Theoretical Justification for the One-Dimensional Geolocation Method

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Abstract. In this article we will discuss the usage feature of the ground penetrating radar (GPR) for the solution underground geotechnologies problems. One of the main problems by the usage GPR method is that the surface of the workings is shielded by metal elements of support (frames, fittings, tightening and other). In this article we suggest to use one-dimensional GPR-scanning method instead of traditional GPR-profiling method. We assume that the scanning will be performed on the development contour in areas free from shielding. For justification one-dimensional GPR method we propose a mathematical model for the propagation of an electromagnetic signal in an inhomogeneous medium based on classical equations of electrodynamics. We also present a numerical implementation of it, which confirms the validity of the accepted problem statement.

1 Introduction

Let us remind how a ground penetrating radar (GPR) works. A GPR unit consists of two objects: a transmitter and a receiver. The transmitter generate electromagnetic (EM) pulse which moves into the studied medium. If the medium contains objects (or layers) with different permittivity, then part of the wave will be reflected and move back to receiver. If we know the EM wave velocity and know the total travel time we can calculate the object distance. The GPR is applied in earth, geological, archaeology science. In each of these application the GPR unit used on the earth's surface and never the under the earth. The full use of GPR is not possible for the following reasons: lack of intrinsically safe equipment; screening of the work-out contour with metal support elements (frames, fittings, tightening); high level of electromagnetic interference in the workings due to the presence of cable power lines and electric power plants; lack of methods and software for interpretation of underground GPR profiles.

Encouraging results were obtained in IGD of North SB RAS [1-9]. It is proposed to use wavelet transformations that include analysis of the signal measured by the GPR at different frequencies and scales when interpreting geo-radar data [1]. This approach is implemented for the Triton GPR with a main frequency of 30 MHz. The developed theoretical foundations and physical modeling in the specified frequency range allowed us to implement this method in the study of a frozen rock mass, a distinctive feature of which is the filling of voids and pores of the rock with both ice that exhibits dielectric properties and an electrically conducting melt solution [2-4]. Also tests of the GPR OKO-2 with an

antenna unit at a main frequency of 400 MHz were performed in the conditions of the Taldinskaya-Zapadnaya mine. Stratifications were recorded in the roof of the preparatory workings. However, it should be noted that when interpreting GPR profiles, the authors did not take into account the above factors that significantly affect the informative value of the GPR method [5, 6]. In [7, 8] the results of using the OKO-2 GPR with the AB-1200 antenna unit are presented. The main problem was the strong influence on the results of probing cavities filled with water with high conductivity. The use of a set of corrective operations (amplitude equalization, Hilbert transformation, horizontal filtering) allowed to increase the information content of monitoring. The authors of [9] proposed to evaluate the quality of compaction of soils by pressure injection of cement-containing rasvor to use integrated GPR profiles that allow monitoring the processing of the bases of technical structures and the degree of strength set by an artificial array according to the final numerical parameters determined using special algorithms.

2 Results and discussion

2.1 Formulation of the problem

It should be noted that the development of the theoretical aspects of GPR is important not only from scientific but also from a practical point of view, as it will allow in the future to automate the processing of GPR.

Let us consider the following model problem. Let us assume that at the initial moment of time on the earth's surface, the GPR transmitter emits an electromagnetic signal that fades over time. The signal reflected from the boundaries of sections of geological media and various foreign inclusions is received and recorded by the GPR receiver. It is necessary to establish a pattern of propagation of an electromagnetic wave deep into space and track the time of return of the wave reflected from the boundaries of layers and inclusions. Next, we will assume that both the transmitter and receiver are located at the same point, that is, the wave propagation occurs along the same coordinate axis. Under this assumption, both geological layers and foreign inclusions will be displayed as one-dimensional segments that have certain physical properties.

Fig.1. Design scheme for three layers medium.

With this in mind, let's consider a simplified model of the geological environment in the form of a three-layer homogeneous isotropic space within each layer with flat borders. The parameters of the layers are known: $\epsilon_{1,2,3}$, $\sigma_{1,2,3}$, $h_{1,2}$ – permittivity, specific electrical conductivity, Sm/m, layer power, m (picture 1), where $h_3 \rightarrow \infty$.

It is known that the most general way to describe any electromagnetic processes is the system of Maxwell's equations, on the basis of which all possible special cases can be obtained.

Let us consider the system of Maxwell's equations

$$\operatorname{rot} \bar{H} = \frac{\partial \bar{D}}{\partial t} + \bar{J}, \operatorname{rot} \bar{E} = -\frac{\partial \bar{B}}{\partial t}, \operatorname{div} \bar{D} = \delta, \operatorname{div} \bar{B} = 0, \quad (1)$$

$$\bar{D} = \varepsilon \varepsilon_0 \bar{E}, \bar{B} = \mu \mu_0 \bar{H}, \bar{J} = \sigma \bar{E}, \quad (2)$$

where δ – 3d-density of third-party current, Кл/м³; $\bar{J}$ – the density of electric current, А/м²; $\bar{E}$ – vector field strength, V/м; $\bar{H}$ – magnetic field strength, А/м; $\bar{D}$ – electrical induction, Кл/м²; $\bar{B}$ – magnetic induction, Тл; $\mu_0 = 1,256 \cdot 10^{-6}$, Hн/м – magnetic constant; $\varepsilon_0 = 8,854 \cdot 10^{-12}$ – electric constant, F/м; ε – relative dielectric permittivity; μ – relative magnetic permittivity; σ – specific conductivity of the medium, См/м.

Since GPR does not track the magnetic properties of the host rocks, there is no need to model the behavior of the magnetic component of the reduced system. Let's write down the electrical component of this system separately. To do this, we make a partial time differentiation of the first and seventh equations in system (1, 2), and also find the rotor from the left and right parts of the second equation of the Maxwell system. Let us find that

$$\operatorname{rot} \left(\frac{\partial \bar{H}}{\partial t} \right) = \frac{\partial^2 \bar{D}}{\partial t^2} + \frac{\partial \bar{J}}{\partial t}, \text{ where } \frac{\partial \bar{J}}{\partial t} = \sigma \frac{\partial \bar{E}}{\partial t},$$

$$\operatorname{rot}(\operatorname{rot} \bar{E}) = \operatorname{rot} \left(\frac{\partial \bar{B}}{\partial t} \right) \Rightarrow \nabla^2(\bar{E}) = \operatorname{rot} \left(\frac{\partial \bar{B}}{\partial t} \right).$$

Then, after simplifications and assuming that $\mu = 1$, we obtain a partial differential equation of the form

$$\nabla^2(\bar{E}) = \mu_0 \left(\varepsilon \varepsilon_0 \frac{\partial^2 \bar{E}}{\partial t^2} + \sigma \frac{\partial \bar{E}}{\partial t} \right). \quad (3)$$

In the case of one-dimensional vertical propagation of an electromagnetic wave, the stress vector will contain only one non-zero component: $E_z = E(z, t)$. Then equation (3) takes the form

$$\frac{\partial^2 E}{\partial z^2} = \mu_0 \left(\varepsilon(z) \varepsilon_0 \frac{\partial^2 E}{\partial t^2} + \sigma(z) \frac{\partial E}{\partial t} \right). \quad (4)$$

Equation (4) will be executed at any time for any point of the considered layers. Let us supplement this equation with initial boundary conditions that reflect the physics of the signal propagation process.

Condition 1. On the ground an electromagnetic signal with a voltage that fades over time is emitted by the law $E = E(z, t)|_{z=0} = \mu_1(t)$, where $t \in [0; 50] \cdot 10^{-9}$ s.

Condition 2. The attenuating effect of the medium on the signal is not so hard that the signal distortion occurs immediately below the earth's surface: $E(z, t + \delta_t)|_{z=\delta_h} = E(z, t)|_{z=0}$, where δ_t, δ_h – small steps in time and space.

Condition 3. The electromagnetic field at the initial moment of time is not present at the entire depth below the point of emission of the pulse $E(z, t)|_{t=0} = 0$, where $z \in [2\delta_h; H]$, where $H = h_1 + h_2 + h_3$ m.

Condition 4. $E(z, t)|_{z=H} = 0$, where $t \in [0; 50] \cdot 10^{-9}$ s – it is mean that there is no electromagnetic field at the lower boundary.

2.2 Numerical solution of the equation

Let us solve equation (4) with boundary conditions by numerical methods. Let us generate two global grids by the spatial $z \in [0; H]$ with the step h_z and by the time $t \in [0; T]$ with the step h_t . On the layers border where $z = h_1$ and $z = h_1 + h_2$ electrical parameters changes as:

$$\varepsilon(z) = \begin{cases} \varepsilon_1, 0 < z < h_1 \\ \varepsilon_2, h_1 < z < h_1 + h_2 \\ \varepsilon_3, h_1 + h_2 < z < h_1 + h_2 + h_3 \end{cases} \quad \text{and} \quad \sigma(z) = \begin{cases} \sigma_1, 0 < z < h_1 \\ \sigma_2, h_1 < z < h_1 + h_2 \\ \sigma_3, h_1 + h_2 < z < h_1 + h_2 + h_3 \end{cases}$$

Let us use the explicit calculation scheme with second-order accuracy of the “cross” type. Here are the difference formulas for partial derivatives of the wave equation (the lower index corresponds to the number of the spatial grid, and the upper index corresponds to the time):

$$\frac{\partial^2 E}{\partial z^2} = \frac{E_{j-1}^i - 2E_j^i + E_{j+1}^i}{h_z^2}, \quad \frac{\partial^2 E}{\partial t^2} = \frac{E_j^{i-1} - 2E_j^i + E_j^{i+1}}{h_t^2}, \quad \frac{\partial E}{\partial t} = \frac{E_j^{i+1} - E_j^{i-1}}{2h_t}.$$

If we insert this expressions in (4) and find the intensity values E_j^{i+1} on the next time layer then we get the expression:

$$E_j^{i+1} \psi = \gamma^2 (E_{j-1}^i - 2E_j^i + E_{j+1}^i) - E_j^{i-1} \psi + 2E_j^i, \quad (5)$$

where $\gamma = \frac{h_t c}{h_z \varepsilon} \leq 1$ – necessary condition for stability of the design scheme (5);

$$\psi = 1 + \frac{\mu_0 \sigma h_t c^2}{2\varepsilon} - \text{auxiliary value.}$$

Since it is impossible to directly model the field strength attenuation at infinity (for $h_3 \rightarrow \infty$) for a numerical solution, we resort to constructing equivalent boundary conditions at the final boundary – absorbing boundary conditions. In the one-dimensional case under consideration, when the electromagnetic wave falls perpendicular to the boundary, it is possible to construct ideal absorbing.

$$\frac{\partial E(z, t)}{\partial t} \Big|_{z=H} + \gamma \frac{\partial E(z, t)}{\partial z} \Big|_{t+h_t} = 0, \quad \frac{\partial E(z, t)}{\partial t} \Big|_{z=H-h_z} + \gamma \frac{\partial E(z, t)}{\partial z} \Big|_t = 0. \quad (6)$$

If we summate this expressions and going to the difference expression, we will get the expression for recalculating the intensity value on the last spatial layer

$$E_H^{i+1} = E_{H-h_z}^i + \frac{(1-\gamma)}{(1+\gamma)} (E_H^i - E_{H-h_z}^{i+1}) \quad (7)$$

To demonstrate the results of using the constructed calculation schemes, let us consider the following test example. Note for readers that a simpler modification of this example is provided on the site www.geo-radar.ru, where a two-layer model was considered without taking into account the absorbing conditions at the lower boundary.

The emitted electromagnetic signal is set using the following formula $\mu_1(t) = A \cos(\omega_0 t) e^{-qt}$, where $A = 250$ – signal amplitude, V/m; $\omega_0 = 100 \div 800$ MGz –

center frequency; $q = (1 \div 9) \cdot 10^8$, $1/c^2$ – attenuation coefficient. The depth of layers are: $h_1 = 1,25$, $h_2 = 0,75$ m. To specify the electrical parameters of the layers, we will assume the following model scheme of the host space: the moistened second layer is located between two drier layers, but the third layer will be wetter than the first. Based on this, we assume that the relative permittivity ε_k and specific conductivities σ_k of layers are: $\varepsilon_1 = 5$, $\varepsilon_2 = 20$, $\varepsilon_3 = 10$ and $\sigma_1 = 0,001$, $\sigma_2 = 0,004$, $\sigma_3 = 0,003$ CM/M.

Figure 2 shows the results of calculations in the form of a map isolines of function $E = E(z, t)$ and the graph received on the surface of the signal reflected from the interface of layers.

Fig. 2. Calculation results for a three-layer medium.

3 Conclusion

The considered test example shows the principle operability of the proposed design scheme: 1) the map of isolines shows graphs of reflected and refracted waves by the boundaries of layers; 2) the efficiency of absorbing boundary conditions is visible – there are no graphs of reflected waves from the lower border; 3) the graph of the received signal in the form of bursts shows reflected waves returned in the opposite phase from both layer boundaries.

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