

Joint adjustment by CPIII precision trigonometric elevation control network assisted with precision levelling observation

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Abstract: Using the precision trigonometric elevation instead of the precision levelling to build a CPIII elevation control network will greatly increase the speed of CPIII control network construction. However, the accuracy of CPIII precision trigonometric elevation control network is still difficult to reach the level of CPIII precision levelling network. Based on the existing parameter method, this paper introduces some precision levelling for joint adjustment, and uses Helmert's variance estimation method to perform strict weight determination. Our experiments show that when the number of precision levelling participating in the joint adjustment exceeds 1/3 of the total number of CPIII precision levelling network observations, the accuracy of the CPIII precision trigonometric elevation control network can be effectively improved.

1 Introduction

A CPIII control network includes a plane control network and an elevation control network, which is the position reference for the construction and operation stages of high-speed railways. The CPIII elevation control network uses electronic level to collect field data. Because the level measurement mode is inefficient, it is a challenges as to how to improve the efficiency of CPIII elevation control network data collection in the current construction of high-speed railway control network.

The CPIII plane control network and the elevation control network have points in common, so some scholars have tried to use the vertical angle and distance in the observation data of the CPIII plane control network to build a CPIII precision trigonometric elevation control network. However, the trigonometric elevation observations are easily affected by systematic errors such as atmospheric refraction, earth curvature, and vertical deviation. In this case, how to eliminate these systematic errors is a key problem to be solved. References [1-5] adopted the observing method to eliminate the influence of atmospheric refraction and the curvature of the earth in the trigonometric elevation observations, so that the precision of the precise trigonometric elevation measurement reached the national second-class requirements. The amount of CPIII control points is huge, and only the measurement prism can be placed near the observation target. The opposite observation method cannot be used to eliminate the effects of atmospheric refraction and earth curvature. Therefore, some

researchers have proposed the difference method [6-7] and the parameter method [8], which have achieved good effect. However, from the calculation of a large amount of measured data, it is found that the probability of gross errors in precision trigonometric elevation observations is much higher than the precision levelling. These gross error observations have a serious impact on the control network. In some CPIII precision trigonometric elevation control, In the net, excluding all the gross error observations will result in poor mesh strength or network construction failure.

This paper studies the introduction of part of the precision levelling in the CPIII precise trigonometric elevation control network to improve the strength and reliability of the CPIII precision trigonometric elevation control network. Since two types of observations are involved, Helmert's variance component estimation method is used for weight determination. Through experiments, it is proved that introducing a certain number of precision levelling can effectively improve the accuracy of the CPIII precise trigonometric elevation control network; but if the number of precision levelling is insufficient, it will lead to the failure of strict weight determination.

2 Joint adjustment model

As shown in Figure 1, the hollow circles are the CPIII points, the solid circle is the precise trigonometric elevation survey site; the solid lines are the precise trigonometric elevation observation routes, and the dotted lines are the precision levelling observation routes.

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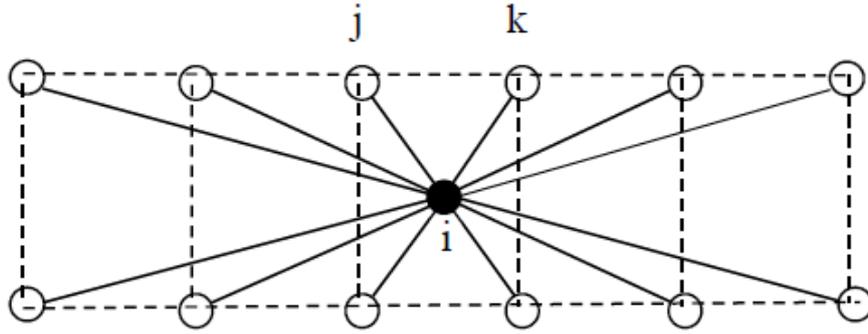


Figure 1: Schematic of Mixed Trigonometric elevation and Precision levelling Observation

Trigonometric elevation measurement is performed on the target point j at the measurement site i . Set the vertical angle to β_{ij} and the oblique observation value to s_{ij} , then we can obtain the trigonometric elevation observation value:

$$h_{ij} = s_{ij} \sin \beta_{ij} \quad (1)$$

If the most probable elevation of the instrument center is y_i and the most probable elevation of the target point j is x_j , then:

$$h_{ij} = x_j - y_i + k_i \times d_{ij}^2 + \Delta_{ij} \quad (2)$$

Where $h_{ij} = d_{ij} \times \tan(\alpha_{ij})$, and Δ_{ij} is the true error of the observed value. When the target point j is the point to be obtained, the error equation of the precise trigonometric elevation observation can be obtained as:

$$v_{ij} = x_j - y_i + k_i \times d_{ij}^2 - h_{ij} \quad (3)$$

When the target point j is a known point and the elevation is H_j , the precise trigonometric elevation observation error equation can be expressed as:

$$v_{ij} = -y_i + k_i \times d_{ij}^2 - (h_{ij} - H_j) \quad (4)$$

Set the error in the angle measurement to m_α and the error in the distance measurement be m_s . Therefore, the error of the precise trigonometric elevation observation h_{ij} is

$$m_{ij}^2 = (\sin(\alpha_{ij}))^2 m_s^2 + (s_{ij} \times \cos(\alpha_{ij}))^2 \frac{m_\alpha^2}{\rho^2}$$

Then its weight $p_i = \frac{m_\alpha^2}{m_i^2}$.

Combine all the precise trigonometric elevation observation error equations and write the matrix form as:

$$V_1 = B_1 X - l_1 \quad (5)$$

Moreover, suppose that level measurement is performed between two points of j and k . The observation value is h_{jk} , the line length is s_{jk} , and the observation error is Δ_{jk} , then:

$$h_{jk} + \Delta_{jk} = x_k - x_j \quad (6)$$

When j and k are both unknown points, the error equation of the precision levelling observation can be expressed as

$$v_{jk} = x_k - x_j - h_{jk} \quad (7)$$

When j is an unknown point, k is a known point, and the elevation is H_k , the error equation of the precision levelling observation can be obtained as:

$$S = \begin{bmatrix} n_{1-2} \text{tr}(\mathbf{N}^{-1} \mathbf{N}_1) + \text{tr}(\mathbf{N}^{-1} \mathbf{N}_1)^2 & \text{tr}(\mathbf{N}^{-1} \mathbf{N}_1 \mathbf{N}^{-1} \mathbf{N}_2) \\ \text{tr}(\mathbf{N}^{-1} \mathbf{N}_1 \mathbf{N}^{-1} \mathbf{N}_2) & n_{2-2} \text{tr}(\mathbf{N}^{-1} \mathbf{N}_2) + \text{tr}(\mathbf{N}^{-1} \mathbf{N}_2)^2 \end{bmatrix}, W = \begin{bmatrix} V_1^T P_1 V_1 \\ V_2^T P_2 V_2 \end{bmatrix}$$

$$v_{jk} = -x_j - (h_{jk} - H_k) \quad (8)$$

When the k point is unknown, the j point is a known point, and the elevation is H_j , the error equation of the precision levelling observation can be obtained as:

$$v_{jk} = x_k - (h_{jk} + H_j) \quad (9)$$

The weight of the precision levelling observation h_{jk} is $p_{jk} = \frac{m_h^2}{m_{jk}^2}$, where $m_{jk} = s_{jk} \times m_h$, and m_h is the round-trip error of per kilometer level observation, which is $\sqrt{2}$ times of the nominal accuracy of the precision levelling. Combine all the precision levelling observation error equations and write the matrix form as

$$V_2 = B_2 X - l_2 \quad (10)$$

3 The tight adjustment of joint adjustment

Combine Eqs.(5) and Eqs.(10), solve the equation using the least square method, and set the estimated parameter value to be X , then:

$$V_1 = B_1 X - l_1$$

$$V_2 = B_2 X - l_2$$

In theory, as the precise trigonometric elevation observations and precision levelling are independent of each other, the addition of precision levelling will not affect the weight of the original CPIII precision trigonometric elevation control network observations. However, the weight ratio relationship between precision levelling and precise trigonometric elevation observations needs to be determined by Helmert variance estimation method.

Suppose the unit weight variances of the precise trigonometric elevation observations and precision levelling are δ_1^2, δ_2^2 respectively, and let $\theta = [\delta_1^2 \ \delta_2^2]^T$. Based on the Helmert variance estimation method, it can be obtained

$$\theta = S^{-1} W$$

Where:

$$S = \begin{bmatrix} n_{1-2} \text{tr}(\mathbf{N}^{-1} \mathbf{N}_1) + \text{tr}(\mathbf{N}^{-1} \mathbf{N}_1)^2 & \text{tr}(\mathbf{N}^{-1} \mathbf{N}_1 \mathbf{N}^{-1} \mathbf{N}_2) \\ \text{tr}(\mathbf{N}^{-1} \mathbf{N}_1 \mathbf{N}^{-1} \mathbf{N}_2) & n_{2-2} \text{tr}(\mathbf{N}^{-1} \mathbf{N}_2) + \text{tr}(\mathbf{N}^{-1} \mathbf{N}_2)^2 \end{bmatrix}, W = \begin{bmatrix} V_1^T P_1 V_1 \\ V_2^T P_2 V_2 \end{bmatrix}$$

$N_1 = B_1^T P_1 B_1$, $N_2 = B_2^T P_2 B_2$, $N = B^T P B$, n_1 and n_2 are the number of precise trigonometric elevation observations and precision levelling.

By weighting again, we can get

$$P_1 = \frac{\delta_1^2}{\delta_1^2 P_1^{-1}}, \quad P_2 = \frac{\delta_2^2}{\delta_2^2 P_2^{-1}}$$

Parameter estimation is performed again using the newly determined weight matrix. Repeat the above process until $\delta_1^2 = \delta_2^2$ or its ratio is 1 to stop.

4 Experimental verification

A CPIII control network with a total length of 10km was observed using measuring robots and the electronic precision levelling. Among them, 103 stations of precision trigonometric height measurement obtained 1261 observations (with a few encryption points); a total of 604 precision levelling were obtained. Taking this case as an example, the calculation accuracy of CPIII precision levelling network, CPIII precision trigonometric elevation

control network, precision levelling constrained CPIII precision trigonometric elevation control network, precision levelling and precision levelling combined CPIII elevation control network are compared and analyzed.

As shown in Figure 2, after the CPIII precision levelling network is adjusted, the error in the height difference between adjacent points can reach 0.12mm. The longitudinal observations on both sides of the CPIII precision levelling network are selected to participate in the CPIII precision trigonometric elevation control network. The results are shown in Figure 3. The error in the height difference between adjacent points can reach the level of 0.25mm. The lateral observations of the line in the CPIII precision levelling network are selected. Participated in the CPIII precision trigonometric elevation control network. The results are shown in Figure 4. The error in the height difference between horizontally adjacent points reached a level of 0.15mm, and the accuracy of the height difference between adjacent vertical points was small, but only three pairs of height differences exceeded 0.5mm.

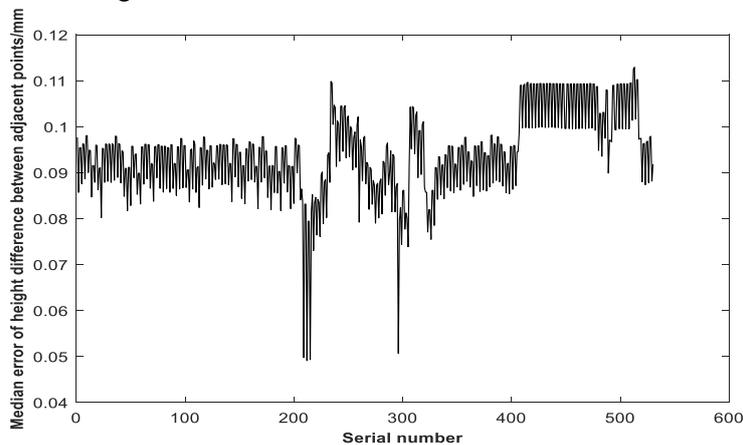


Figure 2: Error curve of height difference between adjacent points of CPIII precision levelling network

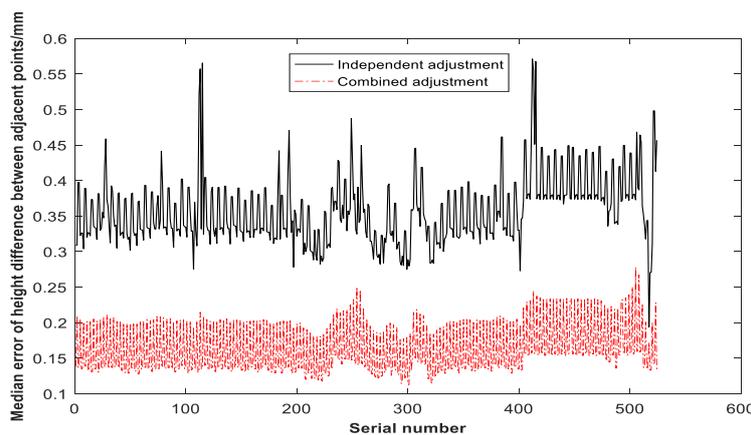


Figure 3: The middle error curve obtained from the joint adjustment of the longitudinal precision levelling and the CPIII precise trigonometric elevation network

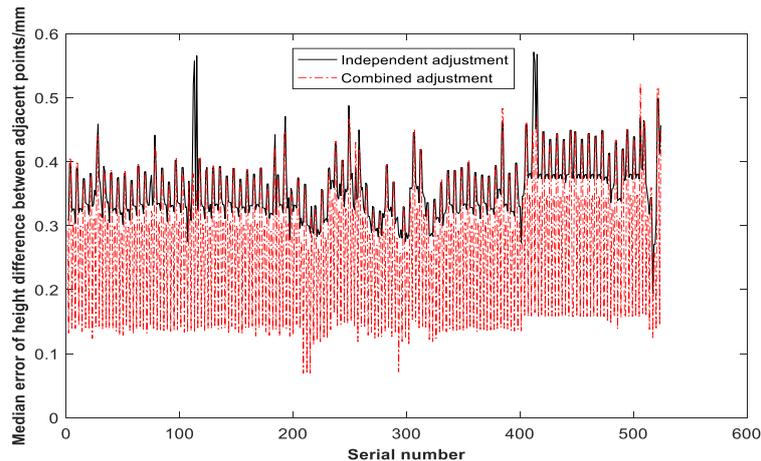


Figure 4: The median error curve obtained from the joint adjustment of the horizontal precision levelling and the CPIII precise trigonometric elevation network

5 Conclusion

This paper studies introducing part of precision levelling in the CPIII precise trigonometric elevation control network for joint adjustment to improve its strength and reliability. It can be concluded through experiments that:

1) Normally, adding the longitudinal precision levelling on both sides of the line can effectively improve the accuracy of the CPIII precise trigonometric elevation control network; if only the lateral precision levelling is introduced, the accuracy of the error in the height difference between the lateral CPIII points can be significantly improved. The improvement of error accuracy in the height difference between longitudinal CPIII points is limited.

2) In the case where the number of precision levelling is small, that is, less than 1/3 of the number of CPIII precision levelling network observations, the effect of estimating weights by using the Helmer test back-difference is not good. It is appropriated to adopt nominal accuracy for fixed weight in this case.

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