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Effect of periodic surface temperature on heat transfer in layered saturated soil

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Abstract. This paper presents numerical analyses of one-dimensional heat transfer in layered saturated soil with effective porosity and under a periodic temperature boundary condition using the numerical model HT1. The model characterizes the soil layer using separate columns to represent solid matrix and mobile pore fluid components, and a series-parallel approach to model soil thermal conductivity. Numerical simulations are presented to illustrate the effect of fluid velocity, thermal retardation factor, thermal conductivity of solid particles, effective porosity and layer heterogeneity. Numerical results indicate that increasing downward fluid velocity and decreasing retardation factor can increase the distance that temperature oscillations from the surface can propagate into the layer. In addition, decreasing fluid velocity, increasing retardation factor, and increasing thermal conductivity of solid particles can decrease the temperature oscillation amplitude in the soil. Temperature profiles also indicate the significance of soil effective porosity and multiple soil layers on heat transfer behavior.

1 Introduction

Many engineering applications require the analysis and modeling of heat transfer through soil, including energy piles, landfill liner systems, nuclear water disposal and buried power cables. Diurnal temperature changes on the soil surface may significantly affect heat transfer behavior, and thus an investigation of heat transfer in soil under periodic temperature boundary condition can be important. A number of solutions for heat transfer in soil with sinusoidal surface temperature are available [1-3]. However, these solutions are limited to a single homogeneous soil layer. In addition, effective porosity (i.e., the fraction of total porosity which contributes to fluid flow) is often assumed equal to total porosity. Many studies [4,5] indicate that effective porosity can be significantly smaller than total porosity. Thus, analyses that account for effective porosity and layered soil conditions would represent a useful addition to the literature. This paper presents a one-dimensional heat transfer analysis for saturated soil under a periodic temperature boundary condition using the numerical model HT1 [6], and investigates the effect of several parameters, including fluid velocity, thermal retardation factor, thermal conductivity of solid particles, effective porosity and layer heterogeneity.

2 Numerical Model

Simulations were performed using numerical model HT1 [6] for one-dimensional heat transfer in layered saturated soil with effective porosity and steady fluid flow. For purposes of this paper, HT1 was slightly modified to include a periodic temperature boundary condition. HT1 was developed using the numerical model CST3 as a point of departure [7], which simulates large strain consolidation and coupled transport of a dilute chemical solute in a saturated layered compressible soil stratum. HT1, and its predecessor models CST3, CS2 [8], CST1 [9], and CST2 [10], have been extensively validated using analytical solutions, numerical solutions, and experimental data. A brief summary of the HT1 model is provided below.

A saturated soil stratum with total height H has R_l horizontal layers. Each soil layer i has height H_i and uniform properties. Soil particles have specific gravity G_{sp}, specific heat c_{sp}, and thermal conductivity λ_{sp}. Pore fluid has density ρ_f, specific heat c_f, thermal dispersivity β and thermal conductivity λ_f. Elevation coordinate z is defined positive upward from the base of the stratum. Discharge velocity v is constant and defined as positive downward. Top and bottom boundaries can be prescribed temperature or prescribed temperature gradient, and top boundary temperature can also be prescribed using a periodic trigonometric function. For constant and periodic temperature boundary conditions, the top and bottom temperatures are represented by T_t and T_b, respectively. The initial

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temperature profile can be uniform or vary with elevation.

Soil volume is divided into solid particle, mobile pore fluid, and immobile pore fluid components, as described by the following equation [11],

$$n_{sp} + n = n_{sp} + n_e + n_i = 1 \quad (1)$$

where n_{sp} = solid volume fraction (= volume of solid particles/total soil volume), n = total porosity (= volume of voids/total soil volume), n_e = effective porosity (= volume of mobile pore fluid/total soil volume), and n_i = immobile fluid porosity (= volume of immobile pore fluid/total soil volume). Effective porosity contributes to fluid flow and defines a seepage velocity $v_s = v/n_e$. Immobile fluid porosity is associated with non-interconnected and dead-end pores, and does not contribute to fluid flow. Immobile pore fluid and mobile pore fluid have the same physical and thermal properties. A "solid matrix" (SM) is defined as the solid particles plus immobile pore fluid.

For analysis purposes, the soil stratum is characterized as a fixed column of R_j SM elements and a moving column of R_{mo} mobile pore fluid elements with Lagrangian element-tracking. Heat transfer mechanisms include advection, conduction, and thermal mechanical dispersion in the fluid column, conduction in the SM column, and heat exchange between the two columns assuming local thermal equilibrium (LTE). The LTE assumption is valid for most cases with the possible exception of coarse soils and high discharge velocity. For cases involving heat exchange between columns, higher numerical resolution is needed for fluid column following the method of [9]. Thermal conductivity of the SM λ_s assumes that heat flow through solid particle contacts can be neglected [12,13], and is calculated based on the thermal conductivity and volume fractions of solid particles and immobile pore fluid. Heat capacity of the SM column $\rho_s c_s$ is calculated as the arithmetic mean of solid particle heat capacity $\rho_{sp} c_{sp}$ and fluid heat capacity $\rho_f c_f$.

To simulate diurnal temperature changes on the soil surface, HT1 uses a sinusoidal function to approximate T_t:

$$T_t = T_{to} + A\sin(\omega t + \phi) \quad (2)$$

where T_{to} is the average temperature of the top boundary; A is the amplitude of temperature oscillations; ω is the angular frequency; and ϕ is an initial phase angle.

3 Parametric Study

Several numeric examples were conducted using R_j = 100 to illustrate the effect of various parameters on heat transfer in saturated soil under a periodic temperature top boundary condition, including fluid velocity, thermal retardation factor, thermal conductivity, multiple soil layers, and effective porosity. Material properties for the fluid phase are ρ_f = 1,000 kg/m^3, λ_f = 0.59 W/m-°C, β = 0, and c_f = 4186 J/ kg-°C. Soil surface temperature is assumed to follow Eq. (2). Since one diurnal cycle is 24 hours, the angular frequency ω is calculated as $2\pi / (24\times60\times60) \approx 2.3\times10^{-5}\pi$ rad·s^{-1}. The average temperature T_{to} and amplitude A are equal to 25 °C and 8 °C, and the initial phase angle ϕ is 0 [3]. Thus, the sinusoidal function for top boundary temperature is

$$T_t = 25 + 8\sin(2.3\times10^{-5}\pi t) \quad (3)$$

The bottom temperature boundary condition is specified as a constant zero temperature gradient.

The effects of fluid velocity, thermal retardation factor, and thermal conductivity were investigated for a finite double-drained saturated layer with H = 1 m, v = -5.0 × 10^{-6} m/s, n = n_e = 0.5, λ_{sp} = 2.0 W/m-°C, ρ_{sp} = 2,650 kg/m^3, c_{sp} = 800 J/kg-°C, and initial uniform temperature of 25 °C as a benchmark case. Simulations were performed for four values of each parameter. Material properties were held constant except for the value for the parameter being investigated. Table 1 provides the parameter values for four cases, as shown in Figures 1, 2, and 3.

Table 1. Parameters for heat transfer analyses

Fig. 1	Fig. 2	Fig. 3
Discharge velocity v (m/s)	Volumetric heat capacity of solid particles $\rho_{sp}c_{sp}$ (J/m^3 -°C)	Thermal conductivity of solid particles λ_{sp} (W/m-°C)
0	100	0
2 × 10^{-6}	2.12 × 10^6	1.0
5 × 10^{-6}	4.24 × 10^6	2.5
1 × 10^{-5}	6.36 × 10^6	5.0

3.1. Effect of fluid velocity

Using material properties for the benchmark case and four values of fluid discharge velocity v, ranging from 0 to 1 × 10^{-5} m/s, soil temperature profiles at t = 24 hours are shown in Fig. 1. The profiles indicate that wavelength (i.e., the length between two contiguous wave peaks) is proportional to fluid discharge velocity. This occurs because heat energy can propagate faster with higher fluid velocity for the given elapsed time. In addition, the amplitude of temperature oscillations increases as v increases because the solid matrix retards the propagation of heat energy in the mobile pore fluid

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and higher fluid velocity allows for more heat energy carried by mobile pore fluid at the given elapsed time. Therefore, Fig. 1 reveals that the depth of the affected soil zone and the magnitude of diurnal temperature variations increase as fluid velocity increases.

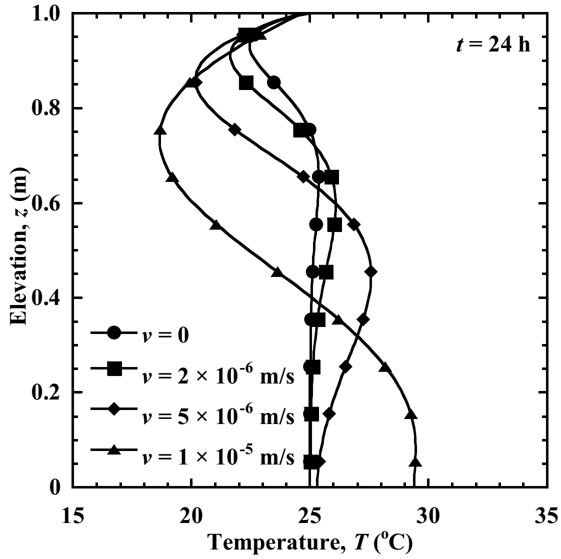

Fig. 1. Temperature profiles for four values of discharge velocity at t = 24 hours

3.2. Effect of retardation factor

Similar to Fig. 1, simulations were conducted using four values of thermal retardation factor, approximately ranging from 1.0 to 2.5, with profiles shown at t = 24 hours in Fig. 2. Thermal retardation factor is defined as [11]

$$R = 1 + \frac{\left(n_{sp} + n_i\right)\rho_s c_s}{n_e \rho_f c_f} \tag{4}$$

where R is a function of soil component porosities and volumetric heat capacities. Four values of R were obtained by changing the volumetric heat capacities of solid particles $\rho_{sp}c_{sp}$, as indicated in Table 1. Fig. 2 indicates that temperature oscillation amplitude increases as R decreases, which occurs because the solid matrix retards the oscillation of mobile fluid, and higher values of volumetric heat capacity of the solid matrix enhance the retardation effect. Fig. 2 also shows that wavelength is reduced with increasing thermal retardation.

3.3. Effect of thermal conductivity

Simulation results at t = 24 hours for four values of solid particle thermal conductivity, ranging from 0 to 5.0 W/m-°C are shown in Fig. 3. For the case λ_{sp} = 0, the thermal conductivity of fluid phase is also set equal to zero to show the temperature profiles in the absence of conduction. Fig. 3 indicates that higher thermal conductivity of solid particles increases the overall heat transfer rate and decreases the temperature oscillation amplitude. For sufficiently large thermal conductivity of the soil matrix, temperature oscillation is not observed because a high temperature gradient leads to fast heat transfer.

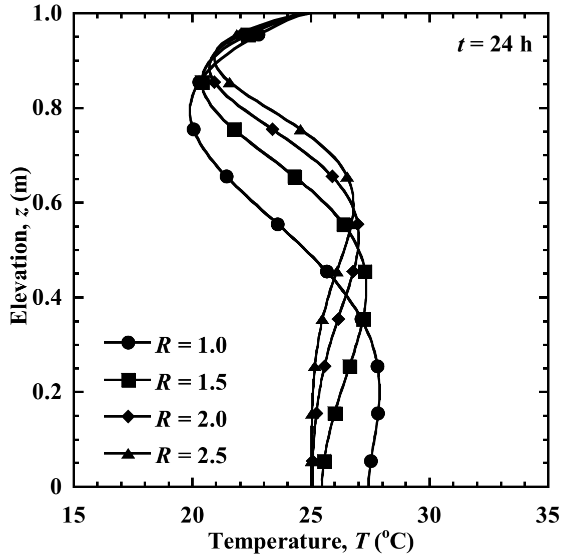

Fig. 2. Temperature profiles for four values of thermal retardation factor at t = 24 hours

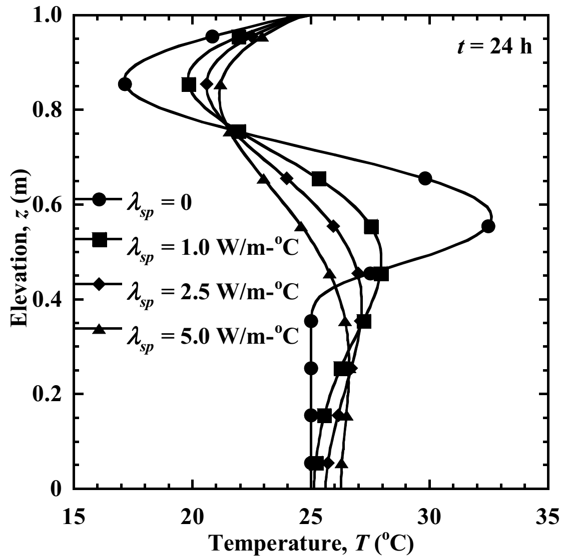

Fig. 3. Temperature profiles for four values of solid particle thermal conductivity at t = 24 hours

3.4. Effect of multiple soil layers and effective porosity

Effects of multiple soil layers and effective porosity on heat transfer behavior under a periodic temperature boundary condition were investigated for a double-drained stratum with H = 1 m, R_i = 3, v = -2.0 × 10^{-6} m/s, and a uniform initial temperature of 25 °C. Material properties for the fluid phase are ρ_f = 1,000 kg/m^3, λ_f = 0.59 W/m-°C, and c_f = 4186 J/ kg-°C. Other material properties are provided in Fig. 4, where each layer has an effective porosity ratio n_e / n = 0.5. Temperature profiles at four values of elapsed time are shown by dashed lines with open symbols in Fig. 5. Temperature at the top boundary follows the sinusoidal relationship given by Eq. (3). The temperature oscillation amplitude gradually decreases with depth to small values near the bottom of the stratum. In addition, the higher value of equivalent thermal conductivity in the silty sand layers produces a higher heat transfer rate than for the clay layers and refraction of temperature profiles at layer interfaces, which indicates that layer heterogeneity can affect the heat transfer behavior in saturated soil significantly.

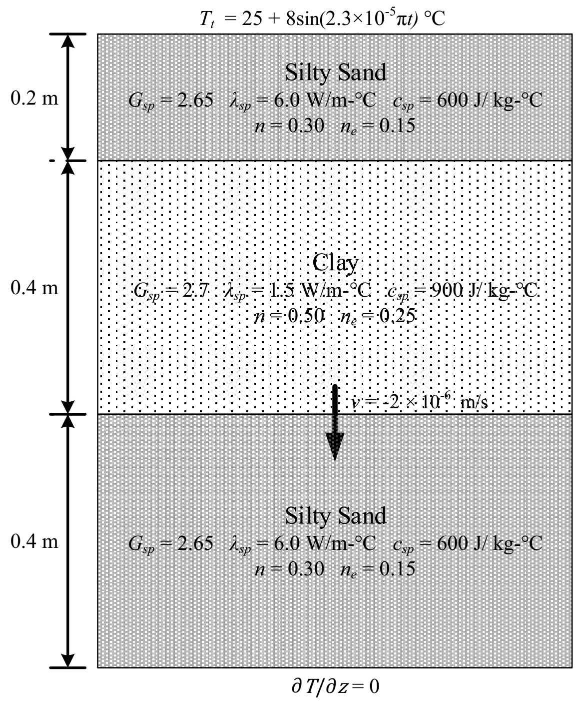

Fig. 4. Soil stratum containing three layers

The simulations were repeated for the same total porosity and no immobile pore fluid (i.e., n_e / n = 1). Temperature profiles for these simulations are shown by solid lines and closed symbols in Fig. 5 and indicate clear differences from dashed temperature profiles. The simulations with $n_e = n$ display sharper refraction at layer interfaces due to the higher contrast of λ_s between layers and the higher heat transfer rate at each layer due to the change of effective porosity. Constant fluid discharge velocity produces the same inflow heat energy by fluid advection for a given elapsed time. However, a larger value of n_e corresponds to less immobile water in the solid matrix. Knowing that solid particles have much higher thermal conductivity than the pore fluid, higher n_e yields a higher equivalent thermal conductivity for the soil.

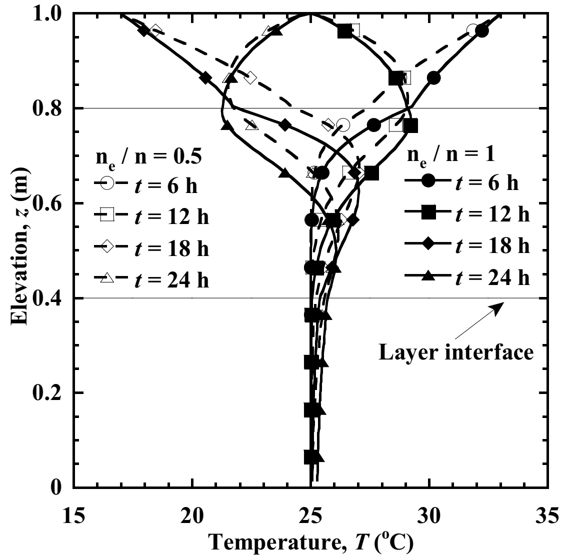

Fig. 5. Simulation results for layered soil stratum with effective porosity

4 Conclusion

One-dimensional analyses of heat transfer in layered saturated soil for a periodic temperature condition at the top boundary has been conducted using the numerical model HT1. A parametric study was performed to illustrate the significance of several variables on heat transfer behavior of soil under diurnal temperature changes, including fluid discharge velocity, thermal retardation factor, thermal conductivity of solid particles, effective porosity and layer heterogeneity. Simulation results indicate that increasing fluid velocity and decreasing retardation factor can increase the depth at which temperature oscillations from surface can influence soil temperature. In addition, decreasing fluid velocity, increasing retardation factor, and increasing thermal conductivity of solid particles can decrease the amplitude of temperature oscillations in the soil. Simulation results also indicate the significance of soil effective porosity and multiple soil layers on soil heat transfer behavior.

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