

Method Prediction for the Environment of Vibration Structure in Consideration with Elastic Joint

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Abstract. In order to accurately and efficiently predict the environment of vibration structures with elastic joints, the BUSH equivalent elastic joints unit of MSC. Nastran software was used to deduce the natural frequencies of structures with elastic joints by using the Fixed Interface Mode Synthesis Method (FIMSM). An example structure with elastic joint is designed, and both FIMSM and traditional FEM are used to calculate the first 30 order natural frequencies of the example structure. The results show that, compared with FEM, the relative calculation error of FIMSM is within $\pm 1.31\%$. Consequently, via using BUSH element to simulate the elastic joint, FIMSM possesses the ability to predict the Environment of Vibration Structure considering elastic joint precisely and efficiently.

1 Introduction

The aircraft will experience complex and cruel vibration environment during launch and flight. For ensuring the high reliability of the aircraft, it must pass the ground vibration environment test. Formulating vibration environmental conditions reasonably is crucial to the development of aircraft. Too low environmental conditions will lead to under-testing, making the aircraft unable to fully expose potential defects during ground tests, and the aircraft may fail to launch and fly in the actual environment. Excessive environmental conditions will lead to over-tests, requiring the aircraft to pass ground tests that are higher than the actual environment, which will increase the difficulty of aircraft development and lead to excessive development cycles and wasted funds^[1,2]. When there is a large amount of measured environmental data, the environmental conditions formulated based on statistical induction methods are closest to the actual environment. However, for the newly-researched aircraft, that exist many uncertain parameters and the lack of measured environmental data, so the need to formulate the environmental conditions itself is often contradictory to the requirements of the aircraft development for environmental conditions. In this case, accurate and efficient prediction for the vibration environment is an inevitable choice to solve the problem^[3].

However, the aircraft is a very complex structural system. There are often a large number of elastic connections (such as bolt connections) in complex structures. The impact of elastic connections on the vibration characteristics of the structure cannot be ignored^[4]. Therefore, a reasonable equivalent elastic connection is an important prerequisite for accurately predicting the structural vibration environment.

Literature [5,6] used the elastic connection in the equivalent structure of a six-degree-of-freedom scalar spring-damping system to conduct a preliminary study on the method of predicting the vibration environment of the structure considering the elastic connection, and obtained certain research results. The equivalent method does not take into account the coupling effect of the elastic connections, which is inconsistent with the actual engineering. Literature [7], through a large number of calculations and comparisons, combined with related finite Meta-theoretical knowledge, the stiffness matrix of the BUSH element was derived from the numerical results. It is found that the stiffness matrix of a single node of a BUSH element is not a diagonal matrix, and there are coupling stiffness terms for vertical displacement and rotation angle, which can be more accurately equivalent to the elastic connection in the actual engineering structure. In view of this, BUSH units have also been widely used in aerospace structural modeling. However, when the traditional FEM is used to predict the aircraft's vibration environment directly, if the prediction accuracy is to be improved, the structure of the aircraft must be finely modeled, which will lead to a sudden increase in calculation costs, so it is necessary to develop a more efficient method for predicting the structural vibration environment. Literature [8,9] pointed out that FIMSM is an efficient method to deal with the dynamic characteristics of complex structures, so using FIMSM to accurately and efficiently predict the structural vibration environment of elastic connections has practical feasibility.

To sum up, this study uses the elastic connection in the equivalent complex structure of the BUSH unit in MSC. Nastran software. Based on the FIMSM, the natural frequency of the structure considering the elastic connection is theoretically derived, and the correctness

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of the method is verified by numerical examples. The conclusions of this study have certain theoretical research and engineering application value.

2 BUSH unit stiffness matrix

Literature [7] based on the back-calculation of numerical results. First, the stiffness matrix of the BUSH unit was output using DMAP language. Then, the input parameters were changed and the stiffness matrix was changed. Finally, an analytical expression of the stiffness matrix K_b of the BUSH unit was obtained by analyzing a large number of calculation results. The stiffness matrix K_b is divided into 2 nodes, as shown below:

$$K_b = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \quad (1)$$

In the above formula, the analytic expressions of the four partitioned matrix K_{11} , K_{12} , K_{21} , and K_{22} are as follows:

$$K_{11} = \begin{bmatrix} k_1 & & & & & \\ 0 & k_2 & & & & \\ 0 & 0 & k_3 & & & \\ 0 & 0 & 0 & k_4 & & \\ 0 & 0 & -\frac{k_3 L}{2} & 0 & k_5 + \frac{k_3 L^2}{4} & \\ 0 & \frac{k_3 L}{2} & 0 & 0 & 0 & k_6 + \frac{k_3 L^2}{4} \end{bmatrix} \quad \text{Symmetry}$$

$$K_{12} = K_{21}^T = \begin{bmatrix} -k_1 & & & & & \\ 0 & -k_2 & & & & \\ 0 & 0 & -k_3 & & & \\ 0 & 0 & 0 & -k_4 & & \\ 0 & 0 & \frac{k_3 L}{2} & 0 & -k_5 + \frac{k_3 L^2}{4} & \\ 0 & -\frac{k_3 L}{2} & 0 & 0 & 0 & -k_6 + \frac{k_3 L^2}{4} \end{bmatrix} \quad \text{Anti-Symmetry}$$

$$K_{22} = \begin{bmatrix} k_1 & & & & & \\ 0 & k_2 & & & & \\ 0 & 0 & k_3 & & & \\ 0 & 0 & 0 & k_4 & & \\ 0 & 0 & \frac{k_3 L}{2} & 0 & k_5 + \frac{k_3 L^2}{4} & \\ 0 & -\frac{k_3 L}{2} & 0 & 0 & 0 & k_6 + \frac{k_3 L^2}{4} \end{bmatrix} \quad \text{Symmetry}$$

(2)

In the formula: k_i ($i = 1, 2, \dots, 6$) is the six input parameters of the BUSH unit; L is the unit length. By consulting the literature [10], it can be seen that the stiffness matrix of the BUSH unit and the shear beam unit have the same form, and there are coupling stiffness terms of vertical displacement and rotation angle. Therefore, compared with the six-degree-of-freedom scalar spring-damping system, the BUSH unit can be more accurately equivalent to the elastic connection in the actual engineering structure.

3 Consider natural frequencies of structures with elastic connections

As shown in Fig. 1, it is assumed that the overall structure is composed of two parts, P and Q, and P and Q are connected through n elastic connections ($p_i \sim q_i$, $i = 1, 2, \dots, n$) of the same specification.

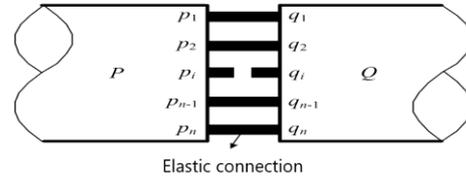


Fig. 1. Schematic chart of overall structure

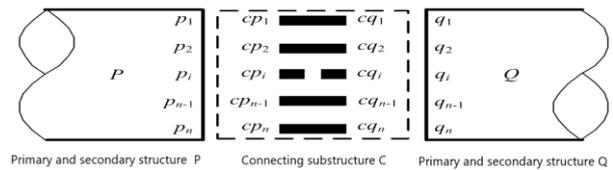


Fig. 2. Schematic chart of substructure partition

In order to improve the calculation efficiency, this study uses FIMSM to calculate the natural frequency of the structure considering elastic connection shown in Fig. 1^[11]. The basic steps of FIMSM are: 1) according to the structural characteristics and analysis requirements, the overall structure is divided into sub-structures with fixed interfaces; 2) extract the modal information of each substructure based on FEM, and calculate the hypothetical mode set that each substructure participates in modal synthesis, which means the low-order retained main mode and the constrained mode; 3) according to the coordinated conditions of the interface displacement, the modal synthesis is performed by using the hypothesized mode set of each substructure, and the vibration equation with reduced degrees of freedom of the overall structure is established, and further calculation and analysis are carried out.

For the overall structure shown in Fig. 1, if the effect of elastic connection is ignored, it can be divided into two substructures P and Q for processing. The corresponding interface node sets P_v and Q_v are:

$$P_v = \{p_i | i = 1, 2, \dots, n\}; Q_v = \{q_i | i = 1, 2, \dots, n\} \quad (3)$$

Literature [11] pointed out that the interfacial displacement coordination between adjacent substructures is required by FIMSM. However, for the substructure P and the substructure Q shown in Fig. 1, since the two are elastically connected, the displacements of the corresponding interface nodes in P_v and Q_v are not continuous. If FIMSM is used to calculate its natural frequency directly, it will cause certain errors or even errors. At this time, if the elastic connection between the substructure P and the substructure Q is divided into a connection substructure C having only interface nodes, not only can it meet the division

principle of the substructure, but it can also coordinate the interface displacement of the substructure P and the substructure Q by connecting the substructure C. Therefore, this study divides the overall structure shown in Fig.1 into P, Q, and C independent substructures for processing, as shown in Fig. 2.

After the sub-structure division is completed, the physical parameters of the main sub-structure λ ($\lambda = P, Q$) are first extracted based on the FEM, that is, the mass matrix M^λ and the stiffness matrix K^λ ; then, according to the internal node u and the interface node v, the mass matrix M^λ and the stiffness matrix M^λ of the main substructure λ are divided into blocks, as shown below:

$$M^\lambda = \begin{bmatrix} M_{uu}^\lambda & M_{uv}^\lambda \\ M_{vu}^\lambda & M_{vv}^\lambda \end{bmatrix};$$

$$K^\lambda = \begin{bmatrix} K_{uu}^\lambda & K_{uv}^\lambda \\ K_{vu}^\lambda & K_{vv}^\lambda \end{bmatrix} \quad (4)$$

At this time, according to the definition of the main mode and the constraint mode of the substructure, the calculation formulas of the main mode φ_n^λ and the constraint mode φ_c^λ of the main substructure λ are^[11]:

$$\varphi_n^\lambda = \begin{bmatrix} \varphi_{un}^\lambda \\ \mathbf{0}_{vn} \end{bmatrix}, \quad (K_{uu}^\lambda - \omega^2 M_{uu}^\lambda) \varphi_{un}^\lambda = \mathbf{0};$$

$$\varphi_c^\lambda = \begin{bmatrix} \varphi_{uc}^\lambda \\ \mathbf{I}_{vc} \end{bmatrix}, \quad \varphi_{uc}^\lambda = -(K_{uu}^\lambda)^{-1} K_{uv}^\lambda \quad (5)$$

In the formula: ω is the angular frequency; I is the identity matrix. Suppose the first r-order ($r \ll n$, so as to reduce the degree of freedom and improve the calculation efficiency) main mode φ_r^λ is selected as the lower-order retained main mode of the substructure λ . Then the hypothetical mode set φ^λ where the substructure λ participates in modal synthesis can be written as:

$$\varphi^\lambda = \begin{bmatrix} \varphi_r^\lambda & \varphi_c^\lambda \end{bmatrix} = \begin{bmatrix} \varphi_{ur}^\lambda & \varphi_{uc}^\lambda \\ \mathbf{0}_{vr} & \mathbf{I}_{vc} \end{bmatrix} \quad (6)$$

For the elastic connection shown in FIG. 1, due to its small mass, the influence on the vibration characteristics of the overall structure can be ignored^[8,9,12]. As for the stiffness of the elastic connection, this study uses the BUSH unit to equivalent it. Therefore, the mass matrix M^C and the stiffness matrix K^C of the connection substructure C can be written as:

$$M^C = \mathbf{0}; \quad K^C = \text{diag}(K_b, \dots, K_n) \quad (7)$$

Similarly, according to equation (5), the hypothetical mode set φ^C connecting the substructure C to participate in modal synthesis can be obtained as:

$$\varphi^C = \varphi_c^C = I \quad (8)$$

In the formula: φ_c^C is the constraint mode of the substructure C. After obtaining the mass matrix, stiffness matrix of each substructure and the set of hypothetical modes involved in modal synthesis, the mass matrix M ,

stiffness matrix K , and modal matrix φ of the overall structure can be written as:

$$M = \begin{bmatrix} M^P & 0 & 0 \\ 0 & M^C & 0 \\ 0 & 0 & M^Q \end{bmatrix} = \begin{bmatrix} M^P & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M^Q \end{bmatrix};$$

$$K = \begin{bmatrix} K^P & 0 & 0 \\ 0 & K^C & 0 \\ 0 & 0 & K^Q \end{bmatrix}; \quad \varphi = \begin{bmatrix} \varphi^P & 0 & 0 \\ 0 & \varphi^C & 0 \\ 0 & 0 & \varphi^Q \end{bmatrix} \quad (9)$$

Let the displacement column vector of the overall structure be X , then:

$$X = [(X_u^P)^T (X_v^P)^T (X_v^{CP})^T [(X_v^{CQ})^T (X_u^Q)^T (X_v^Q)^T]^T]^T \quad (10)$$

In the formula: $X^\lambda u$ and $X^\lambda v$ are displacement vector of internal node and interface node of main substructure λ ; $X^C \lambda v$ is the displacement vector of interface node connecting substructure C and main substructure λ . Let the modal coordinate of the overall structure modal matrix φ be Y , then:

$$X = \varphi Y; \quad Y = [(Y_r^P)^T (Y_c^P)^T (Y_c^{CP})^T (Y_c^{CQ})^T (Y_r^Q)^T (Y_c^Q)^T]^T \quad (11)$$

In the formula: Y_r^λ and Y_c^λ are the mode coordinates of the low-order retained main mode and the constrained mode of the main substructure λ , respectively; $Y_c^{C\lambda}$ is the mode coordinates of the constrained mode connected to the substructure C. At this time, it can be obtained that the modal mass matrix M_Y and the modal stiffness matrix K_Y whose overall structure corresponds to the modal coordinate Y are:

$$M_Y = \varphi^T M \varphi; \quad K_Y = \varphi^T K \varphi \quad (12)$$

The coordination conditions for introducing interface displacements are:

$$X_v^{CP} = L_P X_v^P; \quad X_v^{CQ} = L_Q X_v^Q \quad (13)$$

In the formula: L_P and L_Q are coordinate rotation transformation matrices. Sorting formulas (10) to (12) and formula (13) gives:

$$Y_c^{CP} = L_P Y_c^P; \quad Y_c^{CQ} = L_Q Y_c^Q \quad (14)$$

An independent transformation matrix S is introduced to independently transform Y . The transformation process is as follows:

$$Y = SZ = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & L_P & 0 & 0 \\ 0 & 0 & 0 & L_Q \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} Y_r^P \\ Y_c^P \\ Y_r^Q \\ Y_c^Q \end{bmatrix};$$

$$S = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & L_P & 0 & 0 \\ 0 & 0 & 0 & L_Q \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}; \quad Z = \begin{bmatrix} Y_r^P \\ Y_r^P \\ Y_r^Q \\ Y_c^Q \end{bmatrix}$$

(15)

In the formula: Z is the independent modal coordinate of the overall structure. At this time, it can be obtained that the modal mass matrix M_Z and the modal stiffness matrix K_Z whose overall structure corresponds to the independent modal coordinate Z are:

$$M_Z = S^T M_Y S = S^T \varphi^T M \varphi S; \quad K_Z = S^T K_Y S = S^T \varphi^T K \varphi S \quad (16)$$

Therefore, the undamped free vibration equation of the reduced overall degrees of freedom of the overall structure shown in Fig. 1 based on FIMSM is:

$$M_Z \ddot{Z} + K_Z Z = 0 \quad (17)$$

The modal matrix of the vibration equation shown in formula (17) is Φ , and its calculation formula and expression are:

$$(K_Z - \omega^2 M_Z) \Phi = 0; \quad \Phi = [\Phi_1, \Phi_2, \dots, \Phi_i] \quad (18)$$

At this time, according to the Rayleigh quotient formula, the i -th angular frequency ω_i of the overall structure shown in Fig. 1 is:

$$\omega_i^2 = \frac{\phi_i^T K_Z \phi_i}{\phi_i^T M_Z \phi_i} = \frac{\phi_i^T S^T \varphi^T K \varphi S \phi_i}{\phi_i^T S^T \varphi^T M \varphi S \phi_i} \quad (19)$$

The above process is to use the elastic connection in the equivalent structure of the BUSH unit and express the natural frequency of the structure considering the elastic connection based on the FIMSM derivation. Because FIMSM doesn't consider the influence of the higher-order main modes of each main substructure, the calculation efficiency can be greatly improved.

4 Study Verification

In order to verify the correctness of the above method, an example structure with elastic connection is designed as shown in Fig. 3. Under the premise of using the equivalent elastic connection of the BUSH unit, FIMSM and FEM are used in the first 30 steps of the structure. The frequency is calculated and the calculation results are compared.

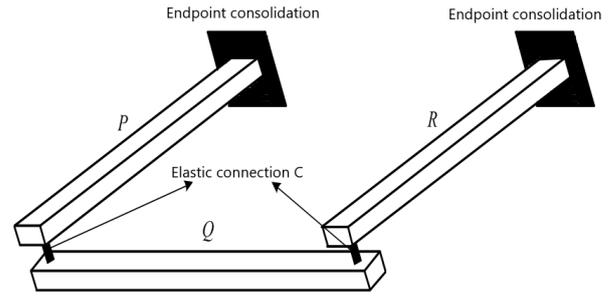


Fig. 3. Schematic structure of example structure

In Fig. 3, P, Q and R are three long beams with the same rectangular cross-section. The section size and beam length are 0.04 m × 0.02 m and 0.5 m, respectively. The elements are divided by using shear beam elements. See Table 1 for element attributes; C is 2 identical elastic connections, which are equivalent using BUSH units. See Table 2 for the unit properties.

Table 1. Element attributes of the shear beam.

Substructure	Elasticity modulus/ Pa	Poisson's ratio	Density/ kg/m ³	Length/ m
P, Q and R	7×10 ¹⁰	0.3	2700	0.05

Table 2. Element attributes of BUSH

Substructure	k ₁ /N /m	k ₂ /N /m	k ₃ /N /m	k ₄ /N ·m/r ad	k ₅ /N ·m/r ad	k ₆ /N ·m/r ad	L/m
C	2.1×10 ⁸	3.8×10 ⁶	3.8×10 ⁶	650	850	850	0.05

When using FIMSM calculations, the first 10 steps of the main substructures P, Q, and R are retained as low-order main modes to participate in modal synthesis. When using FEM for calculations, all modes of the overall structure are selected. The calculation results of the two methods are shown in Table 3.

Table 3. Calculating results from intrinsic frequencies

Order numbe	FIMSM/Hz	FEM/Hz
1	37.033	37.033
2	58.281	58.281
3	75.885	75.885
4	171.817	171.815
5	243.044	243.041
6	338.479	338.464
7	383.041	383.006
8	458.397	458.351
9	462.078	462.039
10	587.736	587.249
11	761.824	761.512
12	835.057	834.904
13	940.449	939.298
14	966.589	965.541
15	1026.760	1026.619

16	1183.474	1182.542
17	1464.751	1462.354
18	1788.334	1787.330
19	1947.586	1940.273
20	1953.285	1945.861
21	2050.529	2048.535
22	2230.566	2226.550
23	2318.983	2306.868
24	2338.818	2327.914
25	2357.539	2330.467
26	2592.201	2577.911
27	2598.795	2587.836
28	3063.926	3057.575
29	3397.919	3394.079
30	3654.106	3607.072

As can be seen from Table 3, compared with the traditional FEM, the relative calculation error of the FIMSM for the first 30 natural frequencies of the structure shown in Figure 3 does not exceed $\pm 1.31\%$. In summary, using the equivalent elastic connection of the BUSH unit, FIMSM can accurately and efficiently calculate the natural frequency of the structure considering the elastic connection.

5 conclusion

In this study, the equivalent elastic connection of BUSH elements in MSC. Nastran software was used. Based on the FIMSM, the natural frequency of the structure considering the elastic connection was theoretically derived, and the correctness of the method was verified by numerical simulation. The results show that using the equivalent elastic connection of the BUSH unit, FIMSM can accurately and efficiently predict the vibration environment with elastic connection structure. The conclusions of this study have certain theoretical research and engineering application value.

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