Uncertainty and sensitivity analysis applied to a rammed earth wall: evaluation of the discrepancies between experimental and numerical data

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Abstract. Due to the environmental impact of building materials, researches on sustainable materials, such as bio-based and earth materials, are now widespread. These materials offer numerous qualities such as their availability, recyclability and their ability to dampen the indoor relative humidity variations due to their hygroscopicity. As these materials can absorb large amount of humidity, numerical and experimental studies of their hygrothermal behaviour are crucial to assess their durability.

To validate a hygrothermal model, numerical and experimental data have to be confronted. Such confrontation must take into consideration the uncertainties related to the experimental protocol, but also to the model. Statistical tools such as uncertainty and global sensitivity analysis are essential for this task. The uncertainty analysis estimates the robustness of the model, while the global sensitivity analysis identifies the most influential input(s) responsible for this robustness. However, these methods are not commonly used because of the complexity of hygrothermal models, and therefore the prohibitive simulation cost.

This study presents a methodology for comparing the numerical and experimental data of a rammed earth wall subjected to varying temperature and relative humidity conditions. The main objectives are the investigation of the uncertainties impact, the estimation of the model robustness, and finally the identification of the input(s) responsible for the discrepancies between numerical and experimental data. To do so, a recent and low-cost global variance-based sensitivity method, named RBD-FAST, is applied. First, the uncertainty propagation through the model is calculated, then the sensitivity indices are estimated. They represent the part of the output variability related to each input variability. The output of interest is the vapour pressure in the middle of the wall to confront it to the experimental measurement. Good agreement is obtained between the experimental and numerical results. It is also highlighted that the sorption isotherm is the main factor influencing the vapour pressure in the material.

1 Introduction

A large amount of earthen constructions is present worldwide. In the context of energy transition, earthen materials offer interesting advantages such as their low embodied energy or their ability to regulate the indoor relative humidity in a passive way, namely their moisture buffering capacity. However, they are relatively sensitive to humidity. Therefore, experimental and numerical studies are crucial to assess their durability.

Some studies focused on measuring the hygrothermal properties of earth. They largely differ from one earth to another due to its variation of composition depending on the soil it was extracted from. A state-of-the-art on the hygrothermal properties of numerous earth may be found in [1]. Also, it is widely acknowledged that the heterogeneity of structure of this material may lead to large experimental uncertainties, and current standards are not adapted for this material [2].

Other studies focused on the hygrothermal modelling of earthen walls. For example, Hall and Allinson [3] used the software WUFI to model three walls

made of stabilized rammed earth. It was concluded that the dry thermal properties did not vary greatly, but the humidity had a significant impact depending on the pore structure. Soudani et al. [4] developed a hygrothermal model taking into account the effect of water phase change. This model considered separately the kinetic of each phase within the wall (i.e, liquid water, water vapour, dry air and solid matrix). The authors highlighted that it is necessary to take into account the impact of temperature on water flow and the variation of the vapour mass due to evaporation-condensation.

However, few studies aimed at taking into consideration the variations of hygrothermal properties of earth in hygrothermal modelling. This is done by applying uncertainty and sensitivity analysis methods. In [5], the authors investigated the impact of the experimental uncertainties on the sorption isotherm and the water vapour permeability on the relative humidity within sprayed hemp concrete. Their impact was non negligible on this specific output. A similar study on hemp concrete was performed in [6]. In both articles, only one property at a time was investigated. Yet, such an approach could

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not take into consideration a large number of hygrothermal properties combinations to cover the whole inputs space. A previous study performed by [7] detailed an efficient method of uncertainty and sensitivity analysis. They applied it to five hygrothermal models to evaluate the cooling demand of a building. To the best of our knowledge, no other studies proposed this approach in hygrothermal modelling.

Consequently, this article aims at proposing a methodology to compare experimental and numerical data on rammed earth. First, the uncertainty and sensitivity analysis methods are introduced. Then, the case study corresponding to an earthen wall and the hygrothermal model are presented. Finally, the measured and calculated water vapour pressure in the middle of the wall are compared and the most influential hygrothermal properties on this specific output are identified.

2 Method

2.1 Uncertainty and sensitivity analysis method

To understand and improve hygrothermal models, it is essential to assess their robustness. It corresponds to the ability of a model to return a stable output in an uncertain environment. In the case of hygrothermal models, knowing the inputs is not straightforward. This lack of knowledge on the results has to be quantified and associated to a confidence bound. The challenge is twofold: firstly, to estimate if the inputs uncertainties lead to differences in the output of interest: this is the uncertainty analysis; and secondly, to identify – on the whole uncertain inputs – which input(s) should be prioritized for a more accurate measurement to limit the variability of the output: this is the sensitivity analysis. For complex models, the tools dedicated to these challenges are global sensitivity analysis based on the variance, known as ANOVA methods [8].

The local sensitivity analysis investigates one input at a time and could lead to an under exploration of the inputs space. It could correspond, in some cases, up to 99% of the space of non-explored parameters according to [9]. In contrast, the global sensitivity analysis evaluates all of the ranges of variation of the uncertain inputs simultaneously. According to Saltelli [10], this allows the interaction phenomena for uncertain inputs to be taken into account in the response of the model. This approach is therefore more relevant in hygrothermal modelling.

The global sensitivity analysis based on the variance consists firstly in sampling N set of values for the p uncertain inputs. Then, this sample is propagated in the model to obtain the N output values qualifying the robustness of the model. Finally, the sensitivity index calculation estimates the most influential input(s) on the output variability.

The ANOVA methods are based on the decomposition of the output variance as the sum of each input variance and interaction of inputs variance. Then, the first order as well as the higher order effects are calculated. The advantage of this decomposition is to obtain an intuitive sensitivity index between 0 and 1,

representing the part of the input variance on the output variance. If the sensitivity index is equal to 0, the input is not responsible for any variability on the output. With a sensitivity index of 1, the input is responsible for all of the variability observed. If there are no interactions between the inputs, the sum of the sensitivity indices should be close to one.

There are many methods for estimating the sensitivity indices based on the variance. The most well-known and time-consuming method is the Sobol method [10]. To drastically reduce the computation time while keeping the reliability of the sensitivity index, the RBD-FAST method was used in [11]. This approach is implemented in the sensitivity analysis package of Python SA-Lib [12]. It rapidly estimates reliable first order indices thanks to the combination of an optimized sampling (LHS) [13], an EASI permutation tip [14], and a bias correction [15]. An example can be found in [7]: this method estimated the influence of 14 inputs on hygrothermal models in a few hundred simulations. Obtaining higher-order sensitivity indices witnessing the interaction effects of inputs into the model is more expensive, but is not necessarily justified. Indeed, if the sum of the first order sensitivity indices is close to 1, this means that the first order effects alone explain all of the output variability. We will therefore calculate this indicator in this study.

2.2 Case study

2.2.1 Experimental set-up

The case study corresponds to a rammed earth wall of dimensions $1.00 \times 1.50 \times 0.30 \text{ m}^3$ enclosed in a climatic chamber of dimensions $1.70 \times 1.20 \times 1.90 \text{ m}^3$. It is illustrated in Fig. 1.

Fig. 1. Scheme of the experimental set-up

The whole experimental set-up was located in a hangar which was not insulated, and its ambient conditions were not controlled. The climatic chamber was insulated with 10 cm of cork. One side was opened to the exterior environment (i.e the hangar air), while the other side was sealed. They are referred to as “exterior” and “interior” side respectively. The interior side was insulated with 10 cm of polystyrene in addition to the cork insulation. Temperature and relative humidity sensors

were placed on the exterior and interior air, as well as in the middle of the wall (meaning at 15 cm from its surfaces). According to the manufacturer, the accuracy of the temperature and relative humidity sensors were ± 0.4 °C and ± 2 % respectively [16].

2.2.2 Material properties

The earth used in this study was extracted from Saint-Antoine l'Abbaye city, next to Lyon (France). Its mineralogical composition can be found in [1]. The hygrothermal properties of the rammed earth were measured in [1] on multiple samples, leading to the definition of intervals of variation for each property. They are summarized in Table 1.

Table 1. Rammed earth hygrothermal properties

Property	Interval of variation
Density ρ [kg/m ³]	1660 – 1820
Heat capacity c_p [J/(kg.K)]	580 – 670
Dry conductivity k [W/(m.K)]	0.64 – 0.90
Vapour resistance factor μ [-]	9.4 – 10.6
Hygric capacity ζ [kg _v /kg] on the interval [0.55; 0.68] ψ	0.015 – 0.021

Instead of the sorption isotherm, the hygric capacity of the material was used for practical reasons. In [1], three sorption isotherms were measured: one obtained with the saturated salt solutions method, and the other two with a Dynamic Vapour Sorption. The GAB model [17] was then applied to the three isotherms (see Fig. 2) according to Eq. (1):

$$w = \frac{C_1 C_2 \psi}{(1 - C_2 \psi)(1 - C_2 \psi + C_1 C_2 \psi)} w_m \quad (1)$$

where ψ is the relative humidity [-], while C_1 , C_2 and w_m are fitting parameters.

Fig. 2. Sorption isotherms of rammed earth extracted from [1], and fitted with GAB model – Whole isotherm (top) and zoom on the interval [0.55; 0.68] ψ (bottom)

Finally, the hygric capacity was calculated on the interval [0.55; 0.68] ψ . While this interval is rather narrow, it was selected for consistency reason as it corresponds to the range of humidity variations during the whole experiment. This approximation was deemed acceptable as the sorption isotherm was assumed straight on this interval (see Fig. 2).

For this study, the liquid water permeability was set null to simplify the inputs. It was assumed that mostly vapour transfer occurred on this relative humidity interval. Moreover, according to the scale analysis done in [18], liquid transfer becomes predominant after 0.85 ψ .

2.3 Hygrothermal model

2.3.1 Governing equations

Transfer in porous media are governed by mass and energy conservation equations.

The mass conservation equations accounts for vapour and liquid transfer, described by Fick's and Darcy's law respectively. It was expressed with the capillary pressure as driving potential, based on the literature review presented in [19]:

$$\frac{\partial w}{\partial p_c} \frac{\partial p_c}{\partial t} = -\nabla \cdot \left[\left(\delta_l + \delta_p \frac{\rho_v}{\rho_l} \right) \nabla p_c \right. \\ \left. - \delta_p \left(\psi \frac{\partial p_{sat}}{\partial T} - p_v \frac{\ln \psi}{T} \right) \nabla T \right] \quad (2)$$

where w is the water content [kg/m^3], p_c is the capillary pressure [Pa], which is linked to ψ through the Kelvin's law, t is the time [s], x is the thickness [m], δ_l is the liquid water permeability [$\text{kg}/(\text{m.s.Pa})$], δ_p is the water vapour permeability [$\text{kg}/(\text{m.s.Pa})$], ρ_v is the water vapour density [kg/m^3], ρ_l is the liquid water density in [kg/m^3], p_{sat} is the saturated vapour pressure [Pa] and T is the temperature [K]. Note that the vapour permeability δ_p is directly linked to the vapour resistance factor μ through Eq. (3):

$$\delta_p = \frac{\delta_a(T)}{\mu} = 2 \cdot 10^{-7} \frac{T^{0.81}}{p_{atm} \cdot \mu} \quad (3)$$

where $\delta_a(T)$ is the vapour diffusion coefficient in air [$\text{kg}/(\text{m.s.Pa})$] calculated as a function of the temperature according to the equation given in [20], and p_{atm} is the atmospheric pressure [Pa].

The energy conservation equation is governed by the first law of thermodynamics. Kinetic and potential energy are usually neglected in building physics, and the flux densities considered are the heat conduction, and the latent and sensible heat due to moisture transfer. This leads to Eq. (4):

$$\left(\rho_{mat} c_{p,mat} + w c_{p,l} \right) \frac{\partial T}{\partial t} \\ = \nabla \cdot \left\{ \left[k_{mat} + L_v \delta_p \left(\psi \frac{\partial p_{sat}}{\partial T} - p_v \frac{\ln \psi}{T} \right) \right] \nabla T \right. \\ \left. + \left(\delta_l c_{p,l} T - L_v \delta_p \frac{\rho_v}{\rho_l} \right) \nabla p_c \right\} \quad (4)$$

where ρ_{mat} is the material density [kg/m^3], $c_{p,mat}$ and $c_{p,l}$ are the heat capacity of the material and liquid water respectively, k_{mat} is the material thermal conductivity [$\text{W}/(\text{m.K})$] and L_v is the latent heat of evaporation [J/kg] taken constant.

Eq. (2) and (4) are coupled and non-linear. Hence, there were solved with the finite difference method using an implicit scheme for a one-dimensional case. This model was developed in Python and was validated with the standard EN 15026 [21] and the study from [6] (not presented here).

2.3.2 Boundary conditions

As the wall was inside a hangar, only convective heat and mass transfer were considered between the wall and the air. A value of $8 \text{ W}/(\text{m}^2.\text{K})$ and $25 \cdot 10^{-9} \text{ kg}/(\text{m}^2.\text{s.Pa})$ were chosen for the convective heat and mass transfer coefficient respectively. It was deemed representative of the case study according to [22], in which different values were tested and compared to experimental data.

The exterior and interior air temperature, relative humidity and vapour pressure of the climatic chamber are given in Fig. 3.

Fig. 3. Exterior and interior air temperature, relative humidity and vapour pressure of the climatic chamber

As the exterior side was opened to the hangar air, the temperature, relative humidity and vapour pressure variations were more significant than the ones in the interior side. Note that no control on the air was done from either side of the wall. The differences between the interior and exterior air are a consequence of the insulation applied to the interior side.

2.3.3 Simulations

The simulations were carried on 80 days. The 30 cm thick wall was decomposed in 31 meshes, meaning one node every centimetre, with the first node located on the exterior surface of the wall. A constant time step equal to 5 min was chosen. As we focused on applying the uncertainties and sensitivity analysis on the hygrothermal

properties of the rammed earth wall, these parameters were chosen arbitrarily. The results obtained and the calculation time were deemed acceptable with these parameters. The initial conditions were set constant throughout the wall, and equal to the temperature and relative humidity measured in the middle of the wall, meaning 18.3°C and 0.659 ψ , corresponding to a vapour pressure of 1385 Pa.

First, based on the inputs interval of variation given in Table 1, the Latin Hypercube Sampling method associated with RBD-FAST generated N combinations of rammed earth hygrothermal properties, assuming a uniform distribution for each input [7]. As a large number of simulations might be needed for the model to converge, an arbitrary value of $N=1000$ was chosen. Then, for each of the N parameters combination, the calculation was done with the hygrothermal model. Each calculation took approximately 2.4 min with our computer specifications (CPU i7, 1.80 GHz).

In order to validate the methodology and to investigate the discrepancies between numerical and experimental data, the output of interest was the vapour pressure in the middle of the wall.

3 Results

First, Fig. 4 presents the normalized variance and the sensitivity index of the most influential input to investigate the model convergence. The red dotted lines correspond to a $\pm 5\%$ interval around the normalized variance.

Fig. 4. Convergence plot of the normalized variance and the most influential sensitivity index

The sensitivity index of the most influential parameters becomes stable at $\pm 5\%$ after 200 simulations, while the normalized variance becomes stable from 500 simulations. Similar studies could therefore use 500 simulations instead of 1000.

As the sensors were only placed in the middle of the rammed earth wall, Fig. 5 compares the measured and calculated water vapour pressure at this point.

Fig. 5. Measured and calculated vapour pressure in the middle of the rammed earth wall

For the measured vapour pressure, the filled area corresponds to the sensors accuracy, while for the calculated one it represents its variation related to the 1000 different inputs combination generated with the Latin Hypercube Sampling method. Most of the time, both filled areas overlap each other, meaning that the model is able to accurately reproduce the experimental data. For the first 15 days, the discrepancies between the measured and calculated vapour pressure are relatively significant. As only the temperature and relative humidity on the exterior and interior air, and in the middle of the wall were available, the simulations initialisation was proved difficult.

One interesting result to note is that the vapour pressure variation due to the inputs variation is less significant than the one measured. This means that for this specific output, the accuracy of the hygrothermal properties measurements has less impact than the sensors accuracy.

As explained in Section 2.1, in order to investigate the dependency between the different inputs, the sum of the sensitivity indices is calculated and plotted in Fig. 6.

Fig. 6. Sum of the sensitivity indices

The sum of the sensitivity indices is close to 1 (≈ 0.96) for most of the 80 days. This means that few interactions exist between the inputs. For the first 15 days, significant variations of the sum can be noticed. As shown in Fig. 5,

few deviations are observed between the different simulations, leading to a small variance and hence to numerical errors on the sensitivity indices that are not relevant to investigate.

Finally, Fig. 7 presents the sensitivity index of each inputs.

Fig. 7. Sensitivity indices S_i for each input at each time step

As explained previously, it is not relevant to investigate the results on the first 15 days as the variations of the sensitivity indices are mostly due to numerical errors. For the remaining days, three inputs influence greatly the vapour pressure uncertainty: the hygric capacity ξ ($\approx 80\%$), the vapour resistance factor μ ($\approx 8\%$) and the material density ρ_{mat} ($\approx 6\%$). As we are looking at the vapour pressure in the middle of the wall, it was expected that the heat capacity and the thermal conductivity would have a negligible impact compared to the other inputs.

The sensitivity indices are strongly dependent on the inputs variation. Hence, they have to be taken into account to investigate the inputs influence. The density was ranging from 1660 to 1820 (average of $1740 \pm 4.6\%$) and the vapour resistance factor value was ranging from 9.4 to 10.6 (average of $10 \pm 6.0\%$), whereas the hygric capacity was ranging from 0.015 to 0.021 (average of $0.018 \pm 16.7\%$). These uncertainties on the experimental measurements are consistent with the ranking of the most influential inputs.

This also shows that the density and the vapour resistance factor measurements give less discrepancies than the sorption isotherm ones. Also, the poor reproducibility of the sorption isotherm was reported in numerous articles ([23-25] among others), and its measurement could lead to large uncertainties [25]. Efforts should therefore be done on the measurement and the modelling of the sorption isotherm to accurately represent the vapour pressure.

4 Conclusion and perspectives

To conclude, this article presented a methodology to validate and assess the robustness of a hygrothermal model. An uncertainty and sensitivity analysis method – RBD-FAST – was applied to a rammed earth wall. The

wall was subjected to environmental conditions and the discrepancies between the measured and calculated vapour pressure in the middle of the wall were investigated.

Few discrepancies were observed, highlighting that the model can accurately replicate the vapour pressure evolution in the wall. Moreover, the sensitivity analysis demonstrated that the main inputs influencing the vapour pressure were the hygric capacity, the vapour resistance factor and the density of the material. This result was expected for this specific output. Particularly, effort should be made on the sorption isotherm measurement as it had the most significant impact ($\approx 80\%$) on this specific output.

Further studies will focus on modelling a wall subjected to realistic solicitations by taking into account the weather and the indoor sources related to the occupants an air-conditioning system. More outputs will also be investigated, like the energy demand, the occupants' comfort and the moisture related risks. The quantification of the error on the hygrothermal model could also be assessed by applying uncertainties and sensitivity analysis on the mesh grid and the time step.

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