

COVID-19 disease transmission model considering direct and indirect transmission

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Abstract. A mathematical model for understanding the COVID-19 transmission mechanism proposed in this article considering two important factors: the path of transmission (direct-indirect) and human awareness. Mathematical model constructed using a four-dimensional ordinary differential equation. We find that the Covid-19 free state is locally asymptotically stable if the basic reproduction number is less than one, and unstable otherwise. Unique endemic states occur when the basic reproduction number is larger than one. From sensitivity analysis on the basic reproduction number, we find that the media campaign succeeds in suppressing the endemicity of COVID-19. Some numerical experiments conducted to show the dynamic of our model respect to the variation of parameters value.

1 Introduction

Formerly come from Wuhan, China at the end of 2019, COVID-19 keep spread through worldwide and threat human respiratory system [1]. Until June 22, 2020, more than nine million people worldwide have been infected by COVID-19, where the USA is the highest contributor of this number of the recorded case [2].

Covid-19 believed to be transmitted originally from animal to human. However, it is more dominant that the infection was coming from direct contact of human-to-human, or from contact with contaminated surfaces which contain droplets from infected individual [3]. Several interventions have been conducted by policymakers in many countries, such as physical distancing, use of medical mask, rapid assessment to trace infected individuals, use of disinfectant, and many more [3].

Mathematical models have been conducted by many authors to understand how COVID-19 spread among the population [5-13]. In this study, continue our previous study on the COVID-19 transmission model [7]. The idea is coming from the fact that media campaigns become one of the most important interventions by policymakers to increase human awareness of the danger of COVID-19. This media campaign will separate the population into aware and non-aware subpopulations. The question that needs to be addressed then: How well is the media campaign could reduce the spread of COVID-19? What is the most influential parameter in determining the existence of COVID-19? Hence, this article will be addressed the above questions based on mathematical model analysis.

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Our article organized as follow. A mathematical model formulation is given in Section 2 and followed with mathematical analysis regarding the steady-state, basic reproduction number, and local stability criteria in Section 3. Furthermore, we conduct some numerical examples in Section 4. Finally, some conclusion is given in Section 5.

2 Model Formulation

Here in this article, we construct our model based on our previous result in [7]. The modification lies in additional awareness of the population due to the media campaign by the government. The purpose of this media campaign is to develop an awareness of susceptible populations to practice a healthy lifestyle against COVID-19 viruses, such as the use of medical mask and routine to wash hands after contact with the suspicious object that might bring virus on the surface.

Hence, let the human population divided into three subpopulations, namely susceptible unaware S_u , susceptible aware S_a , and infected population I . We also include one additional compartment which consider the free viruses in the environment, namely V .

Susceptible unaware population increases due to newborn Λ , drop out from unaware population due to vanishment of awareness u_2 , and recovered from Covid-19 α with a proportion of p . On the other hand, it decreases due to “**direct**” transmission from an infected population with successful infection rate β_1 , and from “**indirect**” transmission from contact with free viruses with successful contact rate β_2 , and transition to the aware population due to media campaign u_1 .

Susceptible aware population increases due to transition from an unaware population with the rate of u_1 , and recovery from the disease at a constant rate of α with proportion of $(1 - p)$. Furthermore, it decreases due to “**direct**” and “**indirect**” transmission with infection probability β_1 and β_2 , and transition to unaware population due to vanishment of awareness with rate of u_2 . Since we assume that aware population implements healthy life-style, the transmission probability reduced by q for the direct transmission, and by r for indirect transmission.

Infected population increases due to infection from an aware and unaware population, and decreases due to recovery from COVID-19. On the other hand, the free Virus increases from infected population which produces viruses from sneezed and decreases due to natural inactivated rate μ_v .

Based on the above assumptions, the COVID-19 transmission model considers direct-indirect transmission and aware population describe by the following four-dimensional ordinary differential equation.

$$\begin{aligned} \frac{dS_u}{dt} &= \Lambda - \beta_1 S_u I - \beta_2 S_u V - \mu_h S_u + p\alpha I - u_1 S_u + u_2 S_a, \\ \frac{dS_a}{dt} &= u_1 S_u - u_2 S_a - q\beta_1 S_a I - r\beta_2 S_a V - \mu_h S_a + (1-p)\alpha I, \\ \frac{dI}{dt} &= \beta_1 S_u I + \beta_2 S_u V + q\beta_1 S_a I + r\beta_2 S_a V - \mu_h I - \alpha I, \\ \frac{dV}{dt} &= \xi I - \mu_v V, \end{aligned} \tag{1}$$

where μ_h is the natural death rate. This system is supplemented by non-negative initial condition:

$$S_u(0) = S_{u0} \geq 0, S_a(0) = S_{a0} \geq 0, I(0) = I_0 \geq 0, V(0) = V_0 \geq 0.$$

It is easy to see that since $\frac{d(S_u+S_a+I)}{dt} = \frac{dN}{dt} = \Lambda - \mu_h N$, we have that $\lim_{t \rightarrow \infty} N = \frac{\Lambda}{\mu_h}$. Using the vector field approach, we can see that S_u, S_a, I , and V always has an inward gradient whenever it approaches 0. Hence, we conclude that our model in (1) is biologically well posed.

3 Model analysis

3.1 The Covid-19 free state and the basic reproduction number

System (1) has a trivial equilibrium when an infected population does not exist in the population. We call this equilibrium as the COVID-19 free state, which given by

$$E_0 = (S_u, S_a, I, V) = \left(\frac{\Lambda(u_1 + u_2)}{\mu_h(\mu_h + u_1 + u_2)}, \frac{\Lambda u_1}{\mu_h(\mu_h + u_1 + u_2)}, 0, 0 \right).$$

Next, we calculate the basic reproduction number using the next-generation matrix approach [14]. System (1) has a basic reproduction number given by

$$R_0 = \frac{\Lambda(\beta_1 \mu_v [\mu_h + u_2 + q u_1] + \xi \beta_2 [\mu_h + u_2 + r u_1])}{\mu_h \mu_v (\mu_h + u_1 + u_2) (\alpha + \mu_h)}.$$

This basic reproduction number could be re-written based on the source of infection (direct and indirect transmission) as follows :

$$R_0 = \underbrace{\frac{\Lambda(\beta_1 [\mu_h + q u_1 + u_2])}{\mu_h (\mu_h + u_1 + u_2) (\alpha + \mu_h)}}_{\text{direct transmission}} + \underbrace{\frac{\Lambda(\xi \beta_2 [\mu_h + r u_1 + u_2])}{\mu_h \mu_v (\mu_h + u_1 + u_2) (\alpha + \mu_h)}}_{\text{indirect transmission}}$$

From the above expression, it can be seen that the component of direct transmission depends on the number of maximum populations $\frac{\Lambda}{\mu_h}$ multiplied with direct transmission rate β_1 . Hence, we conclude that a larger number of the population will increase direct transmission. On the other hand, from indirect transmission component, it appears that the infection depends on the multiplication between transmission rate β_2 , a total of human population $\frac{\Lambda}{\mu_h}$ and a maximum number of free viruses $\frac{\xi}{\mu_v}$. Further discussion on R_0 will be discussed in the later section of this article.

In many epidemiological models, basic reproduction number holds an important role in determining the stability of the disease-free state [15-21]. It is common to find that the disease-free state is locally asymptotically stable if $R_0 < 1$ and unstable otherwise. Similarly, using the linearization approach, we have the following theorem.

Theorem 1. *The Covid-19 free state E_0 of system (1) is locally asymptotically stable (LAS) if $R_0 < 1$, and unstable otherwise.*

Proof. Linearize system (4) at Γ_0 yield

$$J_0 = \begin{bmatrix} J_{1,1} & J_{1,2} \\ J_{2,1} & J_{2,2} \end{bmatrix}.$$

where $J_{1,1} = \begin{bmatrix} -(\mu_h + u_1) & u_2 \\ u_1 & -(\mu_h - u_2) \end{bmatrix}$, $J_{1,2} = \begin{bmatrix} ap - \frac{\Lambda \beta_1 (\mu_h + u_2)}{\mu_h (\mu_h + u_1 + u_2)} & -\frac{\Lambda \beta_2 (\mu_h + u_2)}{\mu_h (\mu_h + u_1 + u_2)} \\ (1-p)\alpha - \frac{\Lambda q \beta_1 u_1}{\mu_h (\mu_h + u_1 + u_2)} & -\frac{\Lambda r \beta_2 u_1}{\mu_h (\mu_h + u_1 + u_2)} \end{bmatrix}$, $J_{2,1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

, and $J_{2,2} = \begin{bmatrix} -(\alpha + \mu_h) + \frac{\beta_1 \Lambda (\mu_h + q u_1 + u_2)}{\mu_h (\mu_h + u_1 + u_2)} & \frac{\beta_2 \Lambda (\mu_h + q u_1 + u_2)}{\mu_h (\mu_h + u_1 + u_2)} \\ \xi & -\mu_v \end{bmatrix}$. J_0 has two negative eigenvalues, i.e $\lambda_1 = -\mu_h$ and $\lambda_2 = -(\mu_h + u_1 + u_2)$. The other two eigenvalues are coming from the root of the following polynomial:

$$a_2 \lambda^2 + a_1 \lambda + a_0 = 0,$$

where $a_2 = -\mu_h (\mu_h + u_1 + u_2)$, $a_1 = \Lambda \beta_1 (\mu_h + q u_1 + u_2) - \mu_h (\mu_h + u_1 + u_2) (\alpha + \mu_h + \mu_v)$, and $a_0 = \mu_h \mu_v (\mu_h + u_1 + u_2) (\alpha + \mu_h) (R_0 - 1)$.

To guarantee this polynomial has a negative real part for all its roots, it should be that $\lambda_3\lambda_4 = \frac{a_0}{a_2} > 0$ and $\lambda_3 + \lambda_4 = -\frac{a_1}{a_2} < 0$. It is easy to see that $\lambda_3\lambda_4 = \frac{a_0}{a_2} > 0 \Leftrightarrow R_0 < 1$. On the other hand, $\lambda_3 + \lambda_4 = -\frac{a_1}{a_2} < 0 \Leftrightarrow \frac{\Lambda\beta_1(\mu_h + qu_1 + u_2)}{\mu_h(\mu_h + u_1 + u_2)(\alpha + \mu_h + \mu_v)} < 1$. Since

$$\frac{\Lambda\beta_1(\mu_h + qu_1 + u_2)}{\mu_h(\mu_h + u_1 + u_2)(\alpha + \mu_h + \mu_v)} < R_0 < 1,$$

Then we have that E_0 is LAS if $R_0 < 1$, and unstable otherwise. Hence, the proof is completed. ■

From the form of the basic reproduction number, when $\beta_v = 0$, which indicate that we neglect the indirect transmission, we have that

$$R_0^* = \frac{\Lambda(\beta_1[\mu_h + qu_1 + u_2])}{\mu_h(\mu_h + u_1 + u_2)(\alpha + \mu_h)} < R_0.$$

Hence, we have the following remark.

Remark 1. *Indirect transmission increases the standard basic reproduction number. The increases of this basic reproduction number highly dependent on the number of human population and the free viruses on the environment. Hence, reducing the number of viruses could using any intervention to kill the free viruses could be considered as an alternative option to eradicate COVID-19 transmission.*

3.2 The Covid-19 endemic state

System (1) has a nontrivial state, which called the endemic state. This state is given by

$$E_1 = (S_u, S_a, I, V) = \left(S_u^*, \frac{u_1 S_u^* + \alpha I^* (1-p)}{u_2 + \mu_h + \beta_2 r V^* + \beta_1 q I^*}, I^*, \frac{\xi I^*}{\mu_v} \right),$$

where S_u^* has a complex expression to be written in this article, while I^* taken from the positive roots of the following polynomial

$$b_2 I^2 + b_1 I + b_0 = 0,$$

where

$$\begin{aligned} b_2 &= -\mu_h(\xi\beta_2 + \beta_1\mu_v)(q\beta_1\mu_v + r\xi\beta_2) \\ b_1 &= (\xi\beta_2 + \beta_1\mu_v)(q\beta_1\mu_v + r\xi\beta_2)\Lambda - \mu_h\mu_v(\alpha\xi\beta_2(1-p) + \alpha\beta_1\mu_v(1-p)) \\ &\quad + \alpha p q \beta_1\mu_v + \alpha p r \xi\beta_2 + q \beta_1\mu_h\mu_v + q \beta_1\mu_h u_1 + r \xi\beta_2\mu_h + r \xi\beta_2 u_1 + \dots \\ &\quad + \xi\beta_2\mu_h\xi\beta_2 u_2 + \beta_1\mu_h\mu_v + \beta_1\mu_v u_2 \\ b_0 &= \mu_v\mu_h\mu_v(\mu_h + u_1 + u_2)(\alpha + \mu_h)(R_0 - 1). \end{aligned}$$

Hence, system (1) has:

- Unique endemic equilibrium if $R_0 > 1$, or $R_0 < 1$, $b_1 > 0$ and $b_1^2 - 4b_0b_2 = 0$, or $R_0 = 1$ and $b_1 < 0$.
- Two endemic equilibrium if $R_0 < 1$, $b_1 > 0$ and $b_1^2 - 4b_0b_2 > 0$.
- No endemic equilibrium otherwise.

Using the linearization of system (1) at E_1 , the Jacobian matrix is given by

$$K_1 = \begin{bmatrix} K_{1,1} & K_{1,2} \\ K_{2,1} & K_{2,2} \end{bmatrix},$$

where $K_{1,1} = \begin{bmatrix} -\beta_1 I^* - V^* \beta_2 - (\mu_h + u_1) & u_2 \\ u_1 & -I^* q \beta_1 - V^* r \beta_2 - (\mu_h + u_2) \end{bmatrix}$, $K_{1,2} = \begin{bmatrix} \alpha p - S_u^* \beta_1 & -S_u^* \beta_2 \\ -q \beta_1 S_a^* + (1-p)\alpha & -r \beta_2 S_a^* \end{bmatrix}$, $K_{2,1} = \begin{bmatrix} \beta_1 I^* + \beta_2 V^* & q \beta_1 I^* + r \beta_2 V^* \\ 0 & 0 \end{bmatrix}$, and $K_{2,2} = \begin{bmatrix} q \beta_1 I^* + \beta_1 S_u^* - \alpha - \mu_h & r \beta_2 S_a^* + S_u^* \beta_2 \\ \xi & -\mu_v \end{bmatrix}$. The characteristic polynomial of K_1 is given by

$$\sum_{i=0}^4 a_i \lambda^i = 0$$

where a_i for $i = 0, 1, \dots, 4$ is a long expression to be shown in this article. Hence, based on Routh-Hurwitz stability criteria, E_1 will locally stable if

$$a_0 > 0, a_3 > 0, a_3 a_2 - a_4 a_1 > 0, (a_3 a_2 - a_4 a_1) a_1 - a_3^2 a_0 > 0.$$

We close this section with the following remark.

Remark 2. *The endemic of COVID-19 will always be established whenever the basic reproduction number is larger than one. Based on the form of the basic reproduction number, we notice that indirect transmission is equally important for the focus of the eradication program compared with an intervention that focuses on minimizing the direct transmission.*

4 Numerical experiments

We focus on two questions in this section. The first question is to what is the most sensitive parameter that determines R_0 ? It is very important to notice what controllable parameter that can be used to modify the size of R_0 most efficiently. The next question is, how is the long-time behavior of our model respect to the change of parameter value? Hence, we conduct an autonomous simulation to answer this question.

4.1 Sensitivity analysis

We have observed from the previous section that the basic reproduction number holds an essential role in the transmission of COVID-19. Here, we perform the local sensitivity analysis [22] to understand the robustness of the basic reproduction number to the parameter value. The normalized sensitivity analysis of variable c respect to R_0 , denoted by $\Gamma_c^{R_0}$, defined as

$$\Gamma_c^{R_0} = \frac{\partial R_0}{\partial c} \times \frac{c}{R_0}.$$

For an example, since $\frac{\partial R_0}{\partial \beta_1} = \frac{\Lambda(q u_1 + \mu_h + u_2)}{\mu_h(\mu_h + u_1 + u_2)(\alpha + \mu_h)}$, then we have

$$\begin{aligned} \Gamma_{\beta_1}^{R_0} &= \frac{\Lambda(q u_1 + \mu_h + u_2)}{\mu_h(\mu_h + u_1 + u_2)(\alpha + \mu_h)} \times \frac{\beta_1 \mu_h \mu_v (\mu_h + u_1 + u_2)(\alpha + \mu_h)}{\Lambda(\beta_1 \mu_v [\mu_h + u_2 + q u_1] + \xi \beta_2 [\mu_h + u_2 + r u_1])} \\ &= \frac{(q u_1 + \mu_h + u_2) \beta_1 \mu_v}{(\beta_1 \mu_v [\mu_h + u_2 + q u_1] + \xi \beta_2 [\mu_h + u_2 + r u_1])}. \end{aligned}$$

Using the following parameters

$$\Lambda = \frac{10^6}{65 \times 365}, \mu_h = \frac{1}{65 \times 365}, \mu_v = \frac{1}{3}, \beta_1 = 0.2 \times 10^{-7}, \beta_2 = 0.2 \times 10^{-11},$$

$$u_1 = 0.1, u_2 = 0.05, \alpha = \frac{1}{30}, q = 0.1, r = 0.1, \xi = 10^4,$$

we have that $\Gamma_{\beta_1}^{R_0} = 0.625$. It means that increasing β_1 for 10% will increase R_0 to 6.25%.

Using the same parameters value, we have that $\Gamma_{u_1}^{R_0} = -0.499$, which means that increasing u_1 for 10% will reduce R_0 for 4.99%. With similar approach, the sensitivity index of $\mu_h, \beta_2, u_2, \mu_v, q, r, \xi$ and α is -1.00084, 0.375, 0.4995, -0.375, 0.104, 0.0625, 0.375, and 0.9987, respectively. We have shown this phenomenon for all parameters in Fig.1. It can be seen that u_1 and u_2 are promising to reduce R_0 since the sensitivity index is negative and positive, respectively. Fig.2 shows the contour plot of the basic reproduction number respect

to u_1 and u_2 . It can be seen clearly that for increasing value of u_1 will reduce R_0 . On the other hand, u_2 should be reduced to reduce R_0 .

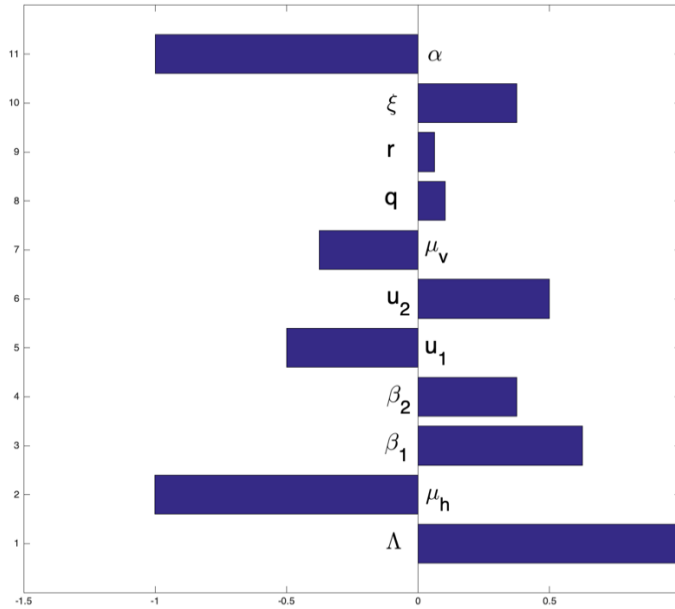


Fig. 1. Tornado plot for the sensitivity of R_0 respect to parameters of model (1).

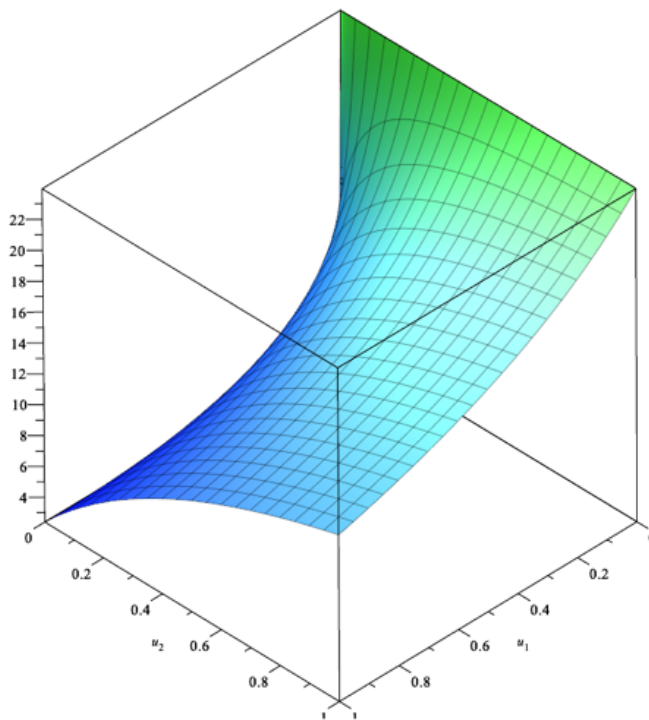


Fig. 2. Contour plot of dependency of R_0 to u_1 and u_2 .

4.2 Numerical simulation of the Autonomous system

As previously mentioned, our goal in this section is to visualize the effect of parameter value to the dynamical solution of our proposed COVID-19 model in (1). For the first example, we will show how our model changed respect to the change of media campaign to transfer unaware subpopulation to the aware population (u_1). To conduct the simulation, we use the following parameters

$$\Lambda = \frac{10^6}{65 \times 365}, \mu_h = \frac{1}{65 \times 365}, \mu_v = \frac{1}{3}, \beta_1 = 0.2 \times 10^{-7}, \beta_2 = 0.2 \times 10^{-11},$$

$$u_2 = 0.05, \alpha = \frac{1}{30}, q = 0.1, r = 0.1, \xi = 10^4,$$

while u_1 are 0, 0.025, 0.05 and 0.4 which give us the basic reproduction number 4.79, 3.35, 2.63 and 0.96, respectively. The result is shown in Fig. 3. Based on Theorem 1, we have that only when $u_1 = 0.4$ will give us a stable COVID-19 free state, while the other value of u_1 will make the COVID-19 exist in the population.

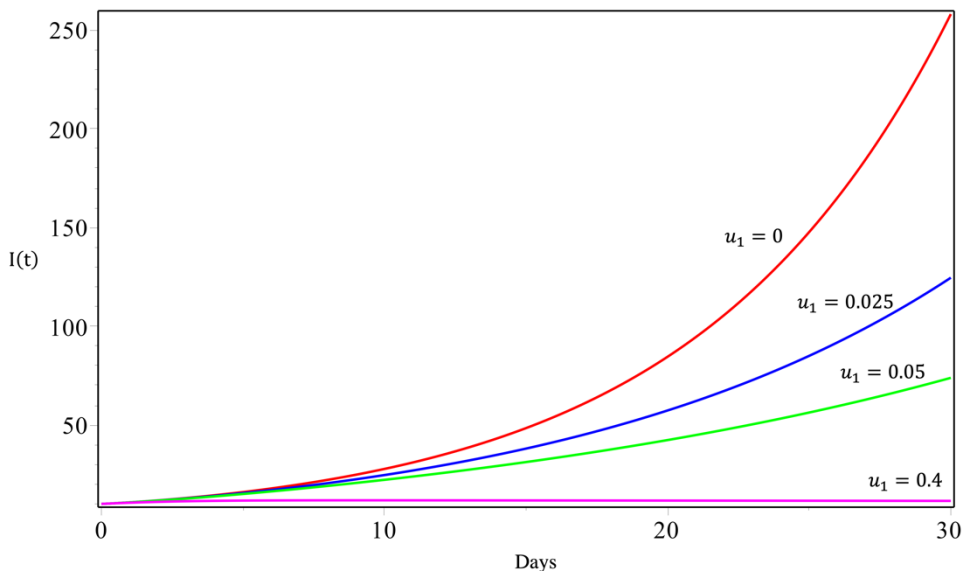


Fig. 3. Solution of trajectories of infected population of model (1) for various value of u_1 .

The next simulation conducted using the following parameters value

$$\Lambda = \frac{10^6}{65 \times 365}, \mu_h = \frac{1}{65 \times 365}, \mu_v = \frac{1}{3}, \beta_1 = 0.2 \times 10^{-7}, \beta_2 = 0.2 \times 10^{-11},$$

$$u_1 = 0.1, \alpha = \frac{1}{30}, q = 0.1, r = 0.1, \xi = 10^4,$$

while u_2 are 0.1, 0.05, 0.025 and 0.01 which gave us the basic reproduction number 2.63, 1.91, 1.34 and 0.87, respectively. The result is shown in Fig. 4. Again, based on Theorem 1, we have that only when $u_2 = 0.01$ will give us a stable COVID-19 free state, while the other value of u_2 will make the COVID-19 exist in the population. Based on these simulations, we can conclude that it needs a proper balance of u_1 and u_2 to achieve a condition where COVID-19 no longer exists in the population. It shows us how important is the media campaign to educate the population about the danger of COVID-19. This intervention

expected to increase the awareness and make the aware population reduce the possibility of infection by COVID-19, whether from direct or from indirect transmission pathway.

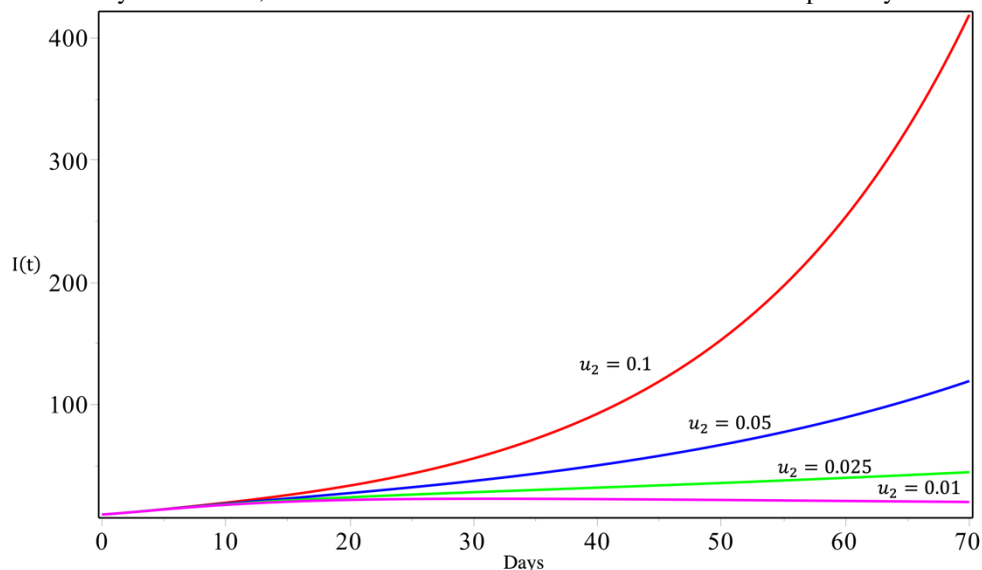


Fig. 4. Solution of trajectories of infected population of model (1) for various value of u_2 .

5 Conclusions

In this paper, we formulate a system of the ordinary differential equation, which describes the transmission of COVID-19 among the human population. The model considers the direct transmission from the contact of human-to-human, and indirect transmission due to contact of human to a contaminated surface with the viruses. The model accounts for the media campaign effect to increase human awareness on the existence of COVID-19.

We show that system (1) always has a unique solution as long as the initial condition is positive. Furthermore, system (1) always has a unique COVID-19 free state, which is locally asymptotically stable (LAS) if the basic reproduction number is less than unity. We find that our model always has a unique COVID-19 endemic state whenever the basic reproduction number is larger than unity.

In the term of the sensitivity analysis of the basic reproduction number, we find that the media campaign succeeds to reduce the value of the basic reproduction number. We also find that we need a proper balance between the media campaign to increase human awareness and the media campaign to reduce the vanishment of people's awareness of the danger of COVID-19, such that we can increase the probability that COVID-19 could be eradicated from the population.

Although our model indicates the promising opportunity to rely on the media campaign for the COVID-19 eradication program, it also comes with a high cost for intervention. Hence, treat the media campaign parameter as a time-dependent variable could be a solution. The aim is to minimize the number of infected populations with minimum cost for the implementation program. This problem could be solved as an optimization problem. Please see [16-18, 21, 23-25] for some examples of optimization problems in epidemiological models.

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