Approximation of limit amplitude diagram for destructive elements of safety devices

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Abstract. The maximum stress of the cycle should be limited not by yield strength but by tensile strength of detail material intended to be destructed. Therefore, such widespread linear approximations of Haigh diagram and conditions of Soderberg and Serensen-Kinasoshvili are not applicable for such details. Safety factor of fatigue strength of safety devices destructive elements should not exceed 1.0. Therefore, it is undesirable to use modified condition of Goodman for destructive elements. Two new variants of linear approximation of Haigh limit amplitudes diagram were developed. Both variants have proven their efficiency in design and operation of safety devices with destructive elements in framework of laboratory 'Protection of metallurgical machines from breakdowns' of the Chief mechanic department of PJSC 'ILYICH IRON AND STEEL WORKS' (Mariupol city, Ukraine).

1 Introduction

Details of metallurgical equipment are exposed to asymmetric loading cycles in vast majority of cases [1-5]. It is known [6-12] that presence of medium tensile stresses reduces and presence of medium compressive stresses increases fatigue limit. However, in case of tangential stresses (e.g., details working under torsion or shear) constant component of stress cycle τ_m always reduces fatigue strength of detail. Decrease of fatigue strength of safety devices destructing elements during their operation is dangerous because safety devices must be triggered (and destructing elements destroyed) at the same stable value of breaking load [1, 13-15].

To determine fatigue strength of detail under action of stresses with asymmetric cycles [16, 17] various types diagrams are developed. The most common diagrams are:

- a) diagram of limit stresses of cycle in coordinates $\sigma_{\max}, \sigma_{\min} - \sigma_m$ (Smith chart);
- b) diagram of limit amplitudes of cycle in coordinates $\sigma_a - \sigma_m$ (Haigh diagram).

Where $\sigma_{\max}$ is the maximum stress of cycle;

$\sigma_{\min}$ is the minimum stress of cycle;

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σ_m is constant component of stress cycle;

σ_a is amplitude (variable component) of stress cycle.

It is necessary to perform big series of experiments to develop such diagrams [18-21]. It is not always technically possible (e.g., for large details such as spindles). In this case it is necessary to use approximate calculations based on simplification of experimental curves [4].

Purpose of this paper is to analyze existing variants for approximating Haigh diagram of limit amplitudes and develop new variants for details intended to be destructed, i.e. for destructing elements of safety devices.

2 Results

Haigh diagram presented on fig. 1 (curve 1) can be considered. Stress state of detail on that diagram is represented by single point with ordinate σ_a and abscissa σ_m . Similar cycles with the same ratio σ_a/σ_m are represented by rays emerging from origin at angle to the abscissa tangent of which is equal to this ratio. Row of points located on curve 1 (refer to fig. 1) shows stress state of the detail at fatigue limit depending on characteristics of the cycle. Value of the fatigue limit for a given ratio σ_a/σ_m is determined by sum of coordinates of point on curve 1. Static load corresponds to point B (intersection of curve 1 with the

abscissa axis): $\sigma_a = 0$; $\sigma_m = \sigma_b$. Fatigue limit of the detail $\sigma_{-1}^{\text{det.}}$ in symmetric cycle ($\sigma_m = 0$) is represented by segment OA on the ordinate axis. Absence of fatigue failure of the detail is guaranteed when point showing the stress state of the detail at given loading cycle lies inside curve AB (e.g., point K on fig. 1). Fatigue failure of the detail can occur when such point on the limit amplitudes diagram lies outside the curve AB (e.g., point P on fig. 1).

Fig. 1. Known linear approximations variants of Haigh limit amplitude diagram: 1 is Haigh diagram; 2 is condition of Soderberg; 3 is modified condition of Goodman; 4 is condition of Serensen-

Kinasoshvili; $\sigma_{-1}^{\text{det.}}$ is fatigue limit of the detail at symmetrical cycle; $\sigma_0^{\text{det.}}$ is fatigue limit of the detail at zero cycle; σ_b is tensile strength of the detail material; σ_y is yield strength of the detail material.

Two types of approximation of the limit amplitudes diagram are possible: linear and nonlinear. Linear ones are more preferred due to their simplicity. The most common are:

a) condition of Soderberg (refer to fig. 1, line 2);

$$\sigma_{\alpha} = \sigma_{-1}^{\text{det.}} \left(1 - \frac{\sigma_m}{\sigma_y} \right), \quad (1)$$

where $\sigma_{-1}^{\text{det.}}$ is fatigue limit of the detail at symmetrical cycle;
 σ_y is yield strength of the detail material.

b) modified condition of Goodman (refer to fig. 1, line 3);

$$\sigma_{\alpha} = \sigma_{-1}^{\text{det.}} \left(1 - \frac{\sigma_m}{\sigma_B} \right), \quad (2)$$

where σ_B is yield strength of the detail material.

c) condition of Serensen-Kinasoshvili (refer to fig. 1, polygonal chain 4):

$$\begin{cases} \sigma_m + \sigma_{\alpha} = \sigma_y; \\ \sigma_{\alpha} = \sigma_{-1}^{\text{det.}} - \psi_{\sigma} \cdot \sigma_m, \end{cases} \quad (3)$$

where $\psi_{\sigma} = \frac{2\sigma_{-1}^{\text{det.}} - \sigma_0^{\text{det.}}}{\sigma_0^{\text{det.}}}$ is coefficient correcting effect of constant component of cycle on fatigue resistance;

$\sigma_0^{\text{det.}}$ is fatigue limit of the detail at zero cycle;

Here, under $\sigma_{-1}^{\text{det.}}$ and $\sigma_0^{\text{det.}}$ should be understood fatigue limits of the particular detail (taking into account stress concentration coefficient, surface quality, etc.) and not laboratory standard sample.

Two of the above given conditions (Soderberg and Serensen-Kinasoshvili) limit the maximum stresses in the detail to yield strength. Therefore, they are completely inapplicable to destructing elements which are designed to be broken and, therefore, work at stresses $\sigma_{\max} \leq \sigma_B$. The third linear approximation (modified condition of Goodman) limits the maximum stresses in the detail to ultimate strength. It can be applied to destructing elements, although it is very approximate. E.g., according to condition of Goodman at point K on fig. 1 fatigue strength of the detail is not provided, whereas it is guaranteed in reality. Indicated approximation of condition of Goodman goes into fatigue reserve of the detail; however, for destructing elements (always calculated according to limiting states) even the maximum fatigue strength safety factor should not exceed 1.0 [1]. In this regard, it seems appropriate to propose new variants for approximating Haigh diagram.

VARIANT 1. Proposed polygonal chain (refer to fig. 2) consists of two straight sections. Line 1 is initial section of condition of Serensen-Kinasoshvili. It is constructed as follows.

Point A is put on ordinate axis corresponding to fatigue limit of the detail at symmetrical stress cycle. OL ray is put from the origin at angle of 45° that characterizes set of zero cycles. Half of fatigue limit of the detail at zero stress cycle is put along the abscissa axis. $\sigma_0^{\text{det.}}/2$ is put parallel to the ordinate axis from obtained point E. Point D is obtained at intersection with the OL ray. Then, value of yield strength of the detail material is plotted on the abscissa axis. TM ray is put at angle of 45° to the abscissa axis from obtained point

T limiting the maximum stresses in the detail to yield strength. Point C is obtained by connecting the points A and D and continuing line to intersection with the TM ray.

Fig. 2. Proposed variant of approximation of the limit amplitudes diagram: 1 is line of Serensen-Kinasoshvili; 2 is new approximation section.

If $\sigma_{\max} \leq \sigma_y$, then according to (3):

$$\sigma_{\alpha} = \sigma_{-1}^{\text{det.}} - \psi_{\sigma} \cdot \sigma_m$$

For tangential stresses it is:

$$\tau_{\alpha} = \tau_{-1}^{\text{det.}} - \psi_{\tau} \cdot \tau_m$$

It is proposed to approximate section of the limit amplitudes diagram of by line CB (line 2 on fig. 2) for maximum cycle stresses exceeding σ_y :

$$\sigma_{\alpha} = \sigma_{\alpha}^c - \text{tg } \beta (\sigma_m - \sigma_m^c) = \sigma_{\alpha}^c - \frac{\sigma_{\alpha}^c}{\sigma_B - \sigma_m^c} (\sigma_m - \sigma_m^c) = \sigma_{\alpha}^c \left(1 - \frac{\sigma_m - \sigma_m^c}{\sigma_B - \sigma_m^c} \right), \quad (4)$$

where σ_{α}^c is ordinate of point C;

σ_m^c is abscissa of point C, moreover:

$$\sigma_m^c + \sigma_{\alpha}^c = \sigma_y, \quad (5)$$

because the point C belongs to the TM ray on which $\sigma_{\max} = \sigma_y$. The point C also belongs to line AC, therefore

$$\sigma_{\alpha}^c = \sigma_{-1}^{\text{det.}} - \psi_{\sigma} \cdot \sigma_m^c. \quad (6)$$

Substituting (6) into (5), it is:

$$\sigma_m^c (1 - \psi_{\sigma}) = \sigma_y - \sigma_{-1}^{\text{det.}}. \quad (7)$$

Counting together (7) and (6), it is:

$$\begin{cases} \sigma_m^c = \frac{\sigma_y - \sigma_{-1}^{\text{det.}}}{1 - \psi_{\sigma}}; \\ \sigma_{\alpha}^c = \sigma_{-1}^{\text{det.}} - \psi_{\sigma} \cdot \frac{\sigma_y - \sigma_{-1}^{\text{det.}}}{1 - \psi_{\sigma}}. \end{cases} \quad (8)$$

Equation of line 2 within CB segment is obtained by substituting (8) into (4):

$$\sigma_{\alpha} = \left(\sigma_{-1}^{\text{det.}} - \psi_{\sigma} \cdot \frac{\sigma_y - \sigma_{-1}^{\text{det.}}}{1 - \psi_{\sigma}} \right) \frac{\sigma_B - \sigma_m}{\sigma_B - \frac{\sigma_y - \sigma_{-1}^{\text{det.}}}{1 - \psi_{\sigma}}} \quad (9)$$

Similarly for tangential stresses:

$$\tau_{\alpha} = \left(\tau_{-1}^{\text{det.}} - \psi_{\tau} \cdot \frac{\tau_y - \tau_{-1}^{\text{det.}}}{1 - \psi_{\tau}} \right) \frac{\tau_B - \tau_m}{\tau_B - \frac{\tau_y - \tau_{-1}^{\text{det.}}}{1 - \psi_{\tau}}}$$

In case when ψ_{σ} and $\psi_{\tau} \rightarrow 0$ (e.g., for low-carbon steel grades)

$$\begin{aligned} \sigma_{\alpha} &= \sigma_{-1}^{\text{det.}} \cdot \frac{\sigma_B - \sigma_m}{\sigma_B - \sigma_y + \sigma_{-1}^{\text{det.}}}; \\ \tau_{\alpha} &= \tau_{-1}^{\text{det.}} \cdot \frac{\tau_B - \tau_m}{\tau_B - \tau_y + \tau_{-1}^{\text{det.}}} \end{aligned} \quad (10)$$

In this case, Serensen-Kinasoshvili line 1 is horizontal section AC (refer to fig. 3). If $\tau_{\max} \leq \tau_y$ then $\tau_{\alpha} = \tau_{-1}^{\text{det.}}$. The point C is positioned at the intersection of the TM ray (drawn from the point T with abscissa τ_y at angle of 45° to the abscissa axis) and horizontal ray (drawn from the point A with ordinate $\tau_{-1}^{\text{det.}}$).

Finally, following linear approximation of the limit amplitudes diagram is obtained.

- when $\sigma_{\max} \leq \sigma_y$:

$$\sigma_{\alpha} = \sigma_{-1}^{\text{det.}} - \psi_{\sigma} \cdot \sigma_m \quad (11)$$

- when $\sigma_y \leq \sigma_{\max} \leq \sigma_B$:

$$\sigma_{\alpha} = \left(\sigma_{-1}^{\text{det.}} - \psi_{\sigma} \cdot \frac{\sigma_y - \sigma_{-1}^{\text{det.}}}{1 - \psi_{\sigma}} \right) \frac{\sigma_B - \sigma_m}{\sigma_B - \frac{\sigma_y - \sigma_{-1}^{\text{det.}}}{1 - \psi_{\sigma}}}$$

For destructive elements from low-carbon steel grades (which is 95% of all destructive elements) a much simpler variant is:

Fig. 3. Variant of approximation of the limit amplitudes diagram at $\psi_{\tau} = 0$: 1 is line of Serensen-Kinasoshvili; 2 is new approximation section; 3 is initial Haigh diagram.

- when $\sigma_{\max} \leq \sigma_y$:

$$\sigma_{\alpha} = \sigma_{-1}^{\det.} \quad (12)$$

- when $\sigma_y \leq \sigma_{\max} \leq \sigma_B$:

$$\sigma_{\alpha} = \sigma_{-1}^{\det.} \frac{\sigma_B - \sigma_m}{\sigma_B - \sigma_y + \sigma_{-1}^{\det.}}$$

Next, several non-linear approximations of Haigh diagram are considered. The most well-known are (refer to fig. 4):

a) Gerber parabola:

$$\sigma_{\alpha} = \sigma_{-1}^{\det.} \left[1 - \left(\frac{\sigma_m}{\sigma_B} \right)^2 \right]; \quad (13)$$

b) condition of Birger-Mavlyutov:

$$\sigma_{\alpha} = \sigma_{-1}^{\det.} \sqrt{1 - \frac{\sigma_m}{\sigma_B}}. \quad (14)$$

It should be noted that (14) is obtained from more general condition:

$$\left(\frac{\sigma_{\alpha}}{\sigma_{-1}^{\det.}} \right)^m = 1 - \left(\frac{\sigma_m}{\sigma_B} \right)^n,$$

moreover, for carbon and alloy steel grades $m=2$ and $n=1$. Graphic plots of these approximation variants intersect in the point C (refer to fig. 4).

Abscissa and ordinate of this point can be found by equating (13) and (14):

$$1 - \left(\frac{\sigma_m}{\sigma_B} \right)^2 = \sqrt{1 - \frac{\sigma_m}{\sigma_B}}.$$

The fourth degree equation can be obtained by designation $\sigma_m/\sigma_B = x$:

$$x^4 - 2x^2 + x = 0. \quad (15)$$

Real roots of (15) are $x_1 = 0$ (point A); $x_2 = 1.0$ (point B) and $x_3 = 0.618$ (point C). Thus,

the point C has an abscissa of $0.618\sigma_B$ and an ordinate of $0.618\sigma_{-1}^{\det.}$.

Fig. 4. Nonlinear approximations of Haigh diagram: 1 is Gerber parabola; 2 is condition of Birger-Mavlyutov.

VARIANT 2. Gerber parabola and condition of Birger-Mavlyutov are convenient to approximate with two line segments, namely, AC and CB, taking the point C as the base point (breaking point). Then equation of the line AC (refer to fig. 5) is:

Fig. 5. Proposed approximation variant of Gerber parabola and condition of Birger-Mavlyutov.

$$\text{tg} \alpha = \frac{\sigma_{-1}^{\text{det.}} - 0.618 \sigma_{-1}^{\text{det.}}}{0.618 \sigma_B} = \frac{0.382 \sigma_{-1}^{\text{det.}}}{0.618 \sigma_B} = 0.618 \frac{\sigma_{-1}^{\text{det.}}}{\sigma_B}.$$

where

After substituting (17) in (16) it is:

$$\sigma_{\alpha} = \sigma_{-1}^{\text{det.}} \left(1 - \frac{0.618 \sigma_m}{\sigma_B} \right). \quad (18)$$

For tangential stresses it is:

$$\tau_{\alpha} = \tau_{-1}^{\text{det.}} \left(1 - \frac{0.618 \tau_m}{\tau_B} \right).$$

Equation of the line CB is obtained from (4) where $\sigma_{\alpha}^c = 0.618 \sigma_{-1}^{\text{det.}}$ and $\sigma_m^c = 0.618 \sigma_B$:

$$\sigma_{\alpha} = 0.618 \sigma_{-1}^{\text{det.}} \left(1 - \frac{\sigma_m - 0.618 \sigma_B}{\sigma_B - 0.618 \sigma_B} \right). \quad (19)$$

Finally, simplifying (19) it is:

$$\sigma_{\alpha} = 1.62 \sigma_{-1}^{\text{det.}} \left(1 - \frac{\sigma_m}{\sigma_B} \right). \quad (20)$$

For tangential stresses it is:

$$\tau_{\alpha} = 1.62 \tau_{-1}^{\text{det.}} \left(1 - \frac{\tau_m}{\tau_B} \right).$$

In this case limit stress is the maximum cycle stress:

$$\sigma_{\max} = 0.618 (\sigma_B + \sigma_{-1}^{\text{det.}}). \quad (21)$$

Thus, the second variant of linear approximation of Haigh diagram proposed in this paper can be reduced to two following equations:

- when

$$\sigma_{\max} \leq 0.618(\sigma_B + \sigma_{-1}^{\det.}) \quad \sigma_{\alpha} = \sigma_{-1}^{\det.} \left(1 - \frac{0.618\sigma_m}{\sigma_B} \right); \quad (22)$$

- when

$$0.618(\sigma_B + \sigma_{-1}^{\det.}) \leq \sigma_{\max} \leq \sigma_B \quad \sigma_{\alpha} = 1.62\sigma_{-1}^{\det.} \left(1 - \frac{\sigma_m}{\sigma_B} \right).$$

Both variants of the linear approximation of Haigh limit amplitude diagram proposed in this paper are shown on fig. 6.

Fig. 6. Variants of the linear approximation of the limit amplitude diagram for destructive elements of safety devices: 1 is linear approximation of Haigh diagram which uses condition of Serensen-Kinasoshvili; 2 is linear approximation of Haigh diagram which considers conditions of Gerber and Birger-Mavlyutov.

Comparative analysis of the proposed approximation variants is given below. Four characteristics of the material σ_B , σ_y , $\sigma_{-1}^{\det.}$, $\sigma_0^{\det.}$ must be known in order to use the first variant. Two characteristics of the material σ_B and $\sigma_{-1}^{\det.}$ must be known in order to use the second variant. Therefore, the second variant is more preferable. Calculation formulas (22) are simpler than (11). However, physical meaning of the breaking point of the diagram is more clearly expressed in the first case where it is a point with $\sigma_{\max} = \sigma_y$. Safety factor of fatigue strength (which for destructive elements should not exceed 1.0) is less for the first approximation. Thus, each of the proposed variants has its own advantages and disadvantages. The first variant of Haigh diagram approximation can be recommended for ductile materials (e.g., low-carbon steel grades) because formulas (12) are quite simple. The second approximation variant is more preferable for brittle (high-strength) materials.

3 Conclusions

1. The maximum stress of the cycle should be limited not by yield strength but by tensile strength of detail material intended to be destructed. Therefore, such widespread linear approximations of Haigh diagram and conditions of Soderberg and Serensen-Kinasoshvili are not applicable for such details.
2. Safety factor of fatigue strength of safety devices destructive elements should not exceed 1.0. Therefore, it is undesirable to use modified condition of Goodman for destructive elements.
3. Two new variants of linear approximation of Haigh limit amplitudes diagram were developed. Both variants have proven their efficiency in design and operation of

safety devices with destructive elements in framework of laboratory ‘Protection of metallurgical machines from breakdowns’ of the Chief mechanic department of PJSC ‘ILYICH IRON AND STEEL WORKS’ (Mariupol city, Ukraine).

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