	$7.72{*}10^{-8}$ $1.82{*}10^{-6}$	0.061 0.116	0.021 0.045	-348.9 219.8
#3	-0.09 0.26	$4.9{*}10^{-4}$ 0.013	$5.86{*}10^{-5}$ $5.87{*}10^{-4}$	$-6.2{*}10^{-8}$ $5.74{*}10^{-6}$	0.241 0.195	0.079 0.101	-1036 422.2

The figures show the implementation of the relative error in estimating the speed (in percent), errors in estimating the attached masses and the moment of inertia of the vessel for the accuracy of Option No. 1.

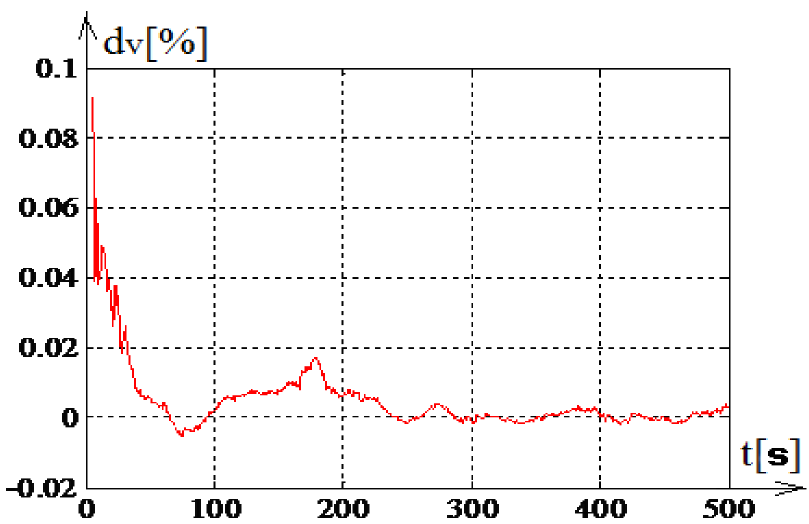

Fig. 1. Relative error of speed estimation

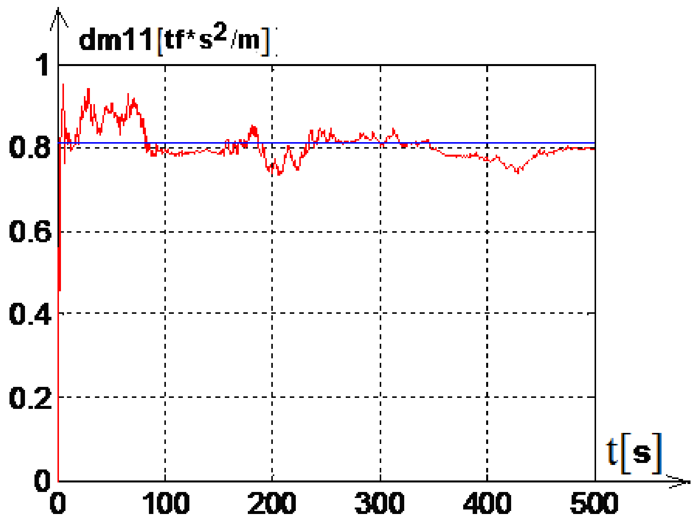

Fig. 2. Assessment of the attached mass

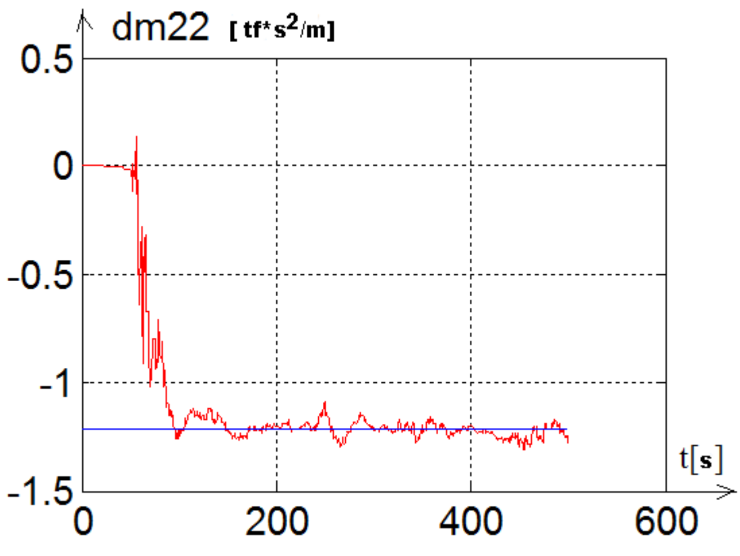

Fig. 3. Assessment of the attached mass

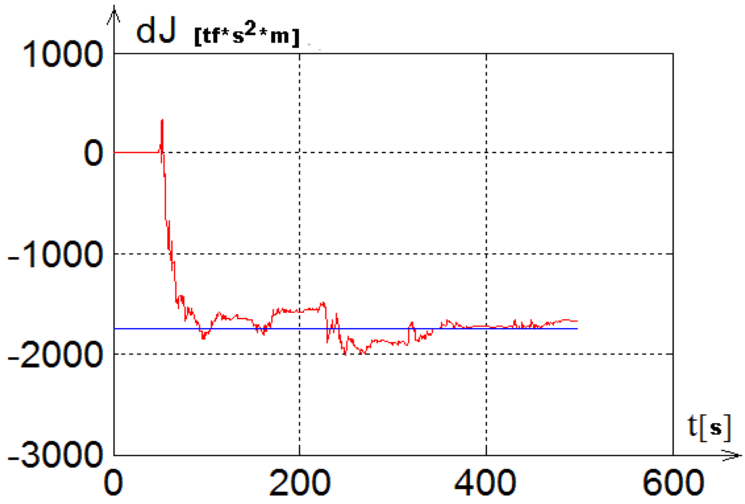

Fig. 4. Assessment of the attached moment of inertia

5 Conclusions

The paper provides a solution to the problem of synthesis of algorithms for identifying random parameters and evaluating the characteristics of the vessel. For synthesized algorithms, statistical modeling of their accuracy estimation has been carried out. The use of on-board high-precision sensors for measuring the characteristics of the vessel's movement, in particular, angular and linear acceleration sensors, allows real-time identification of random parameters and assessment of the current characteristics of the vessel's movement. Having estimates of the true parameters and the current characteristics of the vessel's movement, it is possible to solve the problem of predicting the characteristics of the movement at the moments when the vessel passes through the bottlenecks of the strait and to carry out safer control of the vessel.

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