

Discrete mathematical model of travelling wave of conveyor transport

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Abstract. A mathematical model of a travelling wave in a matrix form is constructed. A degree of discreteness of the travelling wave and corresponding steps in phase and length are introduced. Asymmetric, unified matrices are compiled, which represent a generalized travelling wave, depending on a degree of discreteness. A generalized, dimensionless travelling wave is transformed into a required one with dimensions by specified technical parameters: amplitude and wavelength that is realized. A dependency of coordinates of points of a plane discrete travelling wave and discrete phase angles is established. A dependency of angular (phase) velocity and velocity of the travelling wave, which corresponds to the known results, is established. The presented matrix mathematical model is considered as an initial stage of technical possibility to realize a continuous travelling wave in a discrete form when developing a new type of transportation – wave transport.

1 Introduction

Modern requirements of safety [1-4], performance [5-8] and energy efficiency [9-12] stimulate scientific and technological progress in the field of creation and engineering support of operation of machines [13-16], their equipment [17-20] and mechanisms [21-23] for mining [24-28], transportation [29-34] and mechanical engineering [35-38].

The leading countries are actively pursuing research and development of new types of transportation, based on usage of different physical principles of movement and realized in many design schemes, in order to solve global environmental problems. Such types of transportation include conveyor transport and its constructive realization in forms of a belt [15, 17, 19-20, 34], steel plate, roller, screw conveyor [39], multi-mass vibrating transporters of various modifications, including linear, spiral [40], pneumatic transport and its modern realization in a form of Hyperloop [41].

Designing vehicles that use the principle of travelling wave is of great interest. There is an important property of a travelling wave, which is a transfer of energy and momentum [42]. General ideas and principles of wave phenomena and universal character of their application are illustrated in [43]. Various aspects of the mathematical theory of gravitational waves on a surface of an incompressible heavy fluid and practical applications

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of this theory are described in [44]. Mathematical modelling of the travelling wave in a discrete, matrix form is an important milestone in a process of its technical realization in development of a new type of transportation – wave transport.

2 State of question and statement of research problem

A travelling wave as a physical phenomenon is described by a wave equation, solutions of which are harmonic functions of time and coordinates of space [45]. Technical realization of a continuous travelling wave for transportation purposes necessitates its discretization. A degree of discreteness (step magnitude) of independent arguments by time and coordinates determines the technically necessary accuracy of reproduction of a continuous travelling wave. The problem is to compile a discrete mathematical model of the travelling wave in a form of a square matrix, the order of which is determined by a degree of discreteness. This helps verifying the matrix model of the travelling wave in a graphical form. Establishment of kinematic dependencies, which characterize properties of the travelling wave and introduced parameters of discreteness, is also an actual scientific problem. The purpose of the paper is construction of a discrete mathematical model of a travelling wave in order to ensure a technical possibility of its realization in a development of a new type of conveyor transport.

3 Methods

Construction of a mathematical model of a travelling wave in a matrix form. Introduction of a degree of discreteness of a travelling wave and respective steps by phase and length. Compilation of asymmetric, unified matrices, which represent a generalized travelling wave, depending on a degree of discreteness. Transformation of generalized, dimensionless travelling wave into a required one with dimensions by specified technical parameters. Establishment of a dependency of coordinates of points of a flat discrete travelling wave and discrete phase angles, and a dependency of angular (phase) velocity and velocity of a travelling wave.

4 Results and discussion

4.1 Discrete mathematical model of travelling wave

The simplest harmonic function is selected to be an odd trigonometric function with a period of 2π , the graph of which is a sine wave. Technical parameters of the travelling wave are wave height $2a$, where a is amplitude, m, and wavelength 2λ , where λ is half-wavelength, m.

Degree of discreteness N is a dimensionless integer number defined as a multiple of four, that is $N = 4, 8, 12 \dots$ etc.

Discrete generalized (dimensionless) variables are introduced.

Longitudinal coordinate of the travelling wave

$$\bar{x}_m = \frac{x_m}{\lambda},$$

where x_m is a discrete longitudinal coordinate with dimensions, m,
phase angle of the travelling wave

$$\bar{\varphi}_n = \frac{\varphi_n}{\pi},$$

where φ_n is a discrete phase angle, rad,
 transverse coordinate of the travelling wave

$$\bar{y}_{nm} = \frac{y_{nm}}{a},$$

where y_{nm} is a discrete transverse coordinate with dimensions, m, $n, m = 0, 1, 2, \dots N$.

Discrete, dimensionless, independent variables are defined by an integer argument in a form

$$\bar{x}_m = \frac{2}{N}m; \quad \bar{\varphi}_n = \frac{2}{N}n.$$

Then a dimensionless transverse coordinate of the travelling wave can be represented depending on the introduced variables in the following laconic and discrete form

$$\bar{y}_{nm} = \sin \pi (\bar{x}_m - \bar{\varphi}_n),$$

or

$$\bar{y}_{nm} = \sin \frac{2\pi}{N}(m - n).$$

Thus, a generalized travelling wave is modeled by a matrix $\bar{y}_{nm}$ of the size $n \times m$ depending on a degree of discreteness N .

4.2 Representation of generalized travelling wave in square matrix form

The square matrix of the generalized travelling wave is compiled of elements $\bar{y}_{nm}$, where n is row number ($n = 0, 1, 2, \dots N$), m is column number ($m = 0, 1, 2, \dots N$) and is designated in a form $\bar{Y}_N$, where N is degree of discreteness ($N = 4, 8, 12, \dots$). For a degree of discreteness of $N = 4$ the following is obtained

$$\bar{y} = \sin \frac{\pi}{2}(m - n),$$

or in a detailed matrix representation

$$\bar{Y}_4 = \begin{pmatrix} 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 \end{pmatrix}.$$

For a degree of discreteness of $N = 8$ the following is obtained

$$\bar{y}_{nm} = \sin \frac{\pi}{4}(m-n),$$

or in a matrix representation

$$\bar{Y}_8 = \begin{pmatrix} 0 & \sqrt{2}/2 & 1 & \sqrt{2}/2 & 0 & -\sqrt{2}/2 & -1 & -\sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 & 1 & \sqrt{2}/2 & 0 & -\sqrt{2}/2 & -1 & -\sqrt{2}/2 \\ -1 & -\sqrt{2}/2 & 0 & \sqrt{2}/2 & 1 & \sqrt{2}/2 & 0 & -\sqrt{2}/2 & -1 \\ -\sqrt{2}/2 & -1 & -\sqrt{2}/2 & 0 & \sqrt{2}/2 & 1 & \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & -1 & -\sqrt{2}/2 & 0 & \sqrt{2}/2 & 1 & \sqrt{2}/2 & 0 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 & -1 & -\sqrt{2}/2 & 0 & \sqrt{2}/2 & 1 & \sqrt{2}/2 \\ 1 & \sqrt{2}/2 & 0 & -\sqrt{2}/2 & -1 & -\sqrt{2}/2 & 0 & \sqrt{2}/2 & 1 \\ \sqrt{2}/2 & 1 & \sqrt{2}/2 & 0 & -\sqrt{2}/2 & -1 & -\sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & \sqrt{2}/2 & 1 & \sqrt{2}/2 & 0 & -\sqrt{2}/2 & -1 & -\sqrt{2}/2 & 0 \end{pmatrix}.$$

For a degree of discreteness of $N = 12$ the following is obtained

$$y_{nm} = \sin \frac{\pi}{6}(m-n),$$

or in a matrix representation

$$\bar{Y}_{12} = \begin{pmatrix} A_{12} & -A_{12} \\ -A_{12} & A_{12} \end{pmatrix},$$

where

$$A_{12} = \begin{pmatrix} 0 & 1/2 & \sqrt{3}/2 & 1 & \sqrt{3}/2 & 1/2 \\ -1/2 & 0 & 1/2 & \sqrt{3}/2 & 1 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 & 0 & 1/2 & \sqrt{3}/2 & 1 \\ -1 & \sqrt{3}/2 & -1/2 & 0 & 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -1 & -\sqrt{3}/2 & -1/2 & 0 & 1/2 \\ -1/2 & -\sqrt{3}/2 & -1 & -\sqrt{3}/2 & -1/2 & 0 \end{pmatrix},$$

etc.

The indicated matrices that represent the travelling wave are obliquely symmetric, ordered, and compiled according to an obvious algorithm, using the property of structural symmetry of these matrices, by the first column or the first row.

4.3 Visualization of discrete model of travelling wave

The introduced generalized, discrete variables $\bar{x}_m$ and $\bar{\varphi}_n$ are determined by the linear function of an integer argument m and n , graphical point images of which for the considered degree of discreteness $N = 4, 8, 12$ are given in the figures (Fig. 1).

Generalized discrete transverse coordinate $\bar{y}_{nm}$ of the travelling wave is defined as a harmonic function of two integer arguments m and n , graphical, point image of which for a degree of discreteness of $N = 4$ is given in the figures depending on m and fixed n (Fig. 2a) or depending on n and fixed m (Fig. 2b).

Accordingly, a generalized, discrete transverse coordinate $\bar{y}_{nm}$ is transformed in a similar way into a one with dimensions y_{nm} with known amplitude a and half-wavelength λ depending on a discrete longitudinal coordinate with dimensions x_m of the travelling wave (Fig. 3a) or on a discrete phase angle φ_n of the travelling wave (Fig. 3b).

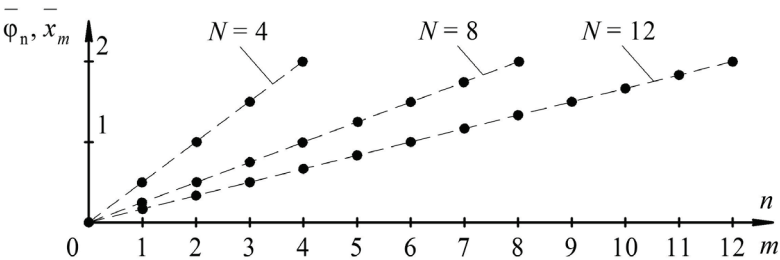


Fig. 1. Dependency of generalized, discrete phase $\bar{\varphi}_n$ and longitudinal coordinate $\bar{x}_m$ on degree of discreteness N .

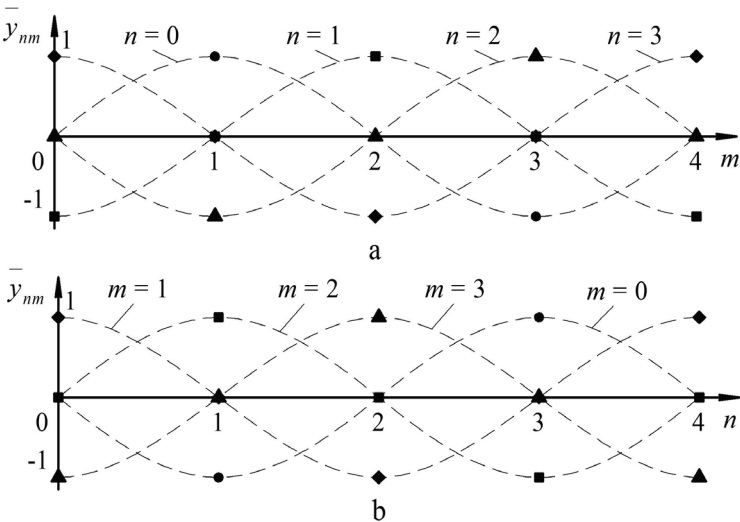


Fig. 2. Dependency of generalized, discrete transverse coordinate $\bar{y}_{nm}$ for degree of discreteness equal to 4: a – with fixed discrete argument n ; b – with fixed discrete argument m .

For a degree of discreteness $N=8$ in a similar way obtain the travelling wave with a fixed phase (Fig. 4a) and with a fixed longitudinal coordinate (Fig. 4b).

A travelling wave for a degree of discreteness $N=12$ can be obtained and illustrated similarly.

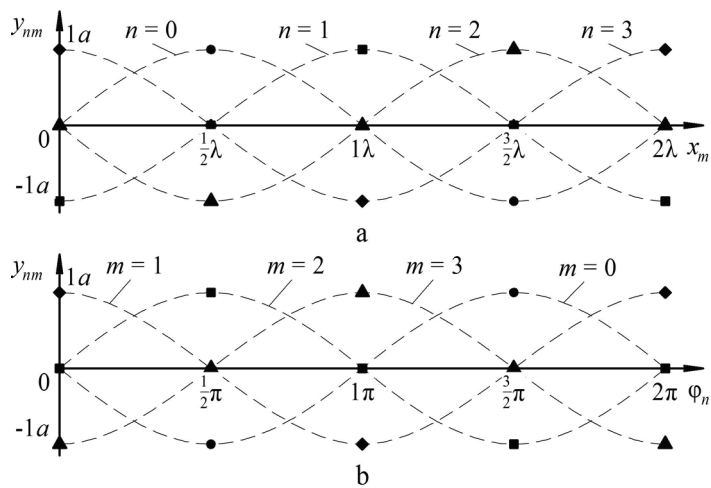


Fig. 3. Dependency of discrete transverse coordinate with dimensions y_{nm} for degree of discreteness equal to 4: a – with fixed discrete phase; b – with fixed discrete longitudinal coordinate.

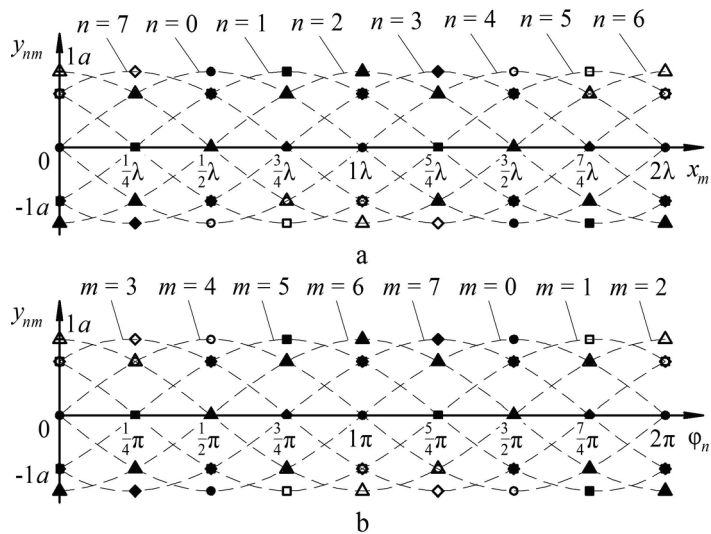


Fig. 4. Dependency of discrete transverse coordinate with dimensions y_{nm} for degree of discreteness equal to 8: a – with fixed discrete phase; b – with fixed discrete longitudinal coordinate.

4.4 Kinematics of discrete travelling wave

According to the specified degree of discreteness a step by phase, rad, is introduced

$$\Delta\varphi = \frac{2\pi}{N},$$

step by longitudinal coordinate, m

$$\Delta x = \frac{2\lambda}{N},$$

step by transverse coordinate, m

$$\Delta y = \frac{2a}{N},$$

step by time, s

$$\Delta t = \frac{2T}{N},$$

where $2T$ is the time, s, of one oscillation in transverse direction or the time of passing of one wave in longitudinal direction. Then the discrete phase φ_n is defined as

$$\varphi_n = \Delta\varphi n,$$

or by introducing an angular (phase) velocity ω , rad/s, obtain the following

$$\varphi_n = \omega t_n,$$

where t_n is time, which corresponds to step n , s

$$t_n = \Delta t n.$$

Similarly, the discrete longitudinal coordinate x_m , m, is defined as

$$x_m = \Delta x m$$

or, by introducing the velocity of travelling wave V , m/s, obtain the following

$$x_m = V t_m,$$

where t_m is time, which corresponds to step m , s

$$t_m = \Delta t m.$$

In particular, when $n = N$, then

$$\varphi_N = 2\pi, t_N = 2T,$$

where φ_N is phase, which corresponds to N , rad, t_N is time, which corresponds to N , s also when $m = N$

$$x_N = 2\lambda, t_N = 2T,$$

where x_N is longitudinal coordinate, which corresponds to N , m.

Or

$$2\pi = \omega 2T ,$$

and also

$$2\lambda = V 2T .$$

Where the known kinematic ratios are established

$$\frac{\pi}{\omega} = \frac{\lambda}{V} = T ,$$

or

$$V = \frac{\lambda}{\pi} \omega .$$

5 Conclusions

A discrete mathematical model of the travelling wave is suggested in order to ensure a technical possibility of its realization in the development of a new type of conveyor transport. A mathematical model of the travelling wave is presented in a form of ordered, asymmetric matrices, the orders of which are determined by the degree of discreteness. A degree of discreteness determines the accuracy of reproduction of a theoretical continuous travelling wave, which is illustrated by detailed figures. The discrete travelling wave is reduced to a generalized, dimensionless form, which is transformed into a one with dimensions, and technically realized form by a required amplitude and wavelength. It is indicated, that the description of kinematics of the travelling wave in a discrete form corresponds to a well-known result of a dependency of phase and linear velocity for a continuous model.

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