Research of some physical and mechanical characteristics of cow's udder nipples

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Abstract. A rational technology for cows' machine milking when tie-up housing should provide for the development of technical equipment that meets the physiological requirements for a dairy cow and ensure complete and safe milking. In this regard, the task of determining and refining certain parameters of the physical-mechanical characteristics of the mammary gland of a cow becomes urgent. When choosing the rational mass of the suspended part of the vacuum milking machine, one should know the pressure transmitted by the teat cup liner to the body of the nipple of the animal when milking, which makes possible to establish the obtained analytical formula. It was found that the higher the deflection of the walls of the stretched teat cup liner, the higher the pressure caused by it on the nipple of the cow's udder. Analysis of the distribution of diameters and lengths of the nipples of the mammary gland of cows showed that their sizes vary in wide ranges. Elongations of nipples depend nonlinearly on the acting tension, and the values of the elastic modulus and the transverse strain coefficient vary over a wide range.

1 Introduction

Physical-mechanical characteristics of nipples of a cow's udder determine not only the suitability of cows for machine milking, but geometric parameters and technological conditions of the milking machine.

The main physical and mechanical characteristics of the mammary gland necessary for the development of technical means of milking include: morphological parameters of the nipples of the mammary gland: the distance between the nipples of the anterior or posterior lobes, between the nipples of the anterior and posterior lobes, the diameter and length of the nipples; characteristics of the elastic properties of the nipples: lateral strain coefficient and elastic modulus; friction coefficient of an interacting pair of the nipple of the udder and the teat cup liner.

The morphological features of the mammary gland affect the choice of the geometric parameters of the hanging part of the milking machine, such as the overall dimensions and shape of the collector, the geometric dimensions of the teat cup liner and the milking cup as

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a whole and, accordingly, the height of the entire hanging part.

The characteristics of the elastic properties of the nipples of the mammary gland are associated with the rationale for their interaction with the actuator of the milking machine. When milking cows with a vacuum apparatus, the nipples experience longitudinal and transverse deformations, significantly changing their geometric parameters such as diameter and length [1,2,3,4,5].

Friction coefficients are also necessary to justify the interaction of the actuator of the milking machine with the mammary gland of the animal, as it affects the choice of operating modes of milking machines.

When milking cows with modern push-pull vacuum milking machines, especially those of the synchronous principle of action, one can observe an excessive crawl of two-chamber milking cups on nipples of the animal's mammary gland, which, therefore, always leads to a premature termination of the milking process. Manual pulling of teat cups prevents crawling, but it requires much working time and causes very great inconvenience in operation.

In order to reduce the crawl of two-chamber teat cups on nipples of the mammary gland, the mass of the suspended part of the vacuum milking machine is usually changed [6,7,8].

The weight of the suspended part of the vacuum milking machine is selected so that on one side the two-chamber teat cups do not crawl on nipples of the mammary gland, and on the other hand, do not fall off them. So far this difficult task has not been completely solved [9,10,11].

The teat cup liner in a two-chamber teat cup is necessarily pinched at the ends and is in a very tight state. During the closure of the walls of the teat cup liner in a two-chamber teat cup, the nipple's body is compressed with the force arising from the pressure of the teat cup liner closure, causing resistance to the elasticity of the body of the nipple of the mammary gland and, as a consequence, the existing reactions to this compression. From the closure of the walls of the highly stretched teat cup liner, forces will also arise that push the nipple of the mammary gland, causing resistance to the elasticity of the body of its end part. According to studies, the pressure of the walls of highly stretched teat cup liner varies in length. The highest pressure is always observed on the top of the nipple of the mammary gland and it causes the movement of the two-chamber teat cup of the vacuum milking machine down the nipple.

When the nipple of the udder is compressed, it deforms, while its cross-section changes from round to oval, and due to a very significant decrease in the suction force, the nipple is somewhat shortened.

When choosing the rational weight of the suspended part of the vacuum milking machine, one should know the pressure that the teat cup liner exerts on the body of the nipple of the udder when milking.

A review of literature showed that the morphological features of the udder of cows were studied by many scientists [12,13,14,15,16,17]. Research was conducted mainly in the seventies and eighties of the last century. After the collapse of the USSR, the situation in agriculture worsened for well-known reasons. Breeding work on the selection of cows for suitability for machine milking in most households was not carried out. The dairy herd was completed with all the cows that could be milked. Therefore, at present, the morphological parameters of the mammary gland of a cow require clarification. The elastic and mechanical properties of the nipples of the mammary gland, as evidenced by a review of the literature, are contradictory and insufficiently studied.

The works of Kartashova L.P., Krasnova I.N. and other scientists [14,15,16] are devoted to analytical dependencies to determine the forces acting on the nipple. The proposed dependencies are sometimes difficult and inconvenient to use.

In this work, an analytical formula is obtained that allows one to establish the pressure

exerted by the teat cup liner on the body of the nipple of the animal’s mammary gland during the suction stroke and, accordingly, during the compression stroke. Morphological characteristics of the nipples of the mammary gland of the cow are specified, the elastic properties of the nipples of the mammary gland and their friction coefficients are determined.

2 Purpose, objectives

Theoretical studies conducted using ideas of classical mechanics and mathematics are aimed at deriving an equation that allows to determine the pressure exerted by the teat cup liner on the nipple of the animal’s mammary gland during a sucking stroke and a compression stroke. Practical studies are aimed at clarifying the morphological characteristics of nipples of the cow’s udder, determining characteristics of elastic properties of nipples of the mammary gland and their friction coefficients.

3 Methods and materials

To determine the pressure of the walls of the stretched teat cup liner on the nipple of the mammary gland, let’s consider the deformation of a thin elastic membrane under the influence of uniformly distributed pressure (the presence of the nipple inside the membrane is not considered). Due to the fact that the teat cup liner of the two-chamber teat cup is pinched at the ends, the shell walls will deflect due to the pressure difference (Fig. 1).

There is a working vacuum inside teat cup liner (teat chamber) 2 with radius r_0 and the wall thickness of h . And there will be either atmospheric pressure or vacuum in a teat cup case (intermural chamber) 1 depending on the pulse cycle. When atmospheric air enters chamber 1, the walls of shell 2 will bend under the influence of the pressure difference. When deflecting (the distance from the surface of the rubber shell in the initial position to the same surface in the current position in the radial direction), $\omega = r_0$ walls will close. In this case, the shell loses stability and the cross section of the shell becomes flat at distance l .

Where: 1 – teat cup case; 2 –teat cup liner.

Fig. 1. The scheme of deflection of the teat cup liner walls.

Consider the cross section at distance l from the edge of the teat cup liner. An element with sides ds and dl_z is cut and the acting tension is applied to it (Fig. 2).

Fig. 2. The scheme of tension on the selected element.

Normal tension σ_z will act in the meridian section (Fig. 3A) on the edge of the selected element and normal tension σ_t will act in a section perpendicular to it (Fig. 3B). The acting tension forces applied to the edges of the selected element are equal to $\sigma_z \cdot dS \cdot h$ and $\sigma_t \cdot dl_z \cdot h$, respectively.

In addition, normal pressure p giving load $p \cdot dS \cdot dl_z$ will act on the surface of the element. Since the teat cup liner undergoes only tension, these normal forces are directed tangentially to the sides of the element faces in the meridian section and in the section perpendicular to.

Fig. 3. The scheme of the action of forces in sections: A – in the meridian section; B – perpendicular to the meridian.

For the projection of total force σ_{zx} on X axis, one can get the expression:

$$\sigma_{zx} = 2\sigma_z \cdot dS \cdot h \cdot \sin \frac{dy}{2} \quad (1)$$

Since for small angles, the sine of the angle is approximately equal to the angle itself, expressed in radians, then: $\sin \frac{dy}{2} \approx \frac{dy}{2}$. Thus, we have:

$$\sigma_{zx} = \sigma_z \cdot dS \cdot h \cdot dy \quad (2)$$

Similarly, it is possible to get the expression for resultant σ_{tx} on X axis:

$$\sigma_{tx} = -2\sigma_t \cdot dl_z \cdot h \cdot \sin \frac{d\phi}{2} = -\sigma_t \cdot dl_z \cdot h \cdot d\phi \quad (3)$$

The general equilibrium equation on X axis written as follows:

$$-\sigma_t \cdot dl_z \cdot h \cdot d\phi - p \cdot dS \cdot dl_z + \sigma_z dS \cdot h \cdot dy = 0 \quad (4)$$

Since $dl_z = \rho dy$ and $dS = r d\phi$, one has:

$$-\sigma_t \cdot \rho \cdot h - p \cdot r \cdot \rho + \sigma_z \cdot r \cdot h = 0 \quad (5)$$

The first element of expression (4) at maximum deflection is equal to zero. Since angle $\frac{d\phi}{2}$ will tend to zero, symmetrically acting forces tend to straighten the walls in cross section $z = l$. External pressure also contributes to this when the teat cup liner loses stability. The shape that the teat cup liner will take in this section is shown in Figure 4.

Fig. 4. The scheme for closing the walls of the teat cup liner.

Since the ends of the teat cup liner are pinched on both sides, it will stretch in the meridian direction. At a certain distance from the edge sealing, their effect on cross section $z = l$ will be insignificant. As already noted, the walls of the teat cup liner in this section will close.

There is a thin teat cup liner with pinched ends. Due to symmetry, the material points of the walls will move along X axis. The walls will stretch in the meridian section, and in the perpendicular section the forces will contribute to the closure of the walls. Moreover, these efforts will be insignificant in cross section $z = l$, since the influence of sealing the ends of

the teat cup liner at a certain distance can be neglected. Then, for simplicity of further analysis of the deformation of the teat cup liner walls, the first element in expression (4) is taken as equal to zero. Then one has:

$$-p \cdot \rho + \sigma_z \cdot h = 0 \tag{6}$$

Expressing σ_z from the last expression, one gets:

$$\sigma_z = \frac{p \cdot \rho}{h} \tag{7}$$

On the other hand, according to Hooke's law:

$$\sigma_z = E \cdot \varepsilon \tag{8}$$

where E – is elastic modulus; ε – is relative deformation of the walls.

Relative strain will be:

$$\varepsilon = \frac{l_T - l_0}{l_0} \tag{9}$$

where l_T – is the current wall length; l_0 – is the wall length before deformation.

Obviously, the deflection of the walls of the teat cup liner occurs along some arc of a circle with radius ρ . Obviously, the deflection of the walls of the teat cup liner occurs along some arc of a circle with radius. Then the length of the wall during deflection will be equal to the length of the arc limited by the sealing zone of the ends of the teat cup liner (Fig. 5).

Fig. 5. The scheme of the deflection of the teat cup wall.

Arc AB length is determined according to recommendations. If one considers a circle with radius ρ centered at point O , then the semicircle has equation $y = \sqrt{\rho^2 - x^2}$. Take a graph of this function on segment $[a; b]$. At that, there is an inequality for values: $-\rho < a < b < \rho$.

The derivative of function $f'(x) = (\sqrt{\rho^2 - x^2})' = -\frac{x}{\sqrt{\rho^2 - x^2}}$ in this section of the segment is continuous, then for

function: $\sqrt{1 + \left((f'(x))^2\right)} = \sqrt{1 + \left(\frac{x}{\sqrt{\rho^2 - x^2}}\right)^2} = \frac{\rho}{\sqrt{1 - \left(\frac{x}{\rho}\right)^2}}$ the antiderivative is
 $F(x) = -\rho \arccos \frac{x}{\rho}$, and the length of the arc is:

$$l_{AB} = F(B) - F(A) = -\rho \arccos \frac{b}{\rho} + \rho \arccos \frac{a}{\rho} = \rho \left(\arccos \left(-\frac{l}{\rho} \right) - \arccos \frac{l}{\rho} \right) = \rho \left(\pi - 2 \arccos \frac{l}{\rho} \right)$$

The relative deformation of the walls of the teat cup liner is expressed, given that $l_0 = 2l$ and $l_T = l_{AB}$, then

$$\varepsilon = \frac{l_T - l_0}{l_0} = \frac{l_{AB}}{2l} - 1 \quad (10)$$

And accordingly, the expression for tension σ_z takes the form:

$$\sigma_z = E \cdot \left(\frac{l_{AB}}{2l} - 1 \right) = E \cdot \left(\frac{\rho \left(\pi - 2 \arccos \frac{l}{\rho} \right)}{2l} - 1 \right) \quad (11)$$

On the other hand, $\sigma_z = \frac{p \cdot \rho}{h}$ and solving the latter together with expression (11) with respect to pressure p , one finally gets:

$$p = \frac{E \cdot h}{\rho} \left(\frac{\rho \left(\pi - 2 \arccos \frac{l}{\rho} \right)}{2l} - 1 \right) \quad (12)$$

The pressure from the closing walls of the rubber is transmitted to the body of the nipple. All other things being equal, the greater the difference, the greater the deflection of the rubber walls of the two-chamber teat cup and, accordingly, the pressure on the udder nipple. Obviously, the reaction of the nipple will be proportional to this effort, but it will have the opposite direction. Based on the above, one can write:

$$p_s = c \cdot p \quad (13)$$

where c is the proportionality coefficient.

Coefficient c takes into account the presence of an udder nipple in the rubber membrane. Assuming that the length of the udder nipple in the stretched teat cup liner takes as a first approximation no more than half the length of the teat cup liner, then the closure of its walls occurs in the middle part. This allows the use of formula (12) to describe the process of deformation of the udder nipple. When milking, the force transmitted to the cow's nipple depends on the deflection of the walls of the stretched teat cup liner. Radius ρ is expressed in terms of the deflection of teat cup liner ω . By the Pythagorean theorem in $\triangle ACO$ (Fig. 5):

$$AO^2 = AC^2 + CO^2 \text{ or } \rho^2 = l^2 + k^2.$$

On the other hand, $\rho = k + \omega$. Then one has the system of equations:

$$\begin{cases} \rho^2 = l^2 + k^2 \\ \rho = k + \omega \end{cases},$$

from which one gets the expression for determining the radius:

$$\rho = \frac{\omega^2 + l^2}{2\omega} \quad (14)$$

Substituting the value of the radius from expression (14) into formula (12), one obtains:

$$p = E \cdot h \cdot \left(\frac{\left(\pi - 2 \arccos \frac{2 \cdot \omega \cdot l}{(\omega^2 + l^2)} \right)}{2l} - \frac{2\omega}{\omega^2 + l^2} \right)$$

and the pressure on the nipple will be:

$$p_s = c \cdot E \cdot h \cdot \left(\frac{\left(\pi - 2 \arccos \frac{2 \cdot \omega \cdot l}{(\omega^2 + l^2)} \right)}{2l} - \frac{2\omega}{\omega^2 + l^2} \right) \quad (15)$$

The value of coefficient c is determined using the experimental data presented by I.N. Krasnov. Thus, by examining the effects of various milking machines on nipples of the cow's udder, he obtained pressures perceived by the nipples of animals during a compression stroke. So for DA-2M and Volga devices, the average pressure of the pre-stretched (60 N force) teat cup liner on the nipple is 0.13-0.15 kp/cm^2 , and the maximum pressure that the nipple perceives is 0.2-0.25 kp/cm^2 .

During the compression stroke, the deflection of the walls of the teat cup liner reaches its maximum value $\omega = r_0$. The working length of the teat cup liner is $l_0 = 140 \text{ mm}$, the elastic modulus for rubber is $E = 80 \text{ kp/cm}^2$, the inner radius of the rubber is $r_0 = 11.5 \text{ mm}$ and the wall thickness is $h = 2.5 \text{ mm}$, so

$$0.2 \dots 0.25 = c \cdot 80 \cdot 0.25 \cdot \left(\frac{\left(3.14 - 2 \arccos \left(\frac{2 \cdot 11.5 \cdot 7}{1.15^2 + 7^2} \right) \right)}{2 \cdot 7} - \frac{2 \cdot 11.5}{1.15^2 + 7^2} \right) = c \cdot 0.01636,$$

from which $c = \frac{0.2 \dots 0.25}{0.01636} \approx 12.2 \dots 15.3$ with arithmetic mean value $c \approx 13.75$.

For further calculations, coefficient c is constant and equal to $c \approx 15$. Formula (15) can be used to determine the pressure on the nipple when various deflections of the teat cup liner.

Due to the existing pressure drop in the intermural chamber of the teat cup during a suction stroke, the walls of the teat cup liner walls bend, affecting the nipple. According to the data of, the difference in vacuum pressure in the intermural and suction chambers of the teat cup is in the range of 11-21 kPa , depending on the operation of the pulsator and the intensity of milk flow. P.I. Ogorodnikov recommends choosing the milking mode so that the pressure drop in the chambers of the teat cup is within 11.3-12 kPa . In this case, the deflection of the stretched teat cup liner will be 4-6 mm , which practically does not affect the intensity of milk excretion.

Let's determine the pressure of the teat cup liner during the suction stroke of a standard two-chamber teat cup. To do it one takes the magnitude of the deflection of walls of the teat cup liner equal to $\omega = 5 \text{ mm}$ and according to formula (15) one gets:

$$p_s = 15 \cdot 80 \cdot 0.25 \cdot \left(\frac{\left(3.14 - 2 \arccos \left(\frac{2 \cdot 0.5 \cdot 7}{0.5^2 + 7^2} \right) \right)}{2 \cdot 7} - \frac{2 \cdot 0.5}{0.5^2 + 7^2} \right) = 0.0207 \text{ kp/cm}^2 \\ \approx 2.030 \text{ Pa}$$

To determine the morphological parameters of the udder nipples, such as the distance between the nipples of the anterior lobe, the distance between the nipples of the posterior lobe, the distance between the nipples of the anterior and posterior lobes, the diameter and length of the nipples, a caliper and a ruler were used.

Direct measurements of the diameter and length of the nipples of a cow's udder were made with a caliper, and measurements of the location of the udder nipples were made with a ruler according to the scheme shown in Figure 6.

Fig. 6. Measurement scheme of the location of the udder nipples.

To determine the characteristics of elastic properties such as the coefficient of transverse deformation and the elastic modulus of the udder nipples, a special device was developed (Fig. 7). The device includes cylindrical rod 1, on which weights 2 can be quickly installed sequentially. Weights 2 are made in the form of metal disks with a slot for the passage of the cylindrical rod.

Where: 1 – rod; 2 – weight; 3 – ruler.

Fig. 7. Determination of the coefficient of deformation of the cow's udder nipples: A – device model; B – device in action.

The methodology for determining the characteristics of elastic properties, such as the coefficient of transverse deformation and a variable modulus of elasticity of the udder nipples, is as follows. Parallel lines were drawn in ink in the middle of the nipple of the cow's udder. Then, cylindrical rod 1 was fixed with an adhesive plaster on the nipple of the udder. The nipple was sequentially loaded stepwise by setting a different number of weights 2 on cylindrical rod 1. After each loading, the nipple was photographed. As a scale factor, additional ruler 3 was used, installed directly at the nipple in one plane. During the experiment, the mass of weights 2 varied from 0.135 to 0.675 kg. After the experiment changes in the length and diameter of the nipples after each loading were measured in photos, taking into account the scale. Further statistical processing was carried out.

Elastic modulus E was determined by the known formula:

$$E = \frac{\sigma}{\epsilon_1}, \tag{16}$$

where σ – is normal tension, N/m²; ϵ_1 – is relative elongation of the nipple.

Accordingly, normal tension σ and relative elongation ϵ_1 are determined from the

following expressions

$$\sigma = \frac{P}{F}, \tag{17}$$

$$\varepsilon_1 = \frac{(l_2-l_1)}{l_1}, \tag{18}$$

where P – is the load, acting on the nipple, N; F – is a sectional area of an elementary site, m²; l_1 and l_2 – are the length of the elementary section before and after loading, respectively, m.

The transverse strain coefficient m of the nipple was determined by the formula:

$$m = \frac{\varepsilon_2}{\varepsilon_1}, \tag{19}$$

where ε_2 – is relative lateral deformation of the nipple.

$$\varepsilon_2 = \frac{(d_1-d_2)}{d_1}, \tag{20}$$

where d_1 and d_2 are the diameter of the nipple before and after loading, respectively, m.

Before fixing the device on nipples and conducting an experiment, stimulation of the reflex of milk ejection by massage of the udder was carried out.

To determine the friction coefficients of an interacting pair of the udder nipple and the teat cup liner, a special device was developed and manufactured (Fig. 8). The device consists of two curvilinear plates 1 in the form of half-cylinders pivotally connected to each other, and equipped with threaded clamp 2 with cylindrical compression spring 3. Fragments of the teat cup liner 4 are fixed on the inner surfaces of curved plates 1. At the bottom, curved plates 1 have hinged suspension 5 with water tank 6.

Threaded clamp 2 serves to move curved plates 1 relative to each other. When plates 1 come together, cylindrical spring 3 is simultaneously compressed. This force is directed perpendicularly to curved plates 1. The force of pressing plates 1 to the surface of the nipple is determined by the length of compressed spring 3.

Where: 1 – curved plates; 2 – threaded clamp; 3 – spring; 4 – teat cup liner; 5 – hinged suspension; 6 – water tank.

Fig. 8. Determination of friction coefficients: A - device diagram; B - the device is in working order when interacting with the nipple of the cow's udder.

Spring 3 was pre-calibrated. The applied force on spring 3 varied from 8.8 to 9.8 N.

After this, a graph of the dependence of the compression force on the working length of the spring was plotted (Fig. 9).

Fig. 9. Graphic dependence of length *l* of the compressed spring on applied force *F*.

The methodology for determining the coefficients is as follows. Initially, semi-cylindrical plates 1 were installed on the udder nipple and freely pressed to each other until they contact the surface of the nipple. Then, using threaded clamp 2, cylindrical spring 3 was compressed, which presses curved plates 1 to the nipple. Due to the friction force, the device was held on the nipple. The length of compressed spring 3 was measured with the help of a caliper. Then, using a piston pump, water was slowly poured into container 6 of the device until curved plates 1 began to move along the nipple. Next, container 6 with water was weighed, and the total gravity of the device was determined. In our case, gravity is the force of friction, since the device moves from the equilibrium condition on the udder nipple.

Friction coefficient *f* was determined by the formula:

$$f = \frac{G}{N} \tag{21}$$

where *G* – is gravity acting on the device, N; *N* – is clamping force of curved plates, N.

Two series of experiments took place. In the first series of experiments, the coefficient of friction of a dry rubbing pair was determined and in the second version, the nipple and fragments of the teat cup liner of the device were moistened with milk before pressing.

In order to study the working surfaces of the teat cup liner (the new, not used one and the one used for some time), which directly affect the coefficient of friction, an experiment was conducted according to the following method. First, samples of the most used in the region domestic and foreign teat cup liner of well-known manufacturers were selected. The following items were included in the list of tested teat cup liner samples: Volga, DA-2M Maiga, ADU-1, DeLaval sample No. 1, DeLaval sample No. 2 and domestic silicone rubber “type” ADU-1 (Fig. 10).

Where: 1 – Volga, 2 – DA-2M Maiga, 3 – ADU-1, 4 – DeLaval sample No. 1, 5 – DeLaval sample No. 2, 6 – Silicone rubber “type” ADU-1.

Fig. 10. Samples of teat cup liner: A - new, not used; B - used in the work.

Then, from the selected teat cup liner samples, their sections of working surfaces with an average size of 20x20 mm in the stocking area at a distance of about 30-40 mm from the top of the suction cup were taken (Fig. 11).

Where: 1 – Volga, 2 – DA-2M Maiga, 3 – ADU-1, 4 – DeLaval sample No. 1, 5 – DeLaval sample No. 2, 6 – Silicone rubber “type” ADU-1.

Fig. 11. The working surface of the teat cup liner samples: A - new, not used; B - used in the work.

The working surfaces of the selected teat cup liner samples were examined using an Altami optical microscope with an integrated digital camera for photo-video recording of the resulting image. In this case, the total increase in the surface area of the samples was 100 times (Fig. 12).

Where: 1 – Volga, 2 – DA-2M Maiga, 3 – ADU-1, 4 – DeLaval sample No. 1, 5 – DeLaval sample No. 2, 6 – Silicone rubber “type” ADU-1.

Fig. 12. The working surfaces of the teat cup liner samples under a 100-fold increase: the top row - a new, not used; the bottom row - used in the work.

4 Results and discussion

As it is shown by a priori data and confirmed by our studies, the vacuum value under the udder nipple decreases by 4.5-11 kPa from the nominal value during a suction stroke, depending on the intensity of the fluid flow. The tension of the teat cup liner with a wall thickness of $h = 2.5\text{-}3.0\text{ mm}$ with a force of 60-70 N compensates for inflation of the rubber from the resulting pressure drop in the chambers of the teat cup.

Figure 13 shows a graphical dependence of the pressure of the teat cup liner exerted on the nipple on the deflection of the walls, calculated by formula 15.

Fig. 13. Graphic dependence of the pressure of the teat cup liner exerted on the nipple on the deflection of the walls.

The graphical dependence shows that with an increase in the deflection of the walls of the teat cup liner, the pressure exerted by the rubber on the udder nipple increases non-linearly and progressively. So, for example, the teat cup liner with a working length of 140 mm, a wall thickness of 2.5 mm, with a deflection of walls of 5 mm, exerts a pressure on the nipple equal to 2,030 Pa. When a deflection of walls of 8 mm the pressure is 8,236 Pa and with a deflection of walls of 12 mm the pressure is 27,266 Pa.

Based on the results of studies of the morphological parameters of the cow's udder nipples, the graphs were plotted as shown in Figures 14 and 15.

Fig. 14. The nature of the distribution of the diameter (A) and length (B) of the udder nipples.

Fig. 15. The nature of the size distribution between the udder nipples.

Analysis of the distribution of diameters and lengths of the cow's udder nipples shows that 80-90% of cases from the sample are in the range from 19 to 34 mm and from 35 to 65 mm, respectively. Analysis of measurements of the udder nipples shows their uneven location on it. Thus, the distance between the nipples of the anterior lobe in most cows is in the range from 9 to 18 cm, and that is in the range from 5 to 11 cm in the posterior lobe, which is almost two times less than in the anterior lobe. The distance between the nipples of the anterior and posterior lobes ranges from 8 to 14 cm.

For clarity of theoretical positions, Figure 16 shows the graphical dependence of the change in the length of the nipples during milking according to the results of different researches.

Fig. 16. The change in the length of the cow's udder nipple during machine milking depending on the value of vacuum: 1 - according to V.F. Korolev; 2 - according to N.I. Pronichev; 3 - according to I.N. Krasnov; 4 - according to formula (15).

Graphic dependence 1 is based on studies conducted by V.F. Korolev, curve 2 is based on the experimental data obtained by N.I. Pronichev, dependence 3 is based on the empirical formula proposed by I.N. Krasnov, and dependence 4 is based on formula (15). The listed dependencies are constructed for the initial length of the udder nipple $l=65$ mm. As it can be seen from the graphs, their convergence with dependence 4, constructed by formula (15) is quite high, which gives a reason to use the results of theoretical studies in the development of milking equipment.

Based on the results of studies of the characteristics of elastic properties, graphical dependences of the transverse strain coefficient and the elastic modulus of the udder nipples on the actual loading of the nipple are constructed and shown in Figure 17.

Fig. 17. Graphic dependences of the lateral strain coefficient and elastic modulus of the udder nipples on the actual loading of the nipple.

An analysis of the obtained experimental data showed that the elongations of the nipples depend nonlinearly on the effective stress, and the values of elastic modulus E and transverse strain coefficient μ vary over a wide range. This is explained by the fact that the udder nipple is a body with a variable mass. When milking, it changes the mode of blood circulation and milk pressure. For nipples with an initial diameter of 24-26 mm and a length of 55-65 mm, the values of elastic modulus E are in the range of $(18.5-51.0) \cdot 10^3$ MPa, and transverse strain coefficient m is in the range of 0.17-0.50.

Based on the results of studies of the friction coefficients of an interacting pair of a cow's udder nipple and a teat cup liner, a graphical dependence of the friction coefficient on the pressing force of curved plates is constructed (Fig. 18).

Fig. 18. Graphical dependence of the coefficient of friction on the pressing force of curved plates.

An analysis of the results showed that the friction coefficients are practically independent of the pressing force (in the biologically acceptable limit) of the teat cup liner to the nipple and are in the range of 0.19-0.23 for dry surfaces of the udder nipple and the teat cup liner and in the range of 0.21-0.24 for wet ones, respectively. A somewhat paradoxical increase in the friction coefficients for the wet surfaces of the udder nipple and the teat cup liner is evident due to the action of surface tension and protein in milk. In the practical plane, the deviation of the value lies within the experimental error.

The study of the working surface of the selected teat cup liner samples showed that new, not previously used samples have a relatively smooth surface in relation to the ones used in the work, respectively (Fig. 12). It should be noted that the working surfaces of new samples No. 1 and No. 6 are smoother than samples No. 2 - No. 5. They have evenly alternating low protrusions and shallow depressions, but sample No. 1 shows small underdeveloped craters, which indicates the destruction of the integrity of the observed surface. Sample No. 2 has pronounced craters, and samples No. 3 – No. 5 have distinct and rather large protrusions, but their surfaces are still smooth and having no craters. The working surfaces of the used teat cup liner samples No. 1 and No. 2 have highly developed craters. Tears and surface cracks are observed in which the remains of milk accumulate, which is a provoking component for strong contamination of the surface by various bacteria. This in turn leads to a decrease in the quality of the milk. The working surfaces of studied samples No. 3 – No. 5 have a smoother surface than their predecessors, their large protrusions are somewhat sharpened, but small craters can be observed in which milk residues may be present, which means that enough bacteria can accumulate. The working surface of silicone teat cup liner, sample No. 6, has no visible significant surface changes. Craters, cracks and grooves are absent. The surface itself is smooth, which contributes to a more complete flushing of the rubber from milk residues, minimizing bacterial contamination.

5 Conclusion

With an increase in the deflection of the walls of the teat cup liner, the pressure exerted by it on the udder nipple increases non-linearly and progressively.

Distributions of diameters and lengths of the cow's udder nipples showed that their sizes are in wide ranges. So the difference between the smallest nipple diameter and the largest one is approximately twofold, as well as the difference for the smallest nipple length and the largest one. Analysis of measurements of the udder nipples shows their uneven location on it.

Elongations of nipples depend nonlinearly on the effective stress and the values of elastic modulus E and transverse strain coefficient μ vary over a wide range.

Friction coefficients practically do not depend on the effort of pressing the teat cup liner to the nipple in the biologically acceptable limit.

The working surface of the new and not used teat cup liner is a priority in relation to the one used before. Moreover, the working surface of silicone teat cup liner has a smoother structure and is more resistant to wear than the one made from food rubber.

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