Research of friction characteristics on the hydraulic transportation in inclined slurry-pipe

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Abstract: By analyzing the momentum transfer and velocity both of solid particles and water over the acceleration time of solid particles, as well as interaction mechanism between water and solid particals, a new model is proposed to predict friction loss for setting slurry flow in inclined pipe. The hydraulic gradient formula for inclined pipes summarized by the author is confirmed by a large amount of experimental data. The results show that the deviation between the theoretical value of the model proposed by the author and the measured value is not more than 13.33%, which is the smallest among all reports.

1 Introduction

With the increasing demand of power plants and chemical companies for mineral resources and the increasingly stringent requirements for the environment, pipeline transportation of coal slurry is more and more widely used due to its economy and environmental friendliness. Generally, the hydraulic gradient of settling slurry in the inclined pipe needs to be calculated to judge the stability of long-distance slurry transportation. However, due to its complexity, there are few reports in this area at present. Rose [1], Kawashima [2] and Worster [3] have studied many transportation parameters in inclined pipelines, but the research on hydraulic gradient and inclined slurry flow characteristics is still incomplete.

Generally, the pressure loss of the slurry directly depends on the state of motion of the solid particles. Based on a new theoretical analysis method[4], a new model is proposed to reveal the relationship between the movement status of solid particles and the pressure loss of setting slurry. In this model, the hydraulic gradient of setting slurry in different motion states (in suspension, by saltation, or in partly suspension and by saltation) in inclined pipe can be predicted clearly.

2 The velocity change of the water during solid particles' accelerating

Using a new theoretical analysis method proposed by the author, it is possible to set the momentum of the solid particles with mass Ms and the water with mass Mw from the pipe outlet A at time t and the balance equation of the continuous equation. And get the following equation [4]:

$$V - V_{\mathrm{w}} = \frac{k_2^2\ \bar{\mathrm{q}}\ \rho_{\mathrm{s}} V_{\mathrm{s}}^2}{k_1 (1 - \bar{\mathrm{q}}) \rho V_{\mathrm{w}}} \tag{1}$$

$$V_{\mathrm{m}} = V_{\mathrm{w}} (1 - \bar{\mathrm{q}}) + V_{\mathrm{s}}\ \bar{\mathrm{q}} \tag{2}$$

where and V is the average rate of the water before momentum transference. ρ and ρ_s are the density of water and solid particles. q, V_w, V_s, V_m are separately in-situ average concentration (by volume) of solid particles, the average velocity of the water and the solid particles and settling slurry in the section A . k_1 and k_2 are coefficient. And the equation to calculate k_1, k_2 are:

$$k_1 = 1/[1 - 0.56\psi(1)\,\varphi(\theta)] \tag{3}$$

$$k_2 = 1 + k_4 \left(33\lambda \cos\theta \pm \sin\theta\right) \frac{k_3 \sqrt{L_a g}}{V_m} \left(1 - \frac{1}{\delta}\right) \tag{4}$$

It can be seen from Rose's experimental results that the value of $\psi(1)$ is closely related to the average velocity V_m of the two-phase flow in the pipeline, the particle size d and the density ratio δ of solid particles to the fluid. According to the experimental data of several scholars, the fitting curve between $\psi(1)$ and Log_{10} （$V_m^2/(gd\delta^2)$ is shown in Figure 1[5]. According to Rose's report[1], $\varphi(\theta)$ is a function of θ in an up-inclined pipe (Figure 2), but in a down-inclined pipe, $\varphi(\theta)$ is smaller than its value in an up-incline pipe. Here we assume that $\varphi(\theta)$ in the up-incline pipe and the down-inclined pipe are symmetric (Figure 3), so that $\varphi(\theta)$ in the down-inclined pipe can be predicted.

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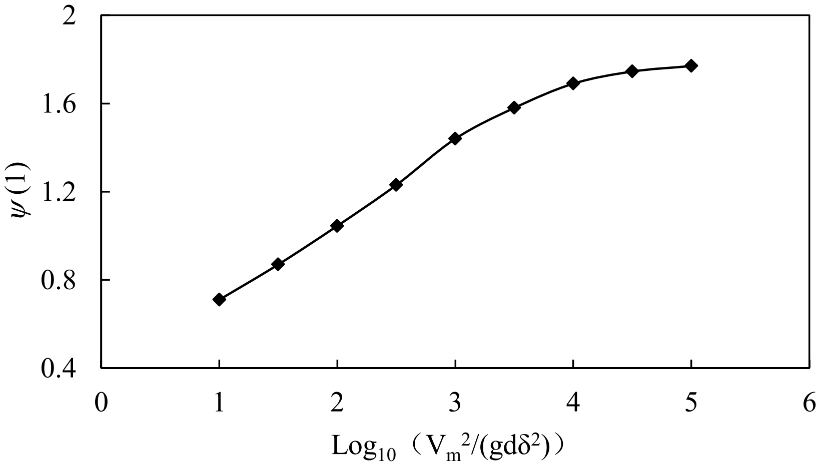


Figure 1. Relationship of $\psi(1)$ with $V_m^2/gd\delta^2$

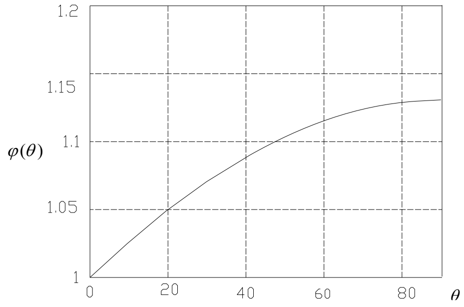


Figure 2. The relationship between φ(θ) and θ for up-inclined pipe

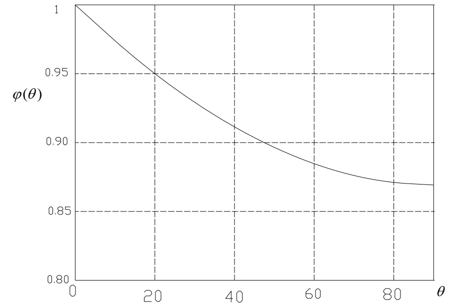


Figure 3. The relationship between φ(*θ*) and *θ* for down-inclined pipe

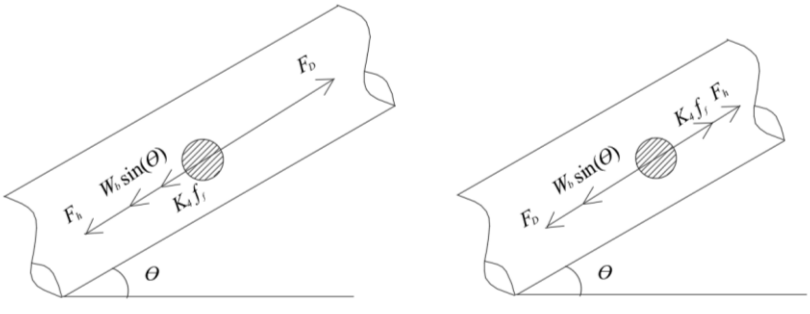
In equation (4), $\delta = \rho_s/\rho$, $k_3 = 3.742$, λ is friction factor. When solid particles are transported in a suspended state, k_4 represents the proportional coefficient of the solid particles in the sliding state to the total weight of the solid particles, so when all solid particles are suspended, $k_4 = 0$; when all solid particles are transported in a saltation state, $k_4 = 1$; When part of the solid particles are transported in a suspended state and another part is transported in a saltation state, $k_4 = V_t/V_m$, where V_m is the average velocity of the condensed slurry, V_t is the final velocity of the particles, and L_a is the length of the pipe required to fully accelerate the solid, the calculation formula is as follows:

$$L_a = 6D\left(\frac{M_s}{\rho\sqrt{g}\sqrt{D^5}}\sqrt{\frac{D}{d}\frac{\rho_s}{\rho}}\right)^{\frac{1}{3}} \qquad (5)$$

Where D represents the diameter of the pipe, d stands for the diameter of the solid particles, M_s is the mass flow rate of the solid, and g stands for the acceleration due to gravity [1].

3 Analysis of the forces that the solid particles exerted

As shown in Figure 4, when the slurry flows in the inclined pipe, the solid particles are subjected to four forces, which are the drag force of the fluid on the particles (F_D), the gravity sliding component of the solid particles ($W_b\sin(\theta)$), and the interference forces of other particles (F_h) and the friction against the particles (K_4f_f).



up-inclined pipe

down-inclined pipe

Figure 4. Diagram of force acting on particles in inclined pipeline

The equation for the acceleration of a solid is as follows:

$$\frac{\pi}{6} d_e^3\left(\rho_s + \frac{\rho}{2}\right)\frac{dV_s}{dt} = F_D - F_h - k_4 f_f \mp W_b \sin\theta \qquad (6)$$

where

$$F_D = \frac{\pi}{4} d^2 C_D \frac{(V_w - V_s)^2}{2}\rho \qquad (7)$$

$$F_h = \left(1-(1-\bar{q})^{2(n-1)}\frac{\left(\sqrt{Rep}\alpha + \sqrt{Rep\,\alpha^2 + 4\sqrt{48}\alpha\beta/(1-\bar{q})^{n-1}}\right)^2}{\left(\sqrt{Rep}\alpha + \sqrt{Rep\,\alpha^2 + 4\sqrt{48}\alpha\beta}\right)^2}\right)W_b \qquad (8)$$

$$f_f = \frac{\pi}{6} d^3 \lambda(\rho_s - \rho)\, g\cos\theta \qquad (9)$$

and F_D represents the drag force of water, F_h stands for the interference force from other solid particles, f_f is the friction force acting on a single solid particle, d is the diameter of the solid particle, W_b is the effective gravity of the solid particle in water, and its expression is: $W_b = \pi d^3(\rho_s - \rho)g/6$; In addition, C_D is the drag coefficient calculated from V_w-V_s, R_{ep} is the Reynolds factor of solid particles, and α, β are Swanson shape factors. n is a prime number calculated using the Sato Hiroshi equation. After the solid particles are accelerated, the setting slurry will become a stable flow state, and the velocities of water, solid particles and slurry will also become the same. At this time, the acceleration of the solid particle $dV_s/d_t=0$, so the equation (6) becomes as follows:

$$F_D - F_h - k_4 f_f \mp W_b \sin(\theta) = 0 \qquad (10)$$

When the fluid moves in the upward inclined pipe, $W_b\sin(\theta)$ is a negative value, otherwise it is a positive value.

Substituting equation (7), (8) into equation (10) and combining equation (9) to obtain equation (11) as follows:

$$V_s = V_w - \sqrt{\frac{8(F_h + k_4 f_f) \mp W_b \sin(\theta)}{\pi d_e^2 C_{Dr}\rho}} \qquad (11)$$

Substituting equation (3) into equation (1) and combing equation (4) to obtain equation (12) as follows:

$$V = V_w + \left[1 - 0.55\psi(1)\varphi(\theta)\right]\frac{\delta\bar{q}}{(1-\bar{q})V_w}V_s^2\left[1 + k_4(33\lambda\cos(\theta) \pm \sin(\theta))\frac{k_3\sqrt{L_a g}}{V_m}\left(1 - \frac{1}{\delta}\right)\right]^2$$

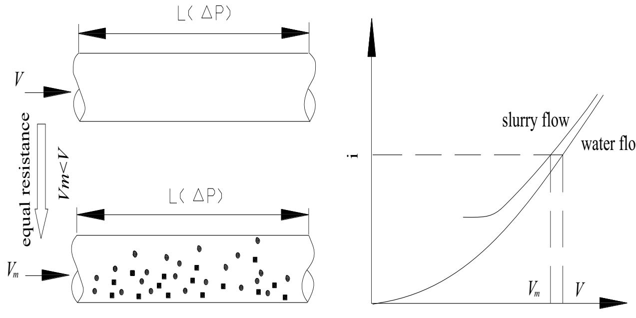
(12)

4 The hydraulic gradient of setting slurry

The hydraulic gradient when the water is in a turbulent state can be calculated by the following equation:

$$i = \frac{\lambda}{2gD}V^2 \qquad (13)$$

where D is the inner diameter of the pipe, V stands for the average velocity of the water. Obviously, when the setting slurry moves in a suspended state, a jumping state or a partly jumping and partly suspended state, equation (13) cannot be used to calculate the hydraulic slope. However, it can be seen from equation (12) that the setting slurry flowing at an average velocity V_m can be regarded as water flowing at an average velocity V. More specifically, if the pressure difference existing in the pipe section can make water move forward at a velocity V, then after being intervened by group of solid particles with the average volume concentration $\bar{q}$, the pressure difference can only make the mixture (setting slurry) move forward with the velocity V_m. Therefore, the hydraulic gradient generated when setting slurry flowing at the velocity V_m flows in the pipeline is equivalent to the hydraulic gradient of the water flowing at the velocity V in the same pipeline. The equivalent resistance model in Figure 5 can illustrate the above point very well. Therefore, only when the water velocity V in the pipeline can be calculated, the hydraulic gradient of the setting slurry can be obtained.



slurry flow in pipe

i-V curve of water and slurry flow

Figure 5. Equivalent Resistance model

According to the previous analysis results, the equation to calculate hydraulic gradient of the setting slurry can be obtained as follows:

$$i = \frac{\lambda}{2gD}\left\{V_w + \left[1 - 0.55\psi(1)\varphi(\theta)\right]\frac{\delta\overline{q}}{\left(1-\overline{q}\right)V_w}V_s^2\left[1 + k_4(33\lambda\cos\theta \pm \sin\theta)\frac{k_3\sqrt{L_a g}}{V_m}\left(1-\frac{1}{\delta}\right)\right]^2\right\}^2 \quad (14)$$

5 The criterion of the solid particles' moving status

As is mentioned above, k_4 represents the proportion coefficient of solid particles in the slip state to the total weight of solid particles, Its value is mainly related to the state of particle movement [6]. When the average velocity of the setting slurry exceeds the limit flow velocity (V_H) of the homogeneous fluid, it can be considered that the particles are transported in a suspended state. When average velocity of setting slurry is between the deposit limit velocity V_{cd} and the float limit velocity V_B, solid particles can be considered as being transported by saltation. When average velocity of setting slurry is between the float limit velocity V_B and the homogeneous limit velocity V_H, solid particles can be considered as being transported in suspension partly and by saltation partly. When the diameter of the solid particles is evenly distributed, it is easy to judge the state of movement. However, when the distribution of the solid particle diameter is within a certain range, it is difficult to determine its motion state. As to the setting slurry when the diameter of the solid particles are rather single, the floating limit velocity V_B and the homogeneous limit velocity V_H can be calculated with the following Newitt and Lazarus' equations [7]:

$$V_B = 17\,V_t \quad (15)$$

$$V_H = 4.4\sqrt{gD}\,C_D^{-0.25}(\delta - 1)^{\frac{1}{5}} \quad (16)$$

6 The comparison of the theoretical results with experimental results

So far, several experts have proposed their calculation models for the friction loss of inclined pipe slurry as follows:

Kawashima's model can be expressed in following:

$$\frac{i - i_w}{i_w C_V} = \pm\frac{3}{2}\frac{C_D}{\lambda}\left(\frac{d}{D}\right)^{-1}\frac{(1-\zeta)^2}{\zeta}$$

(17)

Where i_w stands for hydraulic gradient of water, C_V represents slurry volume concentration, ζ is a variable related to particle size d, specific gravity δ, and pipe inclination θ[2].

When $\mu_s\cos\theta + \sin\theta > 0$, the right sign of equation (17) is positive, and the expression of $\bar{\zeta}$ is as follows:

$$\bar{\xi} = 1 + \frac{1}{V_m}\sqrt{\frac{4(\rho - 1)gd}{3C_D}\left(\mu_s\cos\theta + \sin\theta\right)} \quad (18)$$

When $\mu_s\cos\theta + \sin\theta < 0$, the right sign of equation (17) is negative, and the expression of $\bar{\zeta}$ is as follows:

$$\bar{\xi} = 1 - \frac{1}{V_m}\sqrt{\frac{4(\rho - 1)gd}{3C_D}\left(\mu_s\cos\theta + \sin\theta\right)} \quad (19)$$

When $\mu_s\cos\theta + \sin\theta = 0$, $\bar{\zeta}=1$, $i=i_w$

Where u_s is friction coefficient

Worster's model[2] can be expressed in following:

$$i_m = i_0 + (i_h - i_0)\cos(\theta) \pm C_V(\delta - 1)\sin(\theta) \quad (20)$$

Where i_h stands for the hydraulic gradient of slurry in horizontal pipeline with the same concentration.

In Figure 6, Figure 7 and Figure 8, Huang zhao-lin's experiment data (down inclined) has been used to verify theoretical model[8]. In Huang zhao-lin's experiment, solid particle diameter is 0.2042mm, and relative density is 1.552. The slurry volume concentration is 21.74%. In Figure 6, pipe inclination is 15 degree, pipe inside diameter is 20 mm. In Fig.7, pipe inclination is 30 degree, pipe inside diameter is 15 mm. In Figure 8, pipe inclination is 40 degree, pipe inside diameter is 15 mm.

In figure 6, the maximum deviation between the author formula calculating values and measured values is 9.71%, and the value is 11.09%, 52.91% for Worster and Kawashima formula respectively.

Some important information can be obtained from Figure 7, the maximum deviation between the author formula calculating values and measured values is 10.33%, and 63.44%, 7.07% for Worster and Kawashima formula respectively. Truthfully speaking, the calculation deviation of the author's formula is not obvious.

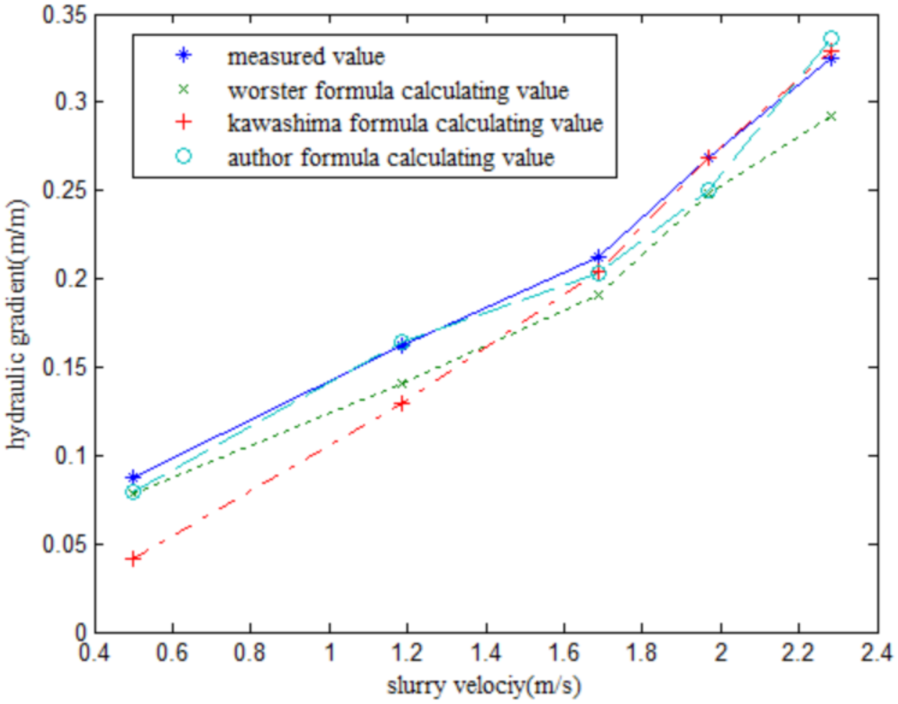


Figure 6. Comparison of Several Calculating Formulas for *D*=20 mm, *θ*=15°

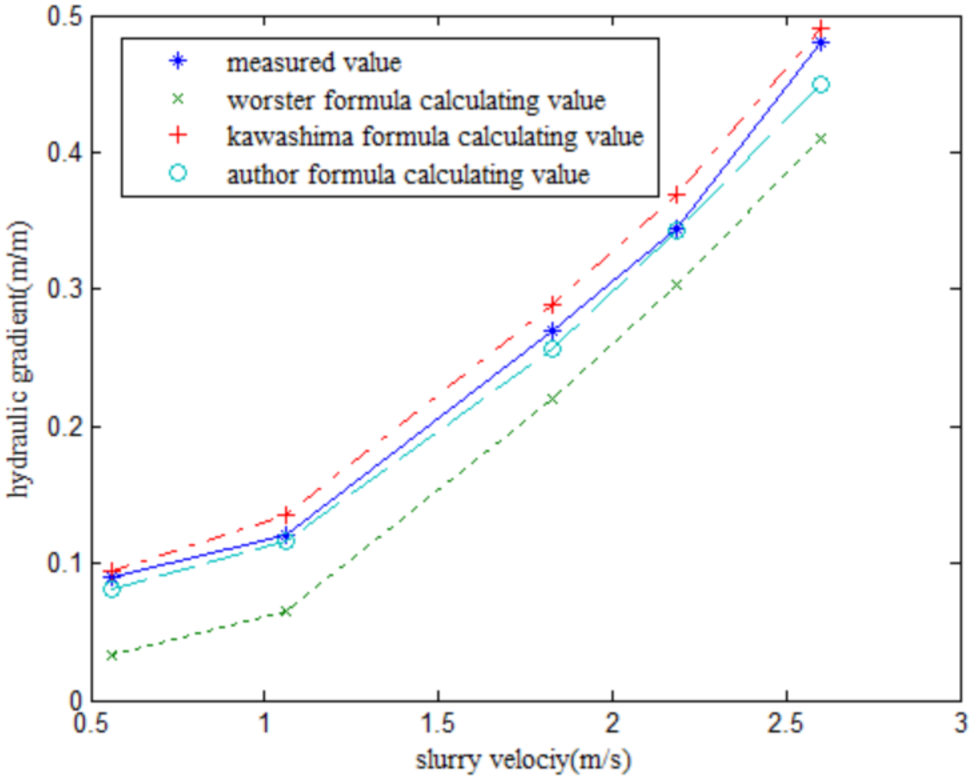


Figure 7. Comparison of Several Calculating Formulas for *D*=15 mm, *θ*=30°

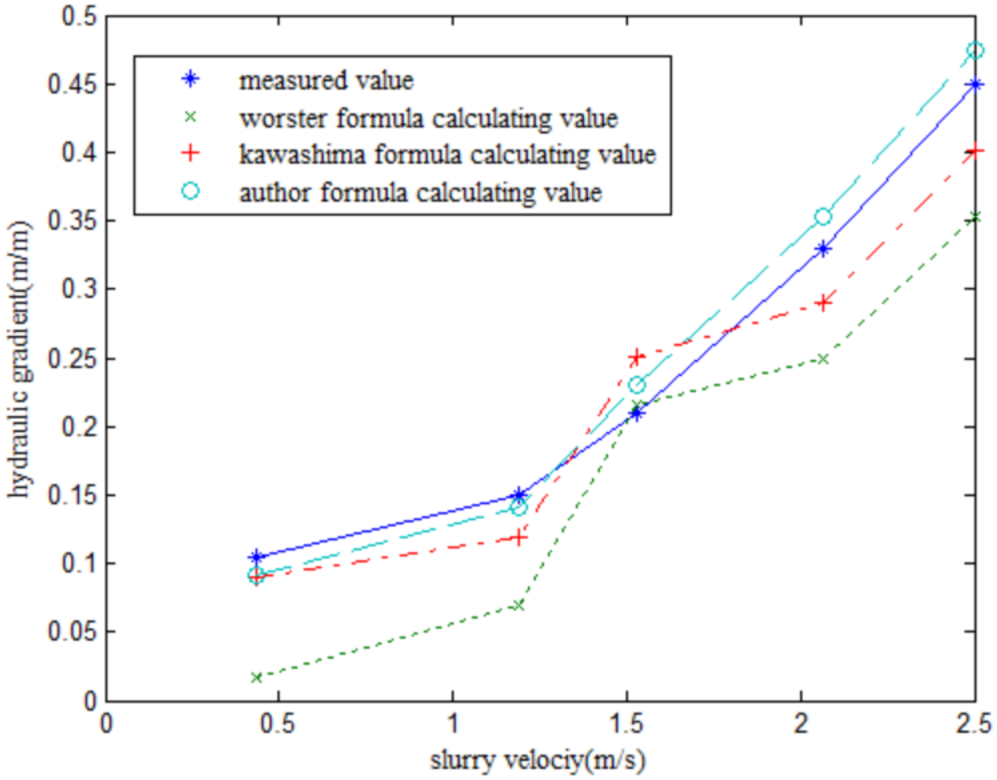


Figure 8. Comparison of Several Calculating Formulas for D=15 mm, θ=40°

In figure 8, the maximum deviation between the author formula calculating values and measured values is 13.33%, and 84.29%, 20.93% for Worster's and Kawashima's formula respectively. Generally speaking, the calculation deviation of the author's formula is small.

Except for individual data points, the deviation calculated by the author's formula is the smallest from Figure 6, Figure 7 and Figure 8.

In Figure 9 and Figure 10, Diniz and Coiado[9] inclined experiment results(up inclined) and detailed experiment procedure are investigatived. In his experiment, solids diameter is 0.20 mm, pipe inside diameter is 75 mm and relative density value is 2.68. The slurry volume concentration is 5%. In Figure 9 and Figure 10, pipe inclination is 11 and 34 degree, respectively.

In Figure 9, the maximum deviation between the author formula calculating values and measured values is 8.71%, 14.21% and 42.36% for Worster's model and Kawashima's formula respectively. In Figure 10, the maximum deviation between the author formula calculating values and measured values is 5.79%, 7.47% and 22.84% for Worster's model and Kawashima's formula respectively. It is clear that in Figure 9 and Figure 10, the deviation of the author's formula is the smallest.

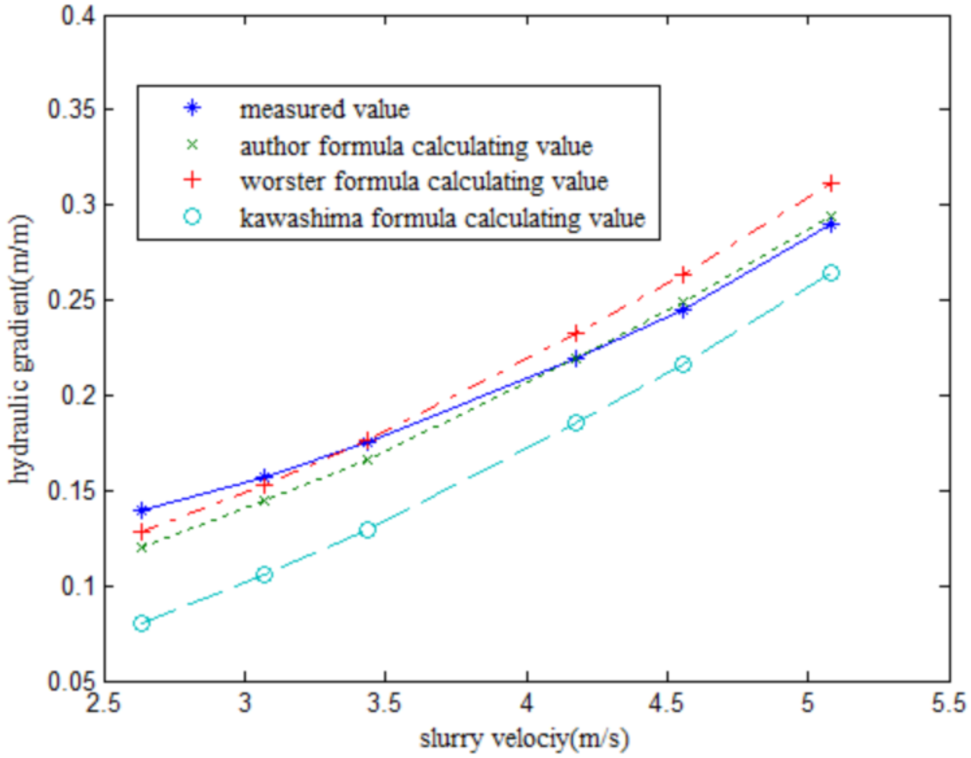


Figure 9. Comparison of several calculating formulas

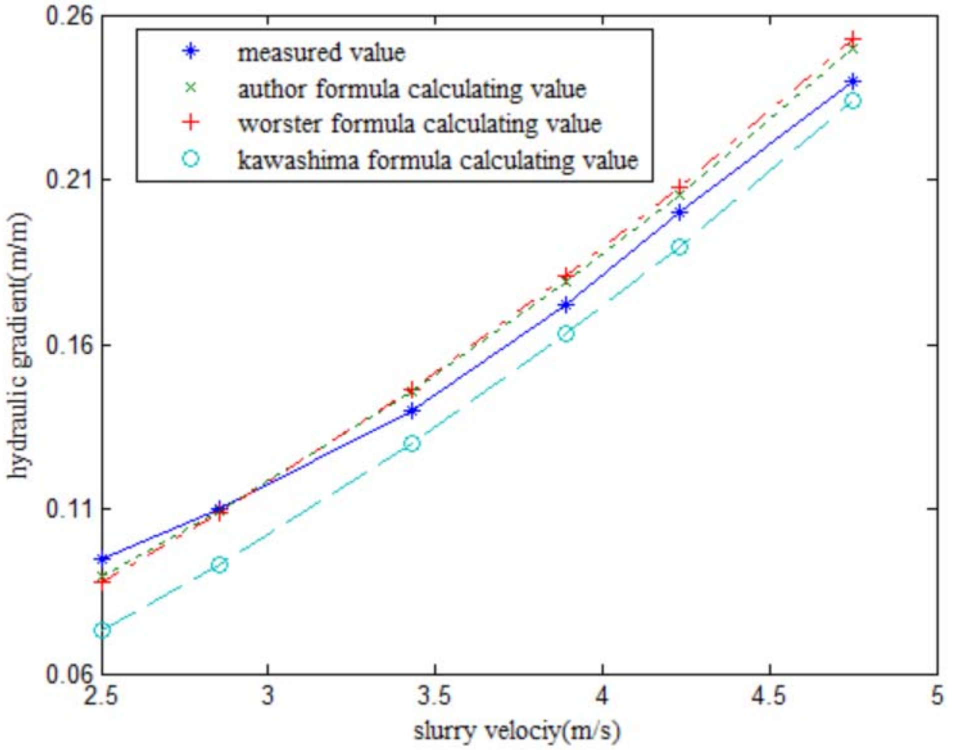


Figure 10. Comparison of Several Calculating Formulas

From Fig.6-10, the author's model is consistent with measured values and the deviation is not exceed 13.33%, while the other two models have larger deviation. It can be concluded that when the slurry velocity is low, all the three model's calculating deviation is large. From the deduction of Kawashima's model, there is not used for solid in suspending state. As for low velocity, the deviation of Kawashima's model is mainly that value of coefficient of pressure loss is not suitable for medium particles. For Worster's model, reason of deviation is probably due to its less influencing factors considered.

In Figure 6, experiment data just for down inclined flow is given out, in order to facilitate comparison and analysis, the upward inclined flow data and the horizontal flow data under the condition of Fig. 6 are also presented in Table 1.

Table 1. Comparison and Analysis of Slurry Inclined Flow and Horizontal Flow

Slurry average V_m /(m/s)	hydraulic gradient *i* / (m/m)				
	up-inclined pipe(down-inclined pipe)				horizontal pipe
	measured value	Author formula	Kawashima	Worster	measured value
0.5	0.1375(0.0875)	0.1375(0.079)	0412	0.1340(0.0778)	0.0145
1.1875	0.1938(0.1625)	0.1874(0.1639)	0.1295	0.2027(0.1405)	0.0820
1.6875	0.2375(0.2125)	0.2312(0.2032)	0.2038	0.2535(0.1913)	0.1657
1.9688	0.2813(0.2688)	0.2988(0.2501)	0.2685	0.3101(0.2479)	0.2255
2.2813	0.3375(0.325)	0.3434(0.3358)	0.3289	0.3544(0.2922)	0.3028

It can be discovered that from Huang zhao-lin's experiment, when the pipe inclination is 15 degree and -15 degree, hydraulic gradient values are completely different. The maximum deviation between the author formula calculating values and measured values is 6.22%, and 70%, 10.24% for Kawashima and Worster formula for up-inclined flow, respectively.

According to Kawashima formula calculated result, $\mu_s \cos\theta + \sin\theta < 0$, hydraulic gradient values of up-inclined and down-inclined are identical, the situation seems unreasonable, it can be seen from Figure 6~Figure 10 and table 1 that there is a large deviation existing between the calculated values of Kawashima formula and the measured values.

Solid diameter and friction factor have not appeared in Worster's formula (20),, but the hydraulic gradient in inclined pipe can be affected by these two factor. So Worster's formula is imperfect, especially for the slurry flow at low speed. As for author's model, possible reason is that the parameter k_4 is not properly determined when flow velocity is low. It is obvious that when slurry velocity is very low, the flow of solid is quit complicated. So discuss the parameter k_4 when slurry velocity at very low level is really essential.

7 Conclusion

By analyzing the change of solid particles and water over the acceleration time of solid particles in momentum, velocity and interplay mechanism of water and solid particles, a new model that can employed to calculate the hydraulic gradient of settling slurry in inclined pipe is put forward in the paper. The proposed new model was verified on the basis of experimental data from several experts, and the calculated values were not much deviation compared with the experimental values. So when solid particle diameter is about 0.2 mm, pipe diameter is in the range of 15-75 mm, and inclination angle is in the range of 10-40 degree. We get reason to believe the author's model is superior to Kawashima and Worster's model. As to other case, more research has to be done.

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