

Development of mathematical model for assessing social and economic state of region

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Abstract. Information systems that imply using mathematical model and enable estimating and forecasting an effect of various factors on the results of social and economic processes may become an effective instrument of comprehensive assessment of territorial development. The present paper considers the elaboration of a mathematical model intended for assessing a region's social and economic state. Elaboration of an approach to assessment of the dynamics of territorial development is considered in this work as well. Based on the statistical research, values of a number of economical, social and ecological indicators were obtained, which play a significant part in assessing the state of a region. The authors suggest an approach to defining an optimal development pathway in case when long-term plans are already formulated.

1 Introduction

Analysis, assessment and forecasting of the advancement of a state of a region as of a complex socio-economic system is an urgent task also being a highly difficult and labor-consuming one. This is associated primarily with the today's lack of unified or at least sufficiently elaborated approaches to the analysis of the problem under consideration and just as much with the need to have a sufficient amount of reliable information about the state of many indicators reflecting the socio-economic situation in a region [1]. Assessment of a socio-economic situation in a region is a component part of the problem under consideration [2-10].

The present work is aimed at developing a mathematical model for assessing socio-economic state of a region, as well as working out an approach to assessing the dynamics of the development of a territory.

The social and economic system of an administrative division is highly sophisticated, and therefore hard to model. The complexity of modeling such a system is associated with a great number of elements comprising it and processes running within it, the impossibility of detailed identification of internal dependences and influences of external factors, ambiguity in margins between phases of the system's development, short duration of processes at bifurcation points of a life cycle [11-15].

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We assume that a set of indicators (economic, social, environmental, etc.), which according to the expert group are considered significant in determining the state of a region, are to compare. We also assume that on the basis of statistical research, the values of these indicators for regions can be obtained. Suppose y_{ij} is values of the i -th indicator for the j -th region, $i = \overline{1, n}; j = \overline{1, m}$. The expert group can decide on assigning each of the regions to one of the two following sets:

A is a set of "prosperous" regions;

B is a set of "disadvantaged" regions.

Thus, a "dividing rule" can be formulated for assigning a certain region under study to a particular set.

2 Materials and Methods

Let us consider the possibility of making a dividing rule for assigning a certain region to a particular set.

For this purposes, we formulate an index space for indicators as follows:

$$V_{i\ m+1} = \frac{y_{m+1} - y_i^{\min}}{y_i^{\max} - y_i^{\min}}, i = \overline{1, n}, \quad (1)$$

when the growth of indicator y_i causes the improvement of the state of a region and growth of its grade:

$$V_{i\ m+1} = \frac{y_i^{\max} - y_{im+1}}{y_i^{\max} - y_i^{\min}}, i = \overline{1, n} \quad (2)$$

when growth of indicator y_i causes the deterioration of the state of a region and decrease in its grade, where:

$$y_i^{\max} = \max_j \{y_{ij}\}, y_i^{\min} = \min_j \{y_{ij}\}, j = \overline{1, m}, i = \overline{1, n}$$

$y_{i\ m+1}$ is for values of indicators for a region under study.

The obtained values of the indices are already normalized, which means that their growth leads only to the increase in a region's grade and $0 \leq V_{m+1} \leq 1, i = \overline{1, n}$. With respect to the initial data on the «reference» regions, these indices are equal to:

$$V_{ij} = \frac{y_{ij} - y_i^{\min}}{y_i^{\max} - y_i^{\min}}, i = \overline{1, n} \quad (3)$$

for case (1) and case (2):

$$V_{ij} = \frac{y_i^{\max} - y_{ij}}{y_i^{\max} - y_i^{\min}}, i = \overline{1, n}. \quad (4)$$

Thus, a set of all normalized vectors representing m regions belongs to hypercube

$K \in R^n$:

$$K = \{V = V_1, \dots, V_n | 0 \leq V_i \leq 1, i = \overline{1, n}\}. \quad (5)$$

in which vectors (or points) $V_j = (V_{1j}, \dots, V_{nj})$, $j = \overline{1, m}$ are somehow distributed, representing regions that have been already studied. Taking into consideration that these regions have already been assigned to set A or set B, it is possible to pose a problem of plotting a dividing hypersurface S so that each of pairs (V_k, V_e) (where $V_k \in A$ and $V_e \in B$) is divided by this surface (Fig. 1).

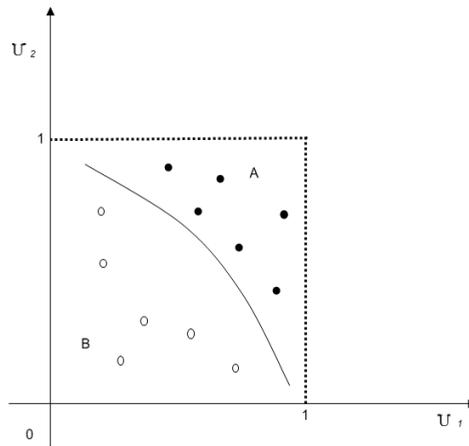


Fig. 1. Visual representation of the dividing surface for sets A and B .

When more than one dividing surface can be plotted, then one of the criteria ensuring neutrality of surface S is formed. As one of such criteria, the following constraint (arbitrary zero potential) is used:

$$\sum_{V_i \in B} \frac{1}{d_i} = \sum_{V_j \in A} \frac{1}{d_j} \quad (6)$$

where d_i is the Euclidean distance from point V_i to surface S . Thus, an equation of surface $S(a, V) = 0$ is determined by a certain category of surfaces (linear, quadratic, etc.) and a vector of parameters $a = (a_1, \dots, a_r)$. The vector's dimension r is determined by the structure of the function; their values are determined from constraint (6).

The surface plotted this way should have certain characteristics. When changing (decreasing) one of the indices while maintaining the values of the rest of them, the grade of the state only decreases, therefore it is possible to write down a constraint for arbitrary points $V_p, V_q \in B$:

$$V = \lambda V_p + (1 - \lambda) V_q, 0 \leq \lambda \leq 1 \Rightarrow V \in B,$$

which defines S as a convex hypersurface that divides hypercube (5) into two intersecting subsets: K_A and K_B so that $K_A \cap K_B = \emptyset, K_A \cup K_B = K$ and $\forall V_i \in A \Rightarrow V_i \in K_A, \forall V_i \in B \Rightarrow V_i \in K_B, i = \overline{1, m}$.

The equation $S(a, V)$ is written in such a way that $S(a, V_i) < 0$ for $\forall V_i \in B$. Then

$S(a, V_i) > 0$ for $\forall V_i \in A$. With these settings, for an arbitrary vector (point) $V^0 \in K$, the following constraints are true:

$$S(a, V^0) > 0 \Rightarrow V^0 \in K_A \quad (7)$$

$$S(a, V^0) < 0 \Rightarrow V^0 \in K_B \quad (8)$$

Constraints (7) and (8) are decisive when assigning any vector. Therefore, for the vector of indices of the region under study (1) - (2) we have:

$$S(a, V_{m+1}) > 0 \Rightarrow V_{m+1} \in K_A - \text{the state of a region is satisfactory;}$$

$$S(a, V_{m+1}) < 0 \Rightarrow V_{m+1} \in K_B - \text{the state of a region is unsatisfactory.}$$

Suppose $V_{m+1} \in K_B$. Then the issue of directions for the development of a region becomes relevant, as well as the issue of the increase in values of indices of a region.

3 Results

As a solution to this problem, the authors suggest the following approach to finding a development pathway for a region in order to achieve the dividing surface $S(a, V)$.

We assume that unit costs for the relative increase in each index $V_{im+1} - q_i(V), i = \overline{1, n}$ are given. Here we should take into account the fact that smaller values V_i entail small values q_i , while with an increase in V_i values q_i also grow, as well as the cost rate, which corresponds to constraints:

$$\frac{\partial q_i}{\partial V_j} > 0, \frac{\partial^2 q_i}{\partial V_j \partial V_k} > 0; i, j, k = \overline{1, n}.$$

The following problem illustrated in Figure 2 is to consider. A starting point V_{m+1} is given that corresponds to the state of the region under study. It is required to find the equation of the shortest line weighted with unit costs $q_i(V), i = \overline{1, n}$ that connects point V_{m+1} and surface $S(a, V)$. The parametric definition of the required curve in n -dimensional space is to be used: $V_i = V_i(t), i = \overline{1, n}$

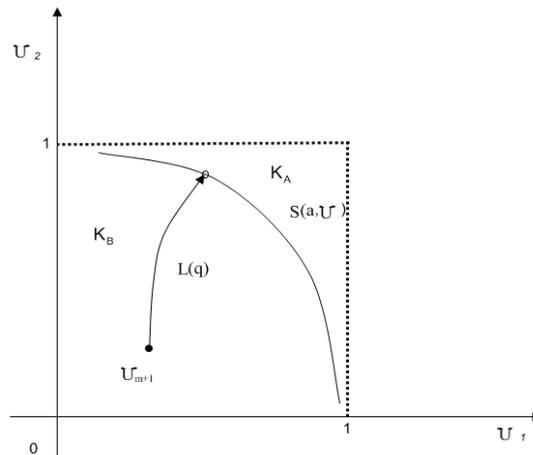


Fig. 2. Search for the optimal pathway for the development of the region.

As a criterion for minimization, we use the composite function corresponding to the weighted line length $L(q)$:

$$Z = \int_{t_0}^{t_f} dL(q) = \int_{t_0}^{t_f} \sqrt{\sum_{i=1}^n \left(\frac{dV_i}{dt} q_i(V) \right)^2} dt \rightarrow \min_V \quad (9)$$

where

$$\begin{aligned} V_i(t_0) &= V_{im+1}, \quad i = \overline{1, n}, \quad V_{m+1} = (V_1(t_0), \dots, V_n(t_0)), \\ S(a, V(t_f)) &= 0, \\ V(t_f) &= (V_1(t_f), \dots, V_n(t_f)), \\ G = V \cdot S(a, V), \quad \frac{\partial G}{\partial V_i(t_f)} + \frac{\partial}{\partial \dot{V}_i} \sqrt{\sum_{i=1}^n \left(q_i(V) \dot{V}_i \right)^2} \Big|_{t_f} &= 0 \quad (10) \\ \dot{V}_i &= \frac{dV_i}{dt}, \quad i = \overline{1, n}, \end{aligned}$$

$$\frac{\partial G}{\partial t_f} - \sum_{j=1}^n \frac{\partial}{\partial \dot{V}_j} \sqrt{\sum_{i=1}^n \left(q_i(V) \dot{V}_i \right)^2} \Big|_{t_f} \dot{V}_j + \sqrt{\sum_{i=1}^n \left(q_i(V) \dot{V}_i \right)^2} \Big|_{t_f} = 0$$

Constraints (9) correspond to the required behavior of the line at the start and end points. Following resulting trajectory $V(t) = (V_1(t), \dots, V_n(t))$, a region will reach the values of indicators that allow classifying it as a prosperous one with minimum costs. When there is a task of transferring a region to a planned state being fulfilled, then the task can be formulated (Fig. 3) as follows:

$$Z = \int_{t_0}^{t_f} dL(q) = \int_{t_0}^{t_f} \sqrt{\sum_{i=1}^n \left(\frac{dV_i}{dt} \cdot q_i(V) \right)^2} dt \rightarrow \min_V, \quad (11)$$

where $V_i(t_0) = V_{im+1}, \quad i = \overline{1, n}, \quad V_{m+1} = (V_1(t_0), \dots, V_n(t_0)),$

$$V_i(t_f) = V_i^f, \quad i = \overline{1, n}, \quad V^f = (V_1^f, \dots, V_n^f),$$

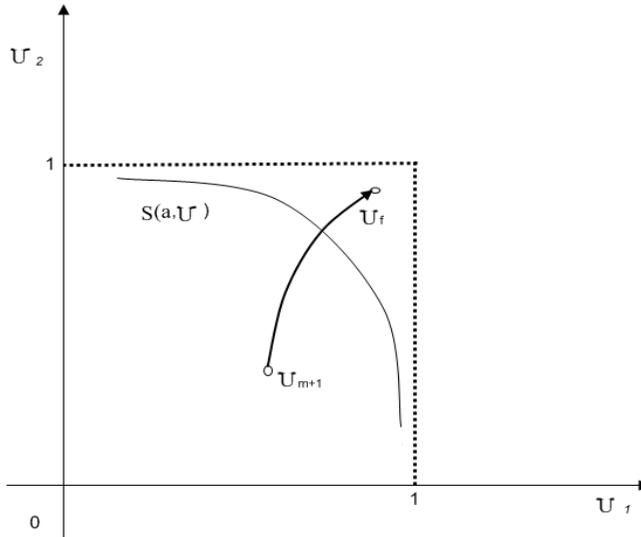


Fig. 3. Transferring a region from the initial state V_{m+1} to the final state V_{m+1} .

When the solution of problem (8)-(9) is near the separating surface, i.e. a region's state is close to satisfactory, it is possible to set the unit costs q_i constant, which significantly reduces the complexity of the problem.

When following the optimal pathway moving to the final state, the components of a region's development vector ΔV_i , $i = \overline{1, n}$ can be obtained from the equation of an optimal pathway:

$$\Delta V_i = \frac{dV_i}{dt} \Delta t, \quad i = \overline{1, n}, \quad \Delta V = (\Delta V_1, \dots, \Delta V_n) \quad (12)$$

However, the process of implementing the transition of a region from the initial state V_{m+1} to the final state V_f along the resulting trajectory can be uneven in time, since the accumulation and implementation of various resources have different time lags. There are both fast and slow processes running in the socio-economic system, which entails complex development pathways for the systems under consideration.

Nevertheless, the considered approach allows assessing the dynamics of the development of a region over certain periods. For these purposes, the state of a region within two consecutive periods V_T and V_{T+1} are to consider. Suppose $V_T \in K_B$, $V_{T+1} \in K_B$. Then the trends of development of a region can be assessed on the basis of the solution of problem (8)-(9) for V_T and V_{T+1} . Substituting optimal solutions $V_T^f(t_0, t_f)$ and $V_{T+1}^f(t_0, t_f)$ in (8), we get cost values L_T and L_{T+1} required to achieve the satisfactory state of a region.

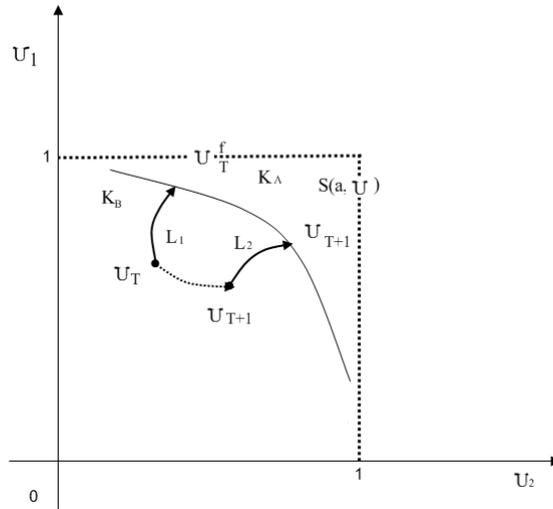


Fig. 4. Assessment of dynamics of regional development.

These values can be perceived as a distance for a region to its satisfactory state. Then, if $L_{T+1} - L_T < 0$, then the dynamics of development is positive. Otherwise it can be said that situation in a region becomes worse.

4 Conclusions

The authors elaborated a mathematical model for assessing social and economic state of a region and suggested approaches to assessing the dynamics of regional development. The paper also suggests an approach to finding an optimal development pathway in case when long-term plans are formulated.

The described model and approaches to assessing the socio-economic state of a region and dynamics of its development should be checked in terms of theory. The adequacy of the models and principles proposed by the work should be assessed using available statistical, ecological, and socio-economic indicators.

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