Optimal control problem on COVID-19 disease transmission model considering medical mask, disinfectants and media campaign

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Abstract. In this paper, a system of ordinary differential equation approach is developed to understand the spread of COVID-19. We first formulate the dynamic model by dividing the human population based on their health status, awareness status, and also including the free virus on the environment. We provide a basic analysis of the model regarding the well-posed properties and how the basic reproduction number can be used to determine the final state of COVID-19 in the population. A Pontryagin Maximum's Principle used to construct the model as an optimal control problem in a purpose to determine the most effective strategies against the spread of COVID-19. Three control strategies involved in the model, such as media campaign to develop an awareness of individuals, medical masks to prevent direct transmission, and use of disinfectant to reduce the number of free virus in the environment. Through numerical simulations, we find that the time-dependent control succeeds in reducing the outbreak of COVID-19. Furthermore, if the intervention should be implemented as a single intervention, then the media campaign gives the most effective cost strategy.

1 Introduction

Covid-19 has been found initially in Wuhan China, at the end of 2019 [1,2]. The symptoms of COVID-19 are like ordinary flu, but it could lead to death caused by difficulties in breathing for patients in chronic stages. Until July 2020, there is no medicine to cure infected individuals, and also no vaccine to prevent infection of COVID-19. However, several candidates for vaccines to prevent COVID-19 infection have been proposed and under consideration in several countries [3-5]. Hence, the most popular way to control the spread of COVID-19 is to prevent infection by introducing a non-pharmaceutical intervention, such as physical distancing, use of medical mask, quarantine, improvement of medical treatment, use of disinfectant, contact tracing and many more [6-8].

Many authors have used mathematical models to understand how diseases may spread among the population [9-13]. For the COVID-19 model, the first mathematical model that has been published to understand how COVID-19 spread in Indonesia is given in [14]. They predict how COVID-19 behave in Indonesia by using some estimated parameters from other

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countries. The author in [7] proposed a $SEA_u A_d IR$ model to describe how COVID-19 spread in Jakarta, where mathematical analysis regarding stability analysis conducted, and parameter estimation provided to see the impact of social distancing in Jakarta. Authors in [14,15] also use a deterministic model to predict the spread of COVID-19 in Indonesia, and several other countries.

In this paper, we develop a mathematical model of COVID-19 spread among population considering two possible sources of infection, namely direct infection due to contact between susceptible human and infected human, and indirect transmission due to susceptible human touch an object which has a COVID-19 virus on it surfaces. Mathematical analysis for some basic properties conducted and followed with an optimal control problem of the model. The main purposes of this article are to determine the best strategies which combine three intervention: media campaign, medical mask, and disinfectant, against COVID-19.

The paper is organized as follows. In section 2, we construct the optimal control model. Mathematical analysis conducted in section 3 regarding the COVID-19 free equilibrium and the basic reproduction number. The optimal control characterization conducted in Section 4, followed by numerical experiments in Section 5. The last section is devoted to our conclusions.

2 The $S_u S_a IRS - V$ model

We consider the $S_u S_a IRS - V$ model in this article, to model how COVID-19 spread among population under three different intervention strategies, namely media campaign to develop awareness of individual, medical mask to prevent human-to-human transmission, and disinfectant to reduce number of free virus in the environment. Let S_u and S_a describe unaware and aware susceptible population, respectively, while I and R describe infected and recovered population, respectively. Furthermore, we denote V as the free virus in the environment. The transmission diagram of the model described in Fig.1 below.

Figure 1. Transmission diagram of COVID-19 for system (1).

The following assumptions used to construct the model.

- The population are closed. All migration is neglected.
- Infected individual may death due to COVID-19.
- Media campaign given in a rate of u_1 to increase human awareness. We assume that aware susceptible individual conduct a perfect self-isolation to avoid human-to-human infection.

- d. Due to assumption (c), only unaware susceptible individual who may get infected by COVID-19.
- e. u_2 proportion of unaware susceptible and infected individual using medical mask to prevent human-to-human transmission, while the rest $(1 - u_2)$ do not.
- f. Disinfectant conducted with a rate of u_3 to reduce number of free virus in the environment.

Based on previous assumptions and transmission diagram in Figure 1, the COVID-19 transmission model is expressed by the following system of ordinary differential equations:

$$\begin{aligned} \frac{dS_u}{dt} &= \Lambda - (1 - u_2(t) + \xi u_2(t))^2 \beta_h S_u I - \beta_v S_u V - \mu_h S_u - u_1(t) S_u + \alpha S_a, \\ \frac{dS_a}{dt} &= u_1(t) S_u - \alpha S_a - \mu_h S_a + \eta R, \\ \frac{dI}{dt} &= (1 - u_2(t) + \xi u_2(t))^2 \beta_h S_u I + \beta_v S_u V - \delta I - \mu_h I - \theta I, \\ \frac{dR}{dt} &= \delta I - \mu_h R - \eta R, \\ \frac{dV}{dt} &= \omega I - \mu_v V - u_3(t) V, \end{aligned} \tag{1}$$

which supplemented with a non-negative initial condition.

Our aim is to minimized number of infected individuals, with as low as possible cost for intervention. The objective functional to be minimized is given by

$$J(u_1, u_2, u_3, I) = \int_0^T (cI + d_1 u_1^2 + d_2 u_2^2 + d_3 u_3^2) dt, \tag{2}$$

where c, d_i for $i = 1, 2, 3$ is the weight parameter for infected individuals and control parameters, respectively, and T is the final time. Description and range of value for each parameter in system (1) is given in Table 1.

Table 1. Description of parameters for COVID-19 model (1).

Par.	Description	Value	Ref.
Λ	Natural recruitment rate due to new born	$\frac{49\,316\,712}{70 \times 365}$	[29]
u_1	Media campaign rate	[0,1]	Assumed
u_2	Proportion of medical mask	[0,1]	Assumed
u_3	Rate of disinfectant	[0,1]	Assumed
β_h	Human-to-human transmission rate	1.26×10^{-8}	[29]
β_v	Transmission rate from free virus in the environment	8×10^{-11}	[8]
μ_h	Natural death rate of human	$\frac{1}{70 \times 365}$	[30]
μ_v	Survival rate of virus without host	$\frac{1}{2}$	[8]
α	Drop-out rate from aware population	0.1	[29]
δ	Natural recovery rate	0.12978	[31]
θ	Death rate due to Covid-19	0.015	[31]
ω	Average production rate of virus from infected individuals	10 000	[8]
η	Transition rate due to temporal immunity vanished	$\frac{1}{30}$	Assumed
ξ	Medical mask efficacy	[0,1]	Assumed

3 Analysis on COVID-19 free equilibrium and $\mathcal{R}_0$

To analyse the existence, and stability of the equilibrium points, let us assume that all intervention strategy are constant, hence $u_j(t) = u_i$ for $j = 1, 2, 3$.

For COVID-19 model (1) to be epidemiologically meaningful, it is important to prove that all its state variables are non-negative for all time $t > 0$, as long as the initial conditions are non-negative. This properties given in the following Lemma 1.

Lemma 1. *Let the initial condition $S_u(0) \geq 0, S_a(0) \geq 0, I(0) \geq 0, R(0) \geq 0$, and $V(0) \geq 0$. Then the solutions for all variables of COVID-19 model in (1) are non-negative for all time $t > 0$.*

Invariant Regions. It can be seen that our COVID-19 model will be analyzed in the following region. Consider the feasible region:

$$\Omega = \Omega_h \times \Omega_v \subset \mathbb{R}_+^4 \times \mathbb{R}_+,$$

where

$$\Omega_h = \left\{ (S_u(t), S_a(t), I(t), R(t)) \in \mathbb{R}_+^4 : N_h(t) \leq \frac{\Lambda}{\mu_h} \right\},$$

with $S_u(t) + S_a(t) + I(t) + R(t) = N_h(t)$ and

$$\Omega_v = \left\{ V \in \mathbb{R}_+ : V \leq \frac{\omega \Lambda}{\mu_h(\mu_v + u_3)} \right\}.$$

Lemma 2. *The region $\Omega \subset \mathbb{R}_+^5$ is positively-invariant for the model (1) with non-negative initial conditions in $\mathbb{R}_+^5$.*

Next, we will determine the form of equilibrium points. The COVID-19 free-equilibrium point of system(1) is given by

$$E_0 = (S_u, S_a, I, R, V) = \left(\frac{\Lambda(\alpha + \mu_h)}{\mu_h(\alpha + \mu_h + u_1)}, \frac{\Lambda u_1}{\mu_h(\alpha + \mu_h + u_1)}, 0, 0, 0 \right).$$

Using the next-generation matrix approach [17], the basic reproduction number of system (1), denoted by $\mathcal{R}_0$, is given by

$$\mathcal{R}_0 = \mathcal{R}_h + \mathcal{R}_v \quad (3)$$

where $\mathcal{R}_h = \frac{\Lambda(\alpha + \mu_h)(1 - u_2(t) + \xi u_2(t))^2 \beta_h}{\mu_h(\alpha + \mu_h + u_1)(\delta + \mu_h + \theta)}$, and $\mathcal{R}_v = \frac{\Lambda(\alpha + \mu_h)\omega \beta_v}{\mu_h(\alpha + \mu_h + u_1)(\delta + \mu_h + \theta)(\mu_v + u_3)}$. Basic reproduction number represent number of new infected individual caused by a primary infected individual during its infection period in a completely susceptible individuals. Note that $\mathcal{R}_h$ represent the basic reproduction number due to human-to-human infection, while $\mathcal{R}_v$ is the basic reproduction number due to contact with free virus in the environment.

Using Theorem 2 in [18], the following results established.

Lemma 3. *The Covid-19 free equilibrium point E_0 of system (1) is locally asymptotically stable (LAS) if $\mathcal{R}_0 < 1$ and unstable if $\mathcal{R}_0 > 1$.*

Lemma 3 is a common result in an epidemiology model for disease spread. Please see [25,26,27] for a similar result regarding the LAS properties of the disease-free equilibrium point for other disease models.

Effect of controls on the basic reproduction number.

From Lemma 3, it can be seen that COVID-19 will disappeared from the population whenever $\mathcal{R}_0 < 1$. Hence, reducing $\mathcal{R}_0$ will increase the chance to eradicate COVID-19 from the environment. Since

$$\frac{\partial \mathcal{R}_0}{\partial u_1} = -\frac{\Lambda(\alpha+\mu_h)(1-u_2(t)+\xi u_2(t))^2 \beta_h}{\mu_h(\alpha+\mu_h+u_1)^2(\delta+\mu_h+\theta)} - \frac{\Lambda(\alpha+\mu_h)\omega\beta_v}{\mu_h(\alpha+\mu_h+u_1)^2(\delta+\mu_h+\theta)(\mu_v+u_3)} < 0,$$

we conclude that increasing u_1 will reduce $\mathcal{R}_0$. Similarly, since

$$\frac{\partial \mathcal{R}_0}{\partial u_2} = -\frac{\Lambda(\alpha+\mu_h)2(1-u_2(t)+\xi u_2(t))(1-\xi)\beta_h}{\mu_h(\alpha+\mu_h+u_1)(\delta+\mu_h+\theta)} < 0,$$

we conclude that increasing u_2 will reduce $\mathcal{R}_0$. However, it can be seen that unlike u_1 where the implementation could reduce $\mathcal{R}_h$ and $\mathcal{R}_v$ simultaneously, implementation of u_2 only reducing $\mathcal{R}_h$. The reason simply because u_2 conducted only to reduce human-to-human infection rate. The other results is regarding the disinfectant intervention. Since

$$\frac{\partial \mathcal{R}_0}{\partial u_3} = -\frac{\Lambda(\alpha+\mu_h)\omega\beta_v}{\mu_h(\alpha+\mu_h+u_1)(\delta+\mu_h+\theta)(\mu_v+u_3)^2} < 0,$$

we also can conclude that disinfectant could reduce $\mathcal{R}_0$. Since the purposes of disinfectant u_3 is to reduce number of free virus on the environment, we have that u_3 only affect $\mathcal{R}_v$.

We close this section with the following remark.

Remark 1. *The basic reproduction number of the COVID-19 model in (1) generated by two source of infection, namely human-to-human and environmental transmission. All intervention (media campaign, medical mask and disinfectant) are promising to control the spread of COVID-19 since they could reduce $\mathcal{R}_0$. However, only intervention of media campaign that could reduce both component of $\mathcal{R}_0$, while medical mask only reducing human-to-human transmission, and disinfectant only reducing environmental transmission.*

4 Optimal control characterization

Our goal is to minimize the cost function in (2) subject to system (1). As mentioned before, the cost function in (2) represent the problem of minimizing number of infected individual $\left(\int_0^T (cI) dt\right)$, and cost for interventions $\left(\int_0^T (d_1 u_1^2 + d_2 u_2^2 + d_3 u_3^2) dt\right)$. In this paper, a quadratic objective functional is implemented to measuring the control cost, since we assume that the cost for intervention are nonlinear. This quadratic cost function has been frequently used by many authors [8,10,19,20,21].

We seek to find optimal controls u_1^* , u_2^* , and u_3^* such that

$$J(u_1^*, u_2^*, u_3^*) = \min_U \{J(u_1, u_2, u_3)\},$$

where the control set U is given by

$$U = \{(u_1, u_2, u_3): [0, T] \rightarrow [0, 1]\}.$$

The necessary condition for control parameters must satisfy the Pontryagin's Maximum principle [22]. First, let us formulate the Hamiltonian from the cost function (2) and the COVID-19 dynamic model (1) to obtain the optimality conditions. Let us define the Hamiltonian for system (1) as follows

$$\begin{aligned}
 H = & cI + d_1 u_1^2 + d_2 u_2^2 + d_3 u_3^2 \\
 & + \lambda_1 \left(\Lambda - (1 - u_2(t) + \xi u_2(t))^2 \beta_h S_u I - \beta_v S_u V - \mu_h S_u - u_1(t) S_u + \alpha S_a \right) \\
 & + \lambda_2 (u_1(t) S_u - \alpha S_a - \mu_h S_a + \eta R) \\
 & + \lambda_3 \left((1 - u_2(t) + \xi u_2(t))^2 \beta_h S_u I + \beta_v S_u V - \delta I - \mu_h I - \theta I \right) \\
 & + \lambda_4 (\delta I - \mu_h R - \eta R) \\
 & + \lambda_5 (\omega I - \mu_v V - u_3(t) V),
 \end{aligned} \tag{4}$$

where λ_i for $i = 1, 2, 3, 4, 5$ are the associated adjoints for state S_u, S_a, I, R, V , respectively.

Theorem 1. Given an optimal controls u_1^*, u_2^*, u_3^* and solutions $S_u^*, S_a^*, I^*, R^*, V^*$ of the corresponding state system (1) that minimizes $J(u_1^*, u_2^*, u_3^*, I^*)$ over U . Then there exists adjoint variables λ_i for $i = 1, 2, 3, 4, 5$ which satisfying

$$-\frac{d\lambda_1}{dt} = \frac{\partial H}{\partial S_u}, -\frac{d\lambda_2}{dt} = \frac{\partial H}{\partial S_a}, -\frac{d\lambda_3}{dt} = \frac{\partial H}{\partial I}, -\frac{d\lambda_4}{dt} = \frac{\partial H}{\partial R}, -\frac{d\lambda_5}{dt} = \frac{\partial H}{\partial V}$$

And with transversality conditions $\lambda_i(T) = 0$ for $i = 1, 2, 3, 4, 5$. The optimality conditions are given as

$$\frac{\partial H}{\partial u_j} = 0, j = 1, 2, 3.$$

Taking the lower bound and upper bound for each control variables as 0 and 1, respectively, the control u_1^*, u_2^* , and u_3^* is given by

$$\begin{aligned}
 u_1^* &= \min \left\{ 1, \max \left\{ 0, \frac{S_u(\lambda_1 - \lambda_2)}{2d_1} \right\} \right\}, \\
 u_2^* &= \min \left\{ 1, \max \left\{ 0, \frac{\beta_h S_u I (1 - \xi)(\lambda_1 - \lambda_3)}{\beta_h S_u I (1 - \xi)^2 (\lambda_1 - \lambda_3) - d_2} \right\} \right\}, \\
 u_3^* &= \min \left\{ 1, \max \left\{ 0, \frac{V\lambda_5}{2d_3} \right\} \right\}.
 \end{aligned} \tag{5}$$

Proof. The existence of an optimal control is guaranteed using the result by author in [23]. The adjoint system satisfying the following ordinary differential equations

$$\begin{aligned}
 \frac{d\lambda_1}{dt} &= (\beta_h I (1 - u_2 + \xi u_2)^2 + \beta_v V)(\lambda_1 - \lambda_3) + u_1(\lambda_1 - \lambda_2) + \mu_h \lambda_1, \\
 \frac{d\lambda_2}{dt} &= \alpha(\lambda_2 - \lambda_1) + \mu_h \lambda_2, \\
 \frac{d\lambda_3}{dt} &= -C + \beta_h S_u (1 - u_2 + \xi u_2)^2 (\lambda_1 - \lambda_3) + \delta(\lambda_3 - \lambda_4) + (\theta + \mu_h) \lambda_3 - \omega \lambda_5, \\
 \frac{d\lambda_4}{dt} &= \eta(\lambda_4 - \lambda_2) + \mu_h \lambda_4, \\
 \frac{d\lambda_5}{dt} &= \beta_v S_u (\lambda_1 - \lambda_3) + (\mu_v + u_3) \lambda_5,
 \end{aligned} \tag{6}$$

With a transversality conditions $\lambda_i(T) = 0$ for $i = 1, 2, 3, 4, 5$. Optimality condition (5) is taken from $\frac{\partial H}{\partial u_j} = 0, j = 1, 2, 3$, solve each of it respect to u_j , and combined it with it correspond lower and upper bound. ■

5. Numerical experiments

To In order to solve the optimal control problem (1) with objective functional (2), we used an iterative forward-backward method. At first, we give an initial guess for control variables u_j for $j = 1,2,3$, and solve the state system (1) forward in time using the Runge-Kutta method with initial condition of each state variables given. Next, we calculate the adjoint system (6) backward in time with the transversality condition. We update the control variables using (5) and iterate this scheme until the convergence criteria achieved. Please see [8,20,21,24-28] for more example on the implementation of this scheme.

To conduct simulation on this section, we use parameter values as in Table 1. When no control intervention given, the basic reproduction number of system (1) is 5.87 which indicates that the COVID-19 free equilibrium is unstable. The dynamic of susceptible unaware, susceptible aware, infected human and free virus is given in Figure (2). It can be seen that the infected individual is increasing and reach the outbreak at $t = 18$ days, and start decreasing to it endemic equilibrium point for larger t .

Figure 2. The dynamic of each variable in system (1) except R , where no control implemented.

The second simulation is conducted to simulate the effect intervention, when only media campaign implemented in the field, while medical mask and disinfectant ignored. Hence, we use $u_1 \geq 0, u_2 = u_3 = 0$. The result is given in Figure 3. It can be seen that by implementing media campaign in it highest rate from the beginning of simulation time, number of infected individual cannot increase significantly. However, the effect of media campaign only succeed at $t \leq 30$ days, since after that, number of infected individual start to increase. Even though number of infected individual increases, but media campaign intervention succeeds to reduce the level of the outbreak, and delay it occurrence time. With this simulation, we have the cost function is 4.86×10^7 , and number of infected individuals avoided are 2.594.699

individuals. Hence, the average cost for each infected individual avoided is $\frac{4.86 \times 10^7}{2.594.699} = 18.76$.

Figure 3. The second simulation. The dynamic of infected individual (above) when only media campaign intervention implemented, while other interventions ignored.

The third simulation is conducted to see the effect when only disinfectant as the only intervention. Hence, we set $u_1 = 0, u_2 = 0, u_3 \geq 0$. The results can be seen in Figure 4. It is easy to see that the effect of disinfectant in reducing number of infected individual is not as good as media campaign. The effect of disinfectant in Figure 4 is that it only reduce the outbreak slightly, and delay the occurrence of the outbreak for a moment. This is as the effect of control profile in Figure (4), With this simulation, number of infected avoided are 36.764 individual, whit cost function is 7.56×10^6 . Hence, average cost for each infected individual avoided is 20.57 .

The last simulation (fourth simulation) is when all intervention implemented. In this simulation, all intervention implemented in the field. The dynamic of infected individuals along with all control variables is given in Figure 5. It can be seen that all interventions should be given it high rates / proportion from the beginning of simulation time, and start to decreasing when the outbreak have been passed. As a result of this scenario, the size of the outbreak has been reduced significantly, and it occurrence time have been delayed. The cost for this scenario is 1.72×10^7 with the total number of infected individual that have been avoided are $2.903.787$ individuals. Hence, the average cost for each infected individual that have been avoided is 5.92 .

Figure 4. The third simulation. The dynamic of infected individual (above) when only disinfectant intervention implemented, while other interventions ignored.

Figure 5. The fourth simulation. The dynamic of infected individual (above) when all interventions implemented.

5 Discussion and concluding remarks

In this article, we introduce a simple deterministic model to understand the spread of COVID-19 under some interventions: media campaign, medical masks, and a disinfectant. Each control has its specific purposes in controlling the spread of COVID-19. The model conducted a five-dimensional system of ordinary differential equations, where free coronavirus in the environment have been considered in the model.

Mathematical analysis regarding the COVID-19 free equilibrium and the basic reproduction number has been shown analytically. We find that COVID-19 will be eradicated from the field whenever the basic reproduction number is less than one. From analysis on the derivation of the basic reproduction number, we find that all derivative of the basic reproduction number to each control parameter is negative, which indicates the increased value of controls will reduce the size of the basic reproduction number. Hence, increasing the rate/proportion of controls will increase the chance to eradicate COVID-19 from the field.

From optimal control simulation, we show how all controls could reduce the number of infected individuals. Each simulation shows how interventions could reduce the size of the outbreak and delay the outbreak's occurrence. However, not all interventions succeed in reducing the spread of COVID-19 significantly. Using a single intervention, we find that the media campaign is more effective in reducing the number of infected individuals compared with using disinfectant only. From an analysis of the average cost for each individual that has been avoided for each scenario, we find that using all interventions is the most effective strategy since the average cost is only 5.92, while using media campaign is 18.76, and disinfectant only is 20.57.

Although our results here indicate a promising intervention strategy to control COVID-19, we still have some limitations on this work. Our work not yet implemented in real incidence data. Hence, real practical in the field still need further justification. The second limitation is that many reports state that there are many undetected cases in the field. These undetected cases contribute to the infection process of COVID-19. Hence, for further work, accommodating undetected cases could be considered to modify our proposed model. For our previous work, which considers an undetected case, can be seen in [7,29].

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