

Hierarchical management model of heat supply to consumers

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Abstract. The article proposes a hierarchical management model for heat supply to consumers based on the method of bilevel programming. An organizational model of heat supply to consumers in the form of a Unified Heat Supply Organization in a two-tier control system is considered, when the regulators (regional tariff service) manages tariffs for consumers, and heat sources cover the given demand from consumers from the condition of obtaining maximum profit. With the help of bilevel approach, the technical and economic indicators of the district heating are calculated. The proposed mathematical model makes it possible to take into account the technical and economic characteristics of heat sources and heat networks, the interests of the participants in the process of heat supply to consumers and to determine the optimal conditions for controlling the functioning of district heating systems.

Keywords: district heating system; mathematical modeling; optimization; optimal operating; theory of hydraulic circuits; bi-level programming.

1 Introduction

District heating system (DHS) in Russian, which accounts for 72% of heat supply, is the main type of heat supply to consumers. The emergence of many owners in this sector of the economy, associated with the process of energy liberalization in the early 90s of the XX century, led to the formation of new economic relations among producers and consumers of heat energy and the creation of a heat market.

Currently more than 50 000 heat markets are operating in Russia. Every market has its features and specificity. This is determined by both a variety of heat sources taking part in the market and by unique configuration of heat networks. In simplified form the markets can be divided into four main categories:

- Extra-large markets - in 15 cities with production and consumption of above 40 million GJ/year;
- Large markets - in 44 cities with consumption from 8 to 40 million GJ/year;

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- Medium-sized markets - in hundreds of cities with consumption from 2 to 8 million GJ/year;

- Small markets - in more than 40 000 settlements that consume less than 2 million GJ/year of heat produced by centralized sources.

Modern DHS, like other energy systems (electric power, oil, gas, etc.) have a hierarchical (vertically integrated) system of organization of operation and development management, which can be represented as a bi-level model [1]. The first (upper) level of the bi-level model of the heat market is represented by the regional tariff service, whose duties include regulation of the tariff for heat energy for consumers, and the second (lower) level is the DHS, which is part of the heat supply company (HSC), which performs the functions of heat supply to consumers.

Such type of heat market is most prevalent in countries with developed of DHS is a natural monopoly with tariff regulation on the heat energy for consumers. They are found in some countries European Union such as Poland [2], Lithuania [3], Latvia [3], Estonia [4], and in Russia [5], China [6] and other country.

The main idea of such an approach to constructing a scheme for management the DHS is to single out the corresponding subsystems into aggregated participants, which makes it possible to specify the goals that each of them pursues. The relationship between the participants of heat supply is as follows. The HSC, based on consumer demand for heat energy, generates heat energy and sells it to consumers from the condition that heat sources produce such amount of heat energy that would maximize their profits, while collectively they should cover the amount set by consumers demand for heat energy, taking into account the physico-technical limitations and optimal flows in the heat network. The regional tariff service, protecting the rights of consumers, sets a level of heat tariff that, on the one hand, would stimulate heat sources to satisfy a given demand from consumers, and on the other hand, would allow them to get maximum profit from the sale of heat energy at observance of the optimum modes in heat networks.

Modeling of such a system is carried out using bi-level programming. The transition to the one-level optimization problem is carried out by replacing the extremal problem of the lower level on their optimality conditions [7].

2 Methods

DHS can be described by a hydraulic network with m nodes and n branches, represented by a directed graph $G = (J, I)$, where $J = J_{HS} \cup J_{CON} \cup J_0$ – a set of nodes, consisting of a sets of heat sources, consumers and branching nodes without heat sources and consumers; I – a set of branches. According to [8], the optimal flow distribution in hydraulic network can be represented by the following set of equations:

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{Q} \quad (1)$$

$$\overline{\mathbf{A}}^T \cdot \overline{\mathbf{P}} = \mathbf{y} \quad (2)$$

$$\mathbf{y} + \mathbf{H} = \mathbf{S} \cdot \mathbf{X} \cdot \mathbf{x} \quad (3)$$

where $\mathbf{A} = \{a_{ij}\}$ - $(m-1) \times n$ incidence matrix for linearly independent nodes, which is obtained on the basis of complete matrix by deleting any of its rows; $\mathbf{x} = (x_1, \dots, x_n)^T$ - vector of flow rates, t/h; $\mathbf{Q} = (Q_1, \dots, Q_{m-1})^T$ - vector nodal sources or sinks, t/h; $\overline{\mathbf{A}}^T$ - transposed complete matrix $\overline{\mathbf{A}}$; $\mathbf{P} = (P_1, \dots, P_m)^T$ - vector of pressures at nodes, Pa; $\mathbf{y} = (y_1, \dots, y_n)^T$ - vector of differences between the pressures at branch nodes, Pa; $\mathbf{H} = (H_1, \dots, H_n)^T$ - vector of

heads, Pa; $\mathbf{S} = \text{diag}(s_1, \dots, s_n)$ and $\mathbf{X} = \text{diag}(|x_1|, \dots, |x_n|)$ - diagonal matrices of hydraulic resistances and flow rates, respectively.

Part of nodes in the hydraulic network is represent of heat sources. The maximum and minimum level of heat production at heat sources is expressed through Q_{j_min} and Q_{j_max} , and is written in the form of system constraints, GJ/h:

$$Q_{j_min} \leq Q_j \leq Q_{j_max}, j \in J_{HS}. \tag{4}$$

Denote by $Z_j(Q_j)$ the costs of heat production by the j -th heat source (EUR). In the market conditions behavior of heat source (lower level) is dictated by its intention to maximize profit $P_j(Q_j)$. Let $c_j, j \in J_{HS}$ be the price per heat unit (EUR/GJ), then the problem of the lower level is written as follows, EUR:

$$P_j(Q_j) = c_j \cdot Q_j - Z_j(Q_j) \rightarrow \max, j \in J_{HS}, \tag{5}$$

$$Q_{j_min} \leq Q_j \leq Q_{j_max}, j \in J_{HS}. \tag{6}$$

Costs in the heat networks is determined by the well-known dependence in [9], EUR:

$$Z^{HN}(\mathbf{x}) = F_1 + F_2 \cdot \sum_{i \in I} x_{it}^2 \cdot |x_{it}| \cdot s_i, \mathbf{x} \in \mathbb{R}^n. \tag{7}$$

where F_1 - semi-fixed costs of heat network, EUR; F_2 - coefficient at semi-variable costs of heat network; \mathbb{R} - a set of real number.

Consumers of heat energy consist of two sets. The first set J_{CON}^{HH} is household consumers with fixed loads $Q_j^{HH} = \text{const}, j \in J_{CON}^{HH}$, GJ/h. The second set J_{CON}^1 is industrial consumers. The behavior of the prosumer is described by the demand function, which was obtained on the basis of real calculations and can be represented as a linear relationship in [10], GJ/h:

$$Q_j^1 = w_j - v_j \cdot c_j^1, j \in J_{CON}^1. \tag{8}$$

where $w_j > 0, v_j > 0$ - coefficients obtained from the approximation of the factual data on the heat volume purchased by an prosumer, depending on price; c_j^1 - purchase price, EUR/GJ.

To simulate the upper level, it is necessary to formalize of the regulator criterion. As noted earlier, the duties of the regulator include the regulation of tariff for household consumers (c^{HH} , EUR/GJ). Next, we will assume that the regulator, defending the interests of household consumers, seeks to determine the minimum tariff for them. Consider the economic balance of DHS:

$$\sum_{j \in J_{HS}} c_j \cdot Q_j + Z^{HN}(\mathbf{x}) = c^{HH} \cdot \sum_{j \in J_{CON}^{HH}} Q_j^{HH} + \sum_{j \in J_{CON}^1} c_j^1 \cdot Q_j^1. \tag{9}$$

Let us express from (9) the tariff of heat energy for household consumers:

$$c^{HH} = \frac{\sum_{j \in J_{HS}} c_j \cdot Q_j + Z^{HN}(\mathbf{x}) - \sum_{j \in J_{CON}^1} c_j^1 \cdot Q_j^1}{\sum_{j \in J_{CON}^{HH}} Q_j^{HH}}. \tag{10}$$

Thus, the mathematical model of the district heating system management with prosumers will be written in the following form:

$$c^{HH} = \frac{\sum_{j \in J_{HS}} c_j \cdot Q_j + Z^{HN}(x) - \sum_{j \in J_{CON}^I} c_j^I \cdot Q_j^I}{\sum_{j \in J_{CON}^{HH}} Q_j} \rightarrow \min, \tag{11}$$

subject to (1)-(3), (7), (8) and

$$Q_j = \arg \max \{P_j(Q_j) : Q_{j_min} \leq Q_j \leq Q_{j_max}\}, j \in J_{HS}. \tag{12}$$

Unlike the flow distribution problem with lumped-parameters [1, 7], in this case the volumes production of heat energy will be variable.

The costs of heat production by a heat source have the form of a squared relationship. They were obtained by approximating actual heat supply data depending on the heat source costs by the method of least squares [11, 12], EUR:

$$Z_j(Q_j) = \alpha_j \cdot Q_j^2 + \beta_j \cdot Q_j + \gamma_j, \alpha_j > 0, \beta_j > 0, \gamma_j > 0, j \in J_{HS}. \tag{13}$$

Due to the fact that the coefficients $\alpha_j, \beta_j, \gamma_j$ are positive, the cost function is a strongly convex, monotonically increasing function that takes positive values at $Q_j \geq 0, j \in J_{HS}$.

Thus, the problem corresponding to expressions (5)-(6) will have the form:

$$-\alpha_j \cdot Q_j^2 + (c_j - \beta_j) \cdot Q_j - \gamma_j \rightarrow \max, j \in J_{HS} \tag{14}$$

$$Q_{j_min} \leq Q_j \leq Q_{j_max}, j \in J_{HS} \tag{15}$$

Dependence of optimal solution to problem of lower level (14)-(15) on heat price is determined by the following relations:

$$Q_j^* = \begin{cases} Q_{j_min}, & c_j < 2 \cdot \alpha_j \cdot Q_{j_min} + \beta_j \\ \frac{c_j - \beta_j}{2\alpha_j}, & 2 \cdot \alpha_j \cdot Q_{j_min} + \beta_j \leq c_j \leq 2 \cdot \alpha_j \cdot Q_{j_max} + \beta_j \\ Q_{j_max}, & c_j > 2 \cdot \alpha_j \cdot Q_{j_max} + \beta_j \end{cases} \tag{16}$$

From the set of equations (16) it follows that if the heat prices vary within the values corresponding to the expression

$$2 \cdot \alpha_j \cdot Q_{j_min} + \beta_j \leq c_j \leq 2 \cdot \alpha_j \cdot Q_{j_max} + \beta_j. \tag{17}$$

the volume of the produced heat providing the maximal profit depends on the price linearly, GJ/h:

$$Q_j = \frac{c_j - \beta_j}{2 \cdot \alpha_j}, j \in J_{HS} \tag{18}$$

In this case the problem optimization of tariff for household consumers will be written in the following form:

$$c^{HH} = \frac{\sum_{j \in J_{HS}} c_j \cdot Q_j + Z^{HN}(x) - \sum_{j \in J_{CON}^I} c_j^I \cdot Q_j^I}{\sum_{j \in J_{CON}^{HH}} Q_j} \rightarrow \min \tag{25}$$

subject to subject to (1)-(3), (7), (8) and (18).

3 Case study

The developed mathematical model was tested for the district heating system with two heat sources (Q_1 , (HS-1), Q_2 (HS-2)), two household consumers ($Q^{HH(1)}$, $Q^{HH(2)}$) and one industrial consumer (Q^I). The design scheme of district heating system is shown in Fig. 1.

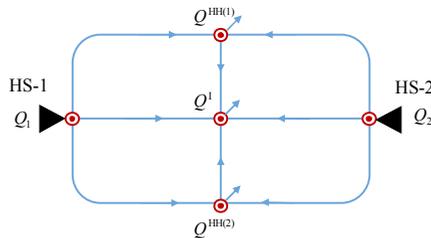


Fig. 1. Design scheme of district heating system

The simulation of the district heating system depicted in Fig. 1 was carried out in GAMS/Conopt computing environments. Table 1 presents the technical and economic indices of DHS obtained in the process of performing optimization calculations.

Table 1. Technical and economic indices

Calculated indices	
Heat production volume of HS-1, GJ/h	3 882
Heat production volume of HS-2, GJ/h	3 493
Heat production costs of HS-1, EUR	6 626
Heat production costs of HS-2, EUR	4 754
Heat energy price of HS-1, EUR/GJ	8.8
Heat energy price of HS-2, EUR/GJ	7.8
Profit of HS-1, EUR	1 503
Profit of HS-2, EUR	1 741
Heat energy price for household consumer, EUR/GJ	18.5
Heat energy price for industrial consumers, EUR/GJ	24.2

From table 1 it can be seen that, heat energy sources produced a total of 7375 GJ/h of heat energy, of which 52% comes from HS-1 and 48% from HS-2. Prices for heat energy

production were 8.8 EUR/GJ and 7.8 EUR/GJ for HS-1 and HS- 2, respectively. Such prices make it possible to profit from the sale of heat energy in the amount of 1503 EUR for the HS-1 and 1741 EUR for the HS-2. The proposed mathematical formulation of the problem of finding the optimal tariff for household consumer allowed determining its minimum value - 18.5 EUR/GJ. In turn, the industrial consumer purchases heat energy from the district heating system at a price of 24.2 EUR/GJ.

4 Conclusion

The paper considers the organizational model of heat supply to consumers in the form of a heat supply company in bi-level system, when the regulator (regional tariff service) manages tariffs for consumers, and sources of heat energy meet specified demand on the part of consumers from the condition for obtaining maximum profit. A criterion for optimizing the regional tariff service is proposed. Using bi-level approach, technical and economic indices of the DHS with prosumers are calculated, ensuring the rational organization of the heating energy market. The proposed mathematical bi-level model to the greatest extent reflects the conditions emerging in the local market for thermal energy. This model reasonably takes into account the established "rules of conduct" in the heat market, as well as the technical and economic constraints of the system under consideration.

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