Experimental and numerical analysis of soil desiccating cracks in compacted and non-compacted specimens

Josbel Cordero¹, Abdallah Najdi¹, David Encalada¹, Pere C. Prat¹ and Alberto Ledesma^{1*}

¹Department of Civil and Environmental Engineering, UPC-BarcelonaTECH, Barcelona, Spain

Abstract. This paper presents the results and analysis of two cracking tests carried on specimens of silty clay. One specimen was prepared in slurry conditions without applying energy and the other specimen was compacted. They were dried in an environmental chamber at a constant temperature and relative humidity to study the effect of the initial consistency on the cracking behaviour. Weight measurements and photographic images taken at regular intervals documented the evolution of the specimens. THM models were then carried to capture the unsaturated flow in the porous medium due to evaporation, and its resulting shrinkage. All the numerical analyses were coupled, incorporating the effect of porosity change on the balance equations and the constitutive model. The transfer coefficients in the imposed drying boundary condition were based on calculations of aerodynamic surface resistances, taking into consideration the new fronts for evaporation created by the cracks. The constructed numerical model results capture the gravimetric water content loss and the occurring shrinkage for both specimen conditions.

1 Introduction

Laboratory experiments on desiccation cracking of soils are commonly conducted to study the impact of cracks on many geotechnical works such as landfills, embankments, earth dams, and waste cover reservoirs. The generation of cracks will eventually induce changes in the hydromechanical behaviour. Currently, the analysis of cracking in desiccating soils is far from being a routine and specific experiments must be developed to understand the processes involved in this phenomenon. Also, cracking alters the boundary conditions between the soil and the atmosphere. This must be taken into account when updating the boundary conditions in numerical models [1,2]. These models must consider the dynamic boundary conditions in the open crack surface, in which the vapour flux varies according to the aerodynamic resistance.

This paper presents part of an ongoing research project exploring the effect of water content and compaction on the crack pattern of desiccating soils. Two specimens of silty clay prepared with different initial conditions are presented. One specimen was prepared in slurry conditions without applying energy and the other specimen was compacted on the dry side of Proctor standard. It appears from the experiments that specimens compacted on the dry side are less sensitive to cracking, which could be related to the stiffness of soils compacted using low water content [3]. However, the complete picture of the relationship between compaction water content and cracking potential will be obtained after performing all the planned experiments.

2 Material and methods

2.1 Soil properties

The soil used in the experiments was a silty clay from the Agròpolis site (Llobregat River delta near Barcelona, Spain), whose more relevant parameters for soil classification are given in Table 1.

The soil used was air dried and carefully crushed to destroy aggregates. Then, a 2 mm sieve was used to remove coarse particles. The resulting material was mainly silt and clay, with a fraction of fine sand also included.

Table 1. Index properties of the Agròpolis silty clay. [4]

Physical property	Experimental value
Sand content ($\leq 2\text{mm}$)	48.3%
Silt content ($\leq 63\mu\text{m}$)	42.1%
Clay content ($\leq 2\mu\text{m}$)	9.6%
Unit weight of solid particles	27 kN/m ³
Liquid limit (w_L)	29%
Plastic limit (w_P)	17%
Unified soil classification system	CL

* Corresponding author: alberto.ledesma@upc.edu

2.2 Soil desiccation tests

Two soil specimens with different initial consistency were dried using an environmental chamber [5] specially designed to study the desiccation response and cracking behaviour of the soil under controlled or imposed environmental conditions controlling temperature (T) and relative humidity (RH).

The specimens were prepared in cylindrical trays with grooved bases that limit the displacements at the bottom. One specimen was prepared in slurry conditions, without applying energy when pouring; the other specimen was compacted. The trays were 10 cm in height and 80 cm in diameter for the slurry specimen but 40 cm in diameter for the compacted specimen. There are plans to perform several experiments with different diameters to check the size effect. Here, for both cases, the circular tray with the specimen was placed in the closed chamber and the desiccating process set by circulating dry air keeping the environmental chamber at $RH = 30\%$ and $T \approx 23^\circ\text{C}$, see Fig. 1. The drying process continued until the weight reached a steady-state condition.

The weight of the specimen was continuously monitored to record the evolution of the global gravimetric water content, which is a measure of the water mass changes in the soil.

Photographs were taken at regular intervals to appraise the shrinkage and crack formation. The descriptor of the superficial cracking is the Crack Intensity Factor (CIF) [6], which is equal to the area of the formed cracks over the intact area. The images were processed using an image analysis technique explained in the literature [7] to determine the area of cracks and then calculate the CIF values. Fig. 2 shows only the initial and final surface of the two analysed tests.

Fig. 1. Environmental conditions setting on the chamber: temperature and relative humidity for slurry and compacted specimen.

2.2.1 Preparation of slurry specimen

The solid particles were mixed with enough water to produce a liquid consistency at a water content higher than the liquid limit. The initial gravimetric water content for the slurry specimen was 43%.

This fluid mixture was carefully poured on the round tray avoiding air bubbles or additional voids. The specimen surface was levelled with a straight edge to get a uniform surface at the beginning of the test, as shown in Fig. 2a.

2.2.2 Preparation of compacted specimen

The solid particles were mixed with enough water to produce the moisture according to the gravimetric water content required for the compaction process. The initial moisture content for the compacted specimen was 12%, below the optimum standard Proctor.

In this case, the specimen was compacted by a dynamic method using the hammer of the Proctor Marshall test. Five layers of 2 cm in high were adopted, and 68 hits on each one were applied. This compaction routine was defined to equalize the energy applied in the Standard Proctor test with that used to prepare the specimen. The surface of the compacted specimen was carefully levelled using a straight edge and filling holes to get a uniform initial surface, as shown in Fig. 2c.

Fig. 2. Surface image from: a) initial condition of the slurry specimen, $w_0 = 43\%$; b) final crack pattern from slurry specimen, $w_0 = 6\%$; c) initial condition of the compacted specimen, $w_0 = 12\%$; d) final crack pattern from compacted specimen, $w_0 = 5\%$.

3 Numerical simulations

The problem is THM-coupled as it incorporates changes in temperature, unsaturated flow in the porous medium due to evaporation, and changes in the stress field due to the resulting shrinkage.

Coupling all the behaviour seems necessary as the evaporation of water from the soil body produces matrix suction, which yields shrinkage and cracking. At the same time, shrinkage and cracking can contribute to the desiccation process as shrinkage forces water out of the soil matrix and cracks create new boundaries where water can evaporate.

3.1 Formulation

The problem was solved with the Finite Element Numerical model CODE_BRIGHT [8]. The theoretical framework consists of a multiphase approach (gaseous, liquid, and solid) and multispecies (dry air, water, and solid). The liquid phase may contain water and dissolved air, and the gas phase may contain water vapour and dry air. The formulation is based on 3 main components: balance equations, constitutive equations and balance restrictions. The state variables are: displacements in each direction, $\mathbf{u}$ (m); liquid pressure, P_l (MPa); gas pressure, P_g (MPa); and soil temperature T (°C). Thermal equilibrium is assumed between phases. Balance momentum for the medium is given by the stress equilibrium equation with the mechanical constitutive equation, while for the fluid phase the constitutive equations are Fick's and Darcy's laws.

The mass balance equations were established following the compositional approach, which is for species rather than phases. The total mass flux of a species in a phase is the sum of non-advective fluxes (diffusive/dispersive), and advective fluxes caused by both fluid and solid motions. An expression for porosity variation caused by volumetric deformation and solid density variation is obtained from the solid mass balance equation.

The basis for the comprehensive mechanical constitutive model are given in details in the papers describing the formulation of CODE_BRIGHT [8][9][10].

3.2. Boundary conditions

For the mechanical part, the classical approach is followed to impose external forces by means of a Cauchy type boundary condition, and a node-release technique to simulate the crack propagation [1]. The bottom boundary limited displacements in horizontal direction to replicate the experimental rough bottom conditions. On the other hand, the imposed drying on the specimens in the laboratory is applied numerically through a hydraulic boundary condition: a flow rate (outflow) of vapour mass fraction that represents the difference in relative humidity, RH,

$$j_g^w = \beta_g \left[(\rho_g \omega_g^w)^0 - (\rho_g \omega_g^w) \right] \quad (1)$$

where j_g^w is the flux of water vapour at the boundary, which depends on the prescribed vapour mass fraction $(\omega_g^w)^0$ and gas density ρ_g^0 and the current values at soil surface. β_g is the gaseous phase transfer coefficient factor. Replicating the experimental results requires

applying similar boundary conditions to the recorded data in the environmental chamber at the laboratory. This process entails the conversion of RH and T values (Fig.1.) into $(\omega_g^w)^0$ and ρ_g^0 values by utilising the psychrometric and ideal gas laws:

$$(\omega_g^w)^0 = \frac{\rho_v^{sat} \cdot RH}{\rho_g^0} \quad (2)$$

$$\rho_g^0 = \frac{P_a M_a}{R(273.15 + T)} \times 10^6 \quad (3)$$

where ρ_v^{sat} is the maximum vapour concentration the air can hold at a given temperature, T , calculated using the vapour pressure at that given temperature and the molecular weight of water; P_a is the air pressure (atm); M_a is the molecular weight of dry air ($\text{kg} \cdot \text{mol}^{-1}$); and R is the gas constant ($\text{J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$).

The transfer coefficient necessary to impose the flux, β_g , is obtained by comparing against the Latent Heat formula in the well-known surface energy equation. It can be then expressed in terms of aerodynamic (r_a) and surface (r_s) resistances ($\text{s} \cdot \text{m}^{-1}$).

$$\beta_g = \frac{1}{r_a + r_s} \quad (4)$$

According to Ohm's Law [11][12], measuring the mean wind velocity $\bar{U}$ in the environmental chamber ($0.3 \text{ m} \cdot \text{s}^{-1}$) at an altitude z (0.7 m) from the soil surface is necessary to calculate the aerodynamic resistance

$$r_a = \frac{1}{k^2 \bar{U}} \left(\ln \frac{z}{z_0} \right)^2 \quad (5)$$

where k is the von Karman's constant (0.41) [13] and z_0 is the roughness length for silty clay (0.001m) [14]. The wind speed is constant inside the chamber as long as the dehumidifier is operating (i.e. when RH inside the chamber is greater than 30%). Alternatively, r_s is dependent on the volumetric water content at the soil surface θ_0 [15]. It gains more significance as the specimen dries, thus hindering the advective fluxes in the soil-atmosphere boundary.

The energy flux caused by mass inflow and outflow through the boundary is reduced to a Cauchy type term in the absence of heat flux. Conversely, by comparing against the Sensible Heat in the surface energy equation, the expression for energy transfer coefficient in the reduced prescribed energy flux equation is obtained.

$$|\gamma_e| = \frac{C_a}{r_h} \quad (6)$$

where C_a is the volumetric heat capacity of air ($1200 \text{ J} \cdot \text{m}^{-3} \cdot \text{K}^{-1}$), and assuming that $r_h = r_a$ [16][17].

The cracks will open a new pathway for soil-water evaporation [18]. The intensity of evaporation is contingent to the transfer coefficients assigned in the flux terms of the newly formed vertical hydraulic boundaries. The magnitude of the coefficients is governed by the wind velocity within the cracks which is expected to decrease with increasing depth, reaching null values at the bottom half of the experiment height. The decreasing crack width with depth furthermore hinders

the wind velocity inside the crack. In the absence of wind, diffusive fluxes become more prominent and the r_a formulation is then dependent upon the diffusivity of water vapour in air, which can be expressed as

$$r_a = \frac{z_h}{D_a} \quad (7)$$

where $z_h = z$, and D_a is the diffusivity of water vapour in air at a temperature T (K).

Consequently, the model's constructed geometry is numerically divided into sequential vertical sections to reproduce the depreciating magnitude of β_g with the increasing crack depth. A wind profile is assumed for each of the experiments, relying on physical observations of crack width and depth propagation with time. The newly assigned boundary condition at the crack vertical surfaces is triggered at specific times coinciding with the first crack initiation for each specimen. The utilised profile is different in each case depending on the explained preceding conditions. The heat transfer coefficient γ_e however was applied equally to all surfaces of the specimen, considering the PVC holding tray to be conductive. The entire procedure and advancements in assigning representative set of governing equations for the soil-atmosphere interaction at the boundary is further explained in separate papers.

3.3 Constitutive model

A set of necessary constitutive laws is associated with the formulation to establish the link between the unknowns in the balance equations and the dependant variables.

For the mechanical part, the volumetric strains are calculated in a reversible manner using a non-linear elasticity equation, based on the concept of state surfaces [19] [20].

$$\frac{\Delta e}{1+e} = a_1 \Delta \ln(-p') + a_2 \Delta \ln\left(\frac{S+0.1}{0.1}\right) + a_3 \left[\Delta \ln(-p') \ln\left(\frac{S+0.1}{0.1}\right) \right] \quad (8)$$

where e is the void ratio, p' is the mean effective stress (MPa), S is the suction (MPa), a_1 and a_2 are constants acquired from $e - \ln p'$ and $e - \ln[(S+0.1)/0.1]$ diagrams respectively. Usually their values are obtained from suction-controlled triaxial and oedometer experiments respectively.

$$a_1 = -\frac{\kappa}{1+e}; a_2 = -\frac{\kappa_s}{1+e} \quad (9)$$

Where κ and κ_s are the slopes of the unload/reload curves in the $e - \ln p'$ and $e - \ln[(S+0.1)/0.1]$ diagrams respectively. Recording the horizontal shrinkage using the Image Analysis technique allowed for estimating the volumetric deformations in the experiments, assuming isotropic conditions. Together with the gravimetric water content, the saturation ratio of the specimen can be computed, allowing determination of suction evolution (Fig.3). As a result, κ_s is obtained

for both experiments. The compacted specimen exhibited a more consistent behaviour ($\kappa_s = -0.0048$) with regards to the slurry which witnessed a double behaviour (from where the specimen's matrix suction was below 100 kPa and above it). An average value was obtained for the slurry ($\kappa_s = -0.0816$) to simplify the solution. As for shear deformations, the variable shear modulus G calculations were based on the constant value of Poisson's ratio $\nu = 0.3$ [21].

The hydraulic component used Generalized Darcy's law to represent the unsaturated advective flow in the deformable porous medium.

$$\mathbf{q}_l = -\frac{k_{rl}}{\mu_l} \mathbf{k} \cdot (\nabla P_l - \rho_l \mathbf{g}) \quad (10)$$

where μ_l is the viscosity, ρ_l is density, k_{rl} is the liquid phase relative permeability defined by the generalized power law based on Corey's functions, and $\mathbf{k}$ is the intrinsic permeability, assumed isotropic and dependent on porosity, ϕ , using the exponential law:

$$k_{rl} = A S_e^r \quad (11)$$

$$\mathbf{k} = \mathbf{k}_0 \exp[b(\phi - \phi_0)] \quad (12)$$

where A and r are empirical parameters based on initial soil conditions, S_e is the effective degree of saturation, $\mathbf{k}_0$ is determined from hydraulic conductivity tests on samples with different ϕ_0 porosity [22], and b is a factor accounting for the porosity influence on permeability.

Fig. 3. $e - \ln[(S + 0.1)/0.1]$ diagram for both experiments showing κ_s values. S , suction in MPa.

To relate the suction ($P_g - P_l$) to the degree of saturation S_l during drying, the van Genuchten model [23] was used, accounting for the porosity effect on the shape parameter.

$$S_e = \frac{S_l - S_{rl}}{S_{ls} - S_{rl}} = \left\{ 1 + \left[\frac{P_g - P_l}{P_0(\sigma/\sigma_0)} \right]^{\frac{1}{1-\lambda}} \right\}^{-\lambda} \quad (13)$$

$$\lambda(\phi) = \lambda \exp[b(\phi_0 - \phi)] \quad (14)$$

Where S_{rl} and S_{ls} are the residual and maximum saturation respectively; P_0 is the air entry value; σ is the surface tension at a temperature T (°C); σ_0 is the surface tension at temperature in which P_0 was measured; and b is a parameter for porosity influence on retention curve. The parameters were then acquired by fitting the curves to the experimental results from specimens with similar initial conditions for the same soil using the hygrometer and T5 Tensiometers in the laboratory (Fig.4).

Fig. 4. Soil Water Retention Curves for both initial conditions showing the fitted models.

Table 2. Parameters used for the numerical simulations.

Parameter	Slurry specimen	Compacted specimen
Initial gravimetric water content, w_0	43%	12%
Void ratio, e_0	1.27	0.54
Dry unit weight, γ_d	1200 kg/m ³	1760 kg/m ³
Hydraulic Conductivity, K	9.5×10^{-6} m/s	3.8×10^{-9} m/s
Air entry value, P_0	2.5 kPa	3500 kPa
Shape parameter, λ	0.085	0.43
Porosity effect, b	4	1
Maximum saturation, S_{ls}	98%	60%
Residual saturation, S_{lr}	2%	
Mechanical parameters, $a_1=a_2$	-0.0367	-0.0031
$\lambda_{dry} / \lambda_{sat}$	0.243 / 1.448 Wm ⁻¹ K ⁻¹	

As for the non-advective fluxes of a species in a phase, they are composed of 2 components: molecular diffusion and mechanical dispersion. Fick's law describes the molecular diffusion of both vapour and air in the gaseous phase in terms of the particle's coefficient of tortuosity, saturation degree and temperature. The

mechanical dispersion tensors for vapour, dissolved air and heat are as well defined by Fick's law. Additionally, the conductive heat fluxes are computed using the geometric weighted mean in Fourier's law, depending on changes in porosity and saturation degree [24][25].

Table 2 sums up the numerical parameters used for each studied experiment.

4 Results

The models are processed based on the applied boundary conditions and the constitutive equations with the corresponding parameters. The gravimetric water content for each of the experiments was evaluated numerically by deducing the water content of each mesh element from the computed vapour content and liquid saturation degree, and compared against the physical results from the laboratory. Figure 5 presents that comparison for both experiments showing a good agreement.

Fig. 5. Gravimetric water content evolution with time for both experiments, showing both the physical and numerical results.

The crack intensity factor is calculated as the area of cracks over the initial specimen area. The numerical model captures the volumetric shrinkage due to desiccation. However, numerically the model assumes isotropic conditions and homogeneity. Consequently, the shrinkage is captured on the perimeter only. For the compacted specimen it did not illustrate a setback, since the physical results suggest the same behaviour of a mere perimeter crack. However, the slurry specimen exhibited internal cracks in the laboratory experiment, whereas the simulation was only able to capture it as a horizontal shrinkage at the perimeter. This affected the results for the slurry (Fig. 6). The simulation results exhibited a double-phase behaviour as suggested by (Fig. 3), where the numerical results undermine the cracking at the first half of the test and elaborates its rate at the second half, but reaching similar residual values.

5 Conclusions

The paper presents two experiments on soil desiccation cracking in an environmental chamber departing from slurry and from compacted conditions, interpreted by means of a numerical finite element model considering the coupled THM phenomena involved. The numerical analyses simulate reasonably the measured desiccation rates and Crack Intensity Factor rates for dry (compacted) and wet (slurry) specimens.

The desiccation and CIF rates are very different for the compacted and the non-compacted specimens. Desiccation rates in dry and wet specimens are affected by soil porosity, the amount of water available for evaporation and the mechanisms providing water to the surface (i.e. advection versus diffusion, which is slower). On the other hand, CIF rate is related to the stiffness of the material, which is much higher for the compacted specimen. This is part of an ongoing research project and further experiments and simulations with different initial conditions are being carried out in order to define a global picture of the desiccation and CIF rates for soils.

Fig. 6. Crack intensity factor evolution with time for both experiments, showing both the physical and numerical results.

Acknowledgement

Financial support from research grant BIA2017-82594-R, awarded by the Spanish Ministry of Science and Innovation (including FEDER funds, European Commission), is gratefully acknowledged.

References

1. H. U. Levatti, P. C. Prat, and A. Ledesma, *Comput. Geotech.* **105**, 155 (2019)
2. R. A. Stirling, S. Glendinning, and C. T. Davie, *Appl. Clay Sci.* **146**, 176 (2017)
3. A. Demagistri, A. Ledesma, J. Cordero, R. Moreno, P. Prat, and A. Jacinto, in *7th Int. Conf. Unsaturated Soils, UNSAT* (Hong Kong, 2018), pp. 1273–1278

4. J. A. Cordero, A. Cuadrado, P. C. Prat, and A. Ledesma, in *3rd Eur. Conf. Unsaturated Soils, E-UNSAT* (Paris, 2016), pp. 12005 (1–6)
5. M. R. Lakshmikantha, Experimental and theoretical analysis of cracking in drying soils. Ph.D. Thesis, UPC-BarcelonaTech, Spain (2009)
6. C. J. Miller, H. Mi, and N. Yesiller, *J. Am. Water Resour. Assoc.* **34**, 677 (1998)
7. M. R. Lakshmikantha, P. C. Prat, and A. Ledesma, *Geotech. Test. J.* **32**, 102216 (2009)
8. S. Olivella, J. Carrera, A. Gens, and E. E. Alonso, *Transp. Porous Media* **15**, 271 (1994)
9. A. Gens and S. Olivella, *Rev. Française Génie Civ.* **5**, 693 (2001)
10. DETCG, *CODE_BRIGHT User's Guide* (UPC-BarcelonaTech, Barcelona, 2016)
11. T. R. Oke, *Boundary Layer Climates*, 2nd ed. (Methuen & Co, 1987)
12. R. B. Stull, *An Introduction to Boundary Layer Meteorology* (Atmospheric Sciences Library, 1988)
13. T. Tagesson, *Turbulent Transport in the Atmospheric Surface Layer* (Stockholm, 2012)
14. A. G. Davenport, C. S. B. Grimmond, T. R. Oke, and J. Wieringa, in *15th Conf. Probab. Stat. Atmos. Sci. Conf. Appl. Climatol.* (American Meteorological Society, Asheville, NC, 2000), pp. 96–99
15. A. A. van de Griend and M. Owe, *Water Resour. Res.* **30**, 181 (1994)
16. J. F. Louis, *Boundary-Layer Meteorol.* **17**, 187 (1979)
17. G. K. Greenhut, *Boundary-Layer Meteorol.* **24**, 253 (1982)
18. A. Cuadrado, *Análisis THM de La Interacción Suelo-Atmósfera en Suelos Arcillosos Sometidos a Desecación*, PhD Thesis (in Spanish) UPC-BarcelonaTech. Barcelona, Spain, (2018)
19. E. E. Alonso, A. Gens, and A. Josa, *Geotechnique* **40**, 405 (1990)
20. A. Lloret and E. E. Alonso, in *Proc. 11th Int. Conf. Soil Mech. Found. Eng. Vol. 2.* (Balkema, San Francisco, 1985), pp. 557–562
21. M. Barrera Bucio, *Estudio Experimental del Comportamiento Hidro-Mecánico de Suelos Colapsables*, PhD Thesis (in Spanish) UPC-BarcelonaTech, Barcelona, Spain, (2002)
22. R. Oorthuis, M. Hürlimann, A. Fraccica, A. Lloret, J. Moya, C. Puig-Polo, and J. Vaunat, *Water (Switzerland)* **10**, 1 (2018)
23. M. T. van Genuchten, *Soil Sci. Soc. Am. J.* **44**, 892 (1980)
24. T. R. Oke, *Boundary Layer Climates*, 2nd ed. (Methuen, 1987)
25. J. F. Villalobos, L. Mateos, F. Orgaz, and E. Fereres, *Fitotecnia : Bases y Tecnologías de La Producción Agrícola* (Mundi Prensa, Madrid, 2002)