

# Distribution of the components of the building mixture in the presence of secondary raw materials during rotary mixing

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**Abstract.** The purpose of this study is a stochastic description of the distribution of solid dispersed components, including those from secondary raw materials, according to the characteristic angle of scattering  $\Theta_{ij}$  when receiving a construction mixture at the first stage of operation of the rotary apparatus. Two stages of the formation of rarefied flows are assumed: when scattering particles of components by elastic blades of a rotating drum and when interacting with the baffle surface. Modeling method this is energy method of Klimontovich Yu.L. The analysis of the efficiency of the first stage (rotary mixing) is carried out based on the obtained distribution functions of the number of particles of bulk components over the scattering angle, taking into account their physical and mechanical properties and a variety of design and operating parameters of the apparatus. The bulk of the particles of the mixed components are scattered at the initial angles of rotation of the mixing drum, when the deformation of the elastic blades is most significant. This is accompanied by the characteristic first bursts of the obtained distribution curves ( $\Theta_{ij} < 0.1$  rad) for the number of particles of the tested bulk materials at the given ranges of parameters.

## 1 Introduction

The constant expansion of the areas of application of dry building mixtures for the needs of the construction industry poses urgent tasks for the designers of equipment for the preliminary processing of bulk materials. On the one hand, there is a requirement to preserve the available natural and energy resources, and, on the other, there is a need to increase the intensity of production of high-quality construction products. Often, the solution to these problems simultaneously leads to a forced compromise in the methods of achieving the required result. The use of materials from the category of secondary raw materials for the preparation of dry building mixtures partially allows solving the first of the listed problems, leaving open questions related to the expenditure of funds for the processing of these products. In construction, industrial waste is actively used [1-3], for example, sand and crushed stone of various chemical compositions from ash and slag after burning solid fuel at thermal power plants (TPP) [4,5]. However, there is a heterogeneity of these slag materials with the instability of their physical and mechanical properties. The expediency of combining several technological operations when performing the specified processing of solid dispersed materials is explained by the possibility of intensifying the flow of several technological processes at the working sites of one device. The need to improve the existing criteria for assessing the quality of dry building mixtures and the development of new relevant criteria leads to an urgent

problem of the development of the theoretical foundations of the process of mixing solid dispersed media in devices of specific types [6]. Obtaining a dry construction mixture in rarefied flows from particles of bulk components is a fairly effective means of combating the unwanted effects of agglomeration and segregation [7] accompanying this technological operation. These circumstances can be considered as advantages in comparison with another mixing method performed in dense layers of bulk components.

The purpose of this study is a stochastic description of the distribution of solid dispersed components, including those from secondary raw materials, according to the characteristic angle of spreading  $\Theta_{ij}$  when receiving a construction mixture at the first stage of operation of the rotary apparatus. Two stages of the formation of rarefied flows are assumed: when scattering particles of components by elastic blades of a rotating drum and when interacting with the baffle surface [8]. In this part of the study, it is of interest to analyze the effectiveness of the first stage (rotary mixing) based on the obtained distribution functions of the number of particles of bulk components over the spreading angle, taking into account their physical and mechanical properties and a variety of design and operating parameters of the apparatus.

Modeling method this is energy method of Klimontovich Yu. L. [9,10] within the framework of the stochastic approach in the approximation of the energetic closeness of macrosystems for each mixed component in their equilibrium state. The application of this approach

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is due to the probabilistic nature of the behavior of material particles in the formed rarefied flows. Note that the classical method of A.A. Markov [11-14] has various uses for describing the process of mixing dispersed media. In particular, the following models are known: birth-death [12], kinetic [13], diffusion [14] and others [15-19]. A more detailed analysis of existing approaches to solving the problem of modeling the mixing of bulk media is contained in [20]. Successful approbation of the energy method [9,10] is observed in the problems of modeling the states of several types of macrosystems, for example, drops when describing liquid dispersion [21], cavitation bubbles when throttling fluid flows [22], particles of rarefied flows when mixing them [18,19] in the working volumes of drum-tape [23,24] and gravity [24-26] devices. These models became the basis for calculating the performance of mixers [27] and assessing the quality of the mixture at various stages of the mixing process [28,29].

Previously, the authors have proposed several stochastic models for mixing bulk components in rarefied flows in interaction with brush elements [20,23-26] and elastic blades [20], fixed on rotating drums. The results of these models allow us to put forward a working hypothesis about the factors that have a significant impact on the studied technological process.

## 2 To the analysis of the kinematic features of the motion of elastic blades

As already noted, in addition to mixing bulk components in dense layers, another method of obtaining a high-quality mixture - in rarefied flows - has proved itself quite successfully. The formation of these streams of solid dispersed media at the first stage of the mixer [8] is carried out using rotary devices located above the conveyor belt. Continuous operation of the rotary device is carried out with vertical loading of each component () from the metering hopper onto a moving belt that feeds layers of mixed materials into the gap with a rotating drum. After interaction with deformed blades, particles of bulk materials are scattered in the form of conical "torches".

For further modeling of the sought distribution functions of the mixture components, we will conduct a brief analysis of the main kinematic features of the motion of elastic rectangular blades with a length  $l_b$ .

For further modeling of the sought distribution functions of the mixture components, the main kinematic features of the motion of elastic rectangular blades with a length  $l_b$ . The attachment  $n_0$  of the blades to the cylindrical surface of the rotating drum is performed perpendicular to its radius  $r_b$ . In this case,  $k$  there are rows of blades installed with an angular displacement  $\beta$  in projection onto the transverse plane of the mixing drum. The row spacing is  $h_b$ . Let the angular speed of rotation of the drum be  $\omega$ , clearance height between drum and conveyor belt  $h_0$ , number of the deformable

elastic blade in the plane of the cross-section of the drum  $j = \overline{1, n_b}$ .

For three successive rows of elastic blades mounted on a drum with angular displacement  $\beta$ , consider in this cross-section the movement of the most deformed blades ( $n_b = 3$ ) when turning the drum at an angle less  $\pi$  rad. Of particular interest is the description on the indicated transverse plane of the position of the ends of the projections of the deformable blades (the point  $U_{ij}$ ,  $i = 1, 2, j = 1, 2, 3$ ). Introduce in the transverse plane the polar coordinate system  $(r_{ij}, \theta_{ij})$  centered at point  $K(x_K, y_K)$ . Coordinates of the point  $K$  relative to the fixed system  $Oxy$ , associated with the axis of rotation of the drum, are  $x_K = r_b \sin \varphi_0, y_K = r_b \cos \varphi_0$ , where  $\varphi_0 = 2\pi / n_0$ . An approximation is adopted about the belonging of the point  $U_{ij}$  of the Archimedes spiral  $r_{A_j}(\theta_{ij})$ . Then the equation  $r_{U_{ij}}(\theta_{ij})$  for the position of the ends of the projections of the deformable blades  $U_{ij}$  has the form

$$r_{U_{ij}}(\theta_{ij}) = r_{A_j}(\theta_{ij}) \cos[3\psi_{ij}(\theta_{ij})/2] + \\ + \{2r_{A_j}(\theta_{ij}) \cos[3\psi_{ij}(\theta_{ij})/2]\}^2 - \\ - 4\{r_{A_j}(\theta_{ij})\}^2 - r_b^2)^{1/2} / 2 \quad (1)$$

where  $\psi_{ij}(\theta_{ij}) = \arctg[b / r_{A_j}(\theta_{ij})]$  is the angle between the vector  $\mathbf{r}_{A_j}(\theta_{U_{ij}})$  and the polar normal at a point  $U_{ij}$  to the tangent for the Archimedes spiral. The equation  $r_{A_j}(\theta_{ij})$  has the form

$$r_{A_j}(\theta_{ij}) = a + b\theta_{ij} \quad (2)$$

where the following notation is accepted

$$a = r_b(1 - \cos \varphi_0) + h_0$$

$$b = \{[r_b^2(1 + \cos \varphi_0)^2 + (r_b + l_b)^2]^{1/2} - a\} / \gamma$$

$$\gamma = \pi + \arctg\{(r_b + l_b) / [r_b(1 + \cos \varphi_0)]\}$$

## 3 Materials and methods

Suppose that at the density of substances of the mixed bulk materials  $\rho_{Ti}$ ,  $i = 1, 2$ , the mass of spherical particles  $m_i$  is determined by the average value of the diameter  $D_{Ti} = n_v^{-1} \sum_{v=1}^{n_v} D_{Tiv}$  over the corresponding. We use the stochastic approach in the formalism of the energy method of Klimontovich Yu.L. [9,10]. It is believed that after the simultaneous dropping from the deformed elastic blades ( $j = 1, 2, 3$ ), the particles of each component ( $i = 1, 2$ ) form energetically closed macrosystems. The realization of such states for each macrosystem is justified by smoothing out small-scale

fluctuations in the form of collisions of particles of one component moving in the formed rarefied flows. According to the indicated approximation, the random process of mixing bulk components refers to the Markov process as a homogeneous, continuous, stationary Gaussian process. In this case, the state of each macrosystem of granular components obeys the solution of the kinetic Fokker-Planck equation [9,10] in a certain phase space. In particular, let the phase volume element have the form

$$d\Gamma_{ij} = dV_{xij}dV_{yij} = -\omega^2 r_{ij}^2 dr_{ij} d\theta_{ij} \quad (3)$$

where  $V_{xij}, V_{yij}$  are velocity vector components  $\mathbf{V}_{C_{ij}}$  for the center of mass particle component  $i=1,2$  when interacting with an elastic blade  $j=1,2,3$  previously selected Cartesian coordinate system  $Oxy$  on the cross-sectional plane of the rotating drum. The direction of the  $Ox$  axis corresponds to the direction of the vertical movement of bulk materials from the metering hopper perpendicular to the conveyor belt. The choice of the axis direction  $Oy$  is consistent with the direction of movement of the conveyor. In the specified polar coordinate system  $(r_{ij}, \theta_{ij})$  centered at the point  $K(x_K, y_K)$  taking into account expressions (1), (2), the following approximation is adopted for the speed of movement of the ends of the projections of deformable blades  $U_{ij}$ , of the mixed components loaded with particles

$$V_{r_{0ij}}(r_{U_{ij}}(\theta_{ij}), \theta_{ij}) = \omega r_{U_{ij}}(\theta_{ij}) / \cos \psi_{ij}(\theta_{ij}) \quad (4)$$

According to (3), the number of particles of the component  $i=1,2$  in the selected elementary phase volume  $d\Gamma_{ij}$  by the form of the stationary solution of the kinetic Fokker-Planck equation [9,10] is given by an expression in the form

$$dN_{ij} = A_{ij} \exp(-E_{ij} / E_{0ij}) d\Gamma_{ij} \quad (5)$$

where  $A_{ij}$  - normalization constant,  $E_{ij}$  - the energy of stochastic motion of the component particle  $i=1,2$  when spreading with deformed elastic blades  $j=1,2,3$ ,  $E_{0ij}$  - value  $E_{ij}$  at the moment of stochastization of the macrosystem. The parameter  $E_{0ij}$  is the main energy characteristic of the mixing process under study and plays a decisive role in the formation of differential distribution functions of the number of particles of the component  $i=1,2$  over the angle of their scattering  $\theta_{ij}$  when interacting with deformed elastic blades  $j=1,2,3$  by

$$g_{ij}(\theta_{ij}) = N_{ij}^{-1} dN_{ij} / d\theta_{ij} \quad (6)$$

When modeling dependency  $E_{ij}(V_{xij}, V_{yij})$  or  $E_{ij}(r_{ij}, \theta_{ij})$  the expression (4) is used. Similar to

approach [23-25] the translational motion of the component particle  $i=1,2$  together with its center of mass is taken into account, rotational motion relative to this center, taking into account the random component of the angular momentum, as well as elastic interaction with a deformable elastic blade  $j=1,2,3$  at a given value of the slope of the stiffness  $k_u$ .

Taking into account (3) - (5) by the definition (6) for the set  $g_{ij}(\theta_{ij})$  of the indicated differential distribution functions by the angle of their spreading  $\theta_{ij}$  the expression is obtained

$$g_{ij}(\theta_{ij}) = \varepsilon_{0ij} f_{3ij}(\theta_{ij}) \exp(-k_u \theta_{ij}^2 / E_{0ij}) \times \left\{ \exp \left[ -10 \left( \varepsilon_{1ij} + \frac{k_u \theta_{ij}^2}{E_{0ij}} \right) \right] - \exp(-\varepsilon_{1ij}) \right\}^{-1} \times \left( \operatorname{erf} \frac{\varepsilon_{2ij} [f_{2ij}(\theta_{ij})]^2}{f_{3ij}(\theta_{ij})} - \operatorname{erf} \frac{\varepsilon_{2ij}}{f_{3ij}(\theta_{ij})} \right) \quad (7)$$

where

$$\varepsilon_{0ij} \equiv [\lambda_0 / (16\lambda_1)] [\pi k_u / (E_{0ij} k_{1ij} k_{2ij})]^{-1/2}$$

$$\varepsilon_{1ij} \equiv 3[k_u k_{1ij} k_{2ij} / E_{0ij}]^{1/2} / (2\lambda_0^2)$$

$$\varepsilon_{1ij} \equiv k_u k_{1ij} k_{2ij} / [E_{0ij} (2P_{0j}^2 + P_{2j}^2)]^{1/2}$$

$$\varepsilon_{2ij} \equiv 3[k_u k_{1ij} k_{2ij} / E_{0ij}]^{1/2} / (2\lambda_0^2), \quad f_{1ij}(\theta_{ij}) \equiv P_{0j} + P_{1j} \theta_{ij}$$

$$f_{2ij}(\theta_{ij}) \equiv \lambda_{0j} + \lambda_{1j} \theta_{ij}, \quad f_{3ij}(\theta_{ij}) \equiv \{ [f_{2ij}(\theta_{ij})]^2 + P_{2j}^2 \}^{1/2}.$$

Note that in [30] preliminary studies of the energy characteristics  $E_{0ij}$  of (5) depending on a number of design and operating parameters of the mixing process of bulk materials related to compositions for the needs of the glass industry. In this case, the set of energy parameters  $E_{0ij}$ ,  $i=1,2$ ,  $j=1,2,3$  is determined from the system of energy balance equations  $E_{bi} = E_{ri}$ . Here, with the total layer thickness  $H_L$  for materials on the tape, the calculation of the total energies of the interaction of particles when they are captured by elastic blades from the drum-tape gap  $E_{bi}$  and during spreading  $E_{ri}$  is carried out according to the formulas

$$E_{bi} = \sum_{i=1}^2 N_{ij} m_i \frac{h_0^2}{2} + \frac{1}{\varphi_0} \int_0^{\varphi_0} \frac{[r_{U_{ij}}(\theta_{ij})]^4 d\theta_{ij}}{C_0^2 + C_{1ij}^2} \quad (8)$$

$$E_{ri} = \sum_{i=1}^2 A_{ij} \int_{\theta_{1ij}}^{\theta_{2ij}} d\theta_{ij} \int_a^{a-H_L/2} E_{ij} [\exp(-E_{ij} / E_{0ij})] r_{ij} dr_{ij} \quad (9)$$

The coefficients included in expressions (7)-(9)  $k_{vij}$ ,  $\tau=1,2$ ,  $P_{vj}$ ,  $\lambda_{vj}$ ,  $v=0,1,2$ ,  $C_{vij}$ ,  $\xi=0,1$  depending on the properties of the materials to be mixed, as well as the design and operating parameters of rotary mixing. The

expressions for these coefficients are not given explicitly due to their cumbersome nature.

So, the results of modeling the set of functions  $g_{ij}(\theta_{ij})$  given in expression (7) allow us to construct complete differential distribution functions of the number of particles of each component along the specified characteristic angle  $\theta_{ij}$ , taking into account the work of elastic blades from three rows on the drum

$$G_i(\theta_{ij}) = \prod_{j=1}^3 g_{ij}(\theta_{ij}) \quad (10)$$

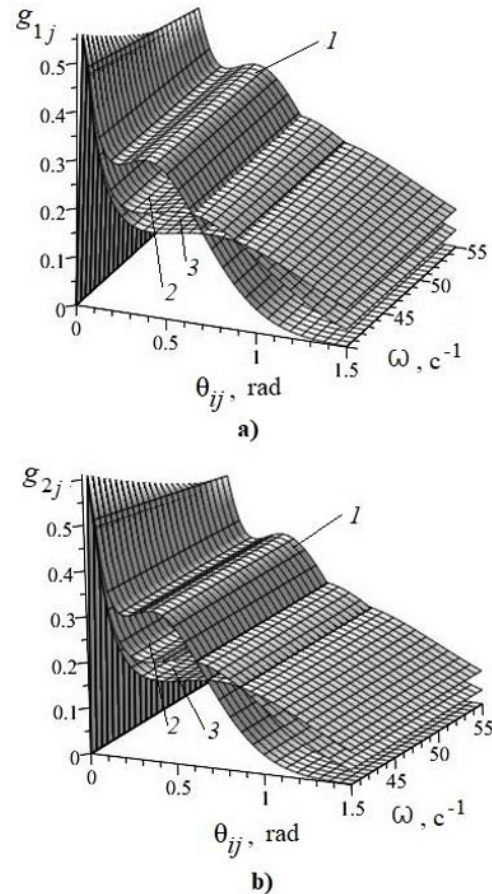
## 4 Results and Discussion

We present the results of modeling the sets of functions  $g_{ij}(\theta_{ij})$  and  $G_j(\theta_{ij})$  according to (7), (10) when obtaining a dry construction mixture, which includes natural sand GOST 8736-93 ( $i=1$ ,  $D_{T1}=2.0 \times 10^{-4}$  m,  $\rho_{T1}=1.8 \times 10^3$  kg/m<sup>3</sup>) and slag sand GOST 3344-83 ( $i=2$ ,  $D_{T2}=2.25 \times 10^{-4}$  m,  $\rho_{T2}=1.3 \times 10^3$  kg/m<sup>3</sup>). Note that “GOST” is an abbreviation of the term “state all-union standard, since 1992 it means “interstate standard” in the CIS countries. We present the values of the main parameters of the mixing process of the indicated components in the rotary apparatus at the first stage of its operation: constructive ( $r_b = 3.0 \times 10^{-2}$  m,  $l_b = 4.5 \times 10^{-2}$  m,  $h_0 = 3.0 \times 10^{-2}$  m,  $n_0 = 8$ ,  $k = 3$ ) and operational ( $H_L = 3.0 \times 10^{-2}$  m,  $\omega = (41.5 - 55.0) \text{ s}^{-1}$ ).

The analysis of the results obtained made it possible to identify a set of the most significant parameters of the rotary mixing process that affect the quality of the resulting mixture at the first stage of the apparatus operation. These parameters, first of all, include: the angular speed of rotation of the drum and the complex characteristic of the deformation of elastic blades  $d = h_0/l_b$ . Dependency Surface Classes  $g_{ij}(\theta_{ij}, \omega)$ , obtained using expression (7) are shown in Fig. 1a, 1b.

For both cases of the formation of rarefied streams, two characteristic bursts are observed (Fig. 1a, 1b) functions  $g_{1j}(\theta_{ij}, \omega)$ ,  $g_{2j}(\theta_{ij}, \omega)$ . The first rise in the values of these functions  $i=1, 2$  is associated with the dropping of bulk components from the deformed elastic blades almost immediately after they leave the gap between the drum and the conveyor belt. (graphs 1-3, Fig. 1a; graphs 1-3, Fig. 1b) for each row. The second spike in values is associated with the ability to remove particles that remain on the blades when restoring their shape (graph 1, Fig. 1a; graph 1, Fig. 1b) and smoothes as flexes straighten out (graph 3, Fig. 1a; graph 3, Fig. 1b). In addition, a comparable variance is noted for both families of graphs, despite the difference in the densities of the mixed components ( $\rho_{T1}/\rho_{T2}=1.38$ ) and the average diameters of their particles ( $D_{T1}/D_{T2}=0.88$ ). Increasing the angular velocity within the selected range  $\omega = (41.5 - 55.0) \text{ s}^{-1}$  does not significantly change the general nature of dependence  $g_{ij}(\theta_{ij}, \omega)$ . In particular,

almost no displacement of the first burst along the abscissa axis is observed, but there is a slight decrease in the values of these functions.  $g_{ij}(\theta_{ij}, \omega)$  no more than 1.3 times for both components with tape (graphs 1-3, Fig. 1a; graphs 1-3, Fig. 1b).



**Fig. 1.** Relationship between the differential distribution functions  $g_{ij}(\theta_{ij}, \omega)$  by the angle of their spreading  $\theta_{ij}$  and the rotation speed of the drum  $\omega$ : a) natural sand GOST 8736-93 ( $i=1$ ); b) slag sand GOST 3344-83 ( $i=2$ );  $d=0.67$ ;  $k=3$ ; 1 -  $j=1$ ; 2 -  $j=2$ ; 3 -  $j=3$ .

An illustration of the significant influence of the complex characteristics of deformation of elastic blades  $d$  on the process of formation of rarefied flows is the families of surfaces for the dependence  $G_i(\theta_{ij})$  in accordance with expression (10), given at Fig. 2a, 2b. For example, the growth of the indicator  $d$  1.27 times is reflected in the fall in values  $G_1(\theta_{ij}), G_2(\theta_{ij})$  5 times and 7 times respectively at  $\omega = 41.5 \text{ s}^{-1}$  (graphs 1, 4, Fig. 2a; graphs 1, 4, Fig. 1b). This fact creates conditions for varying the design parameters included in this complex indicator when finding effective ranges of their change.

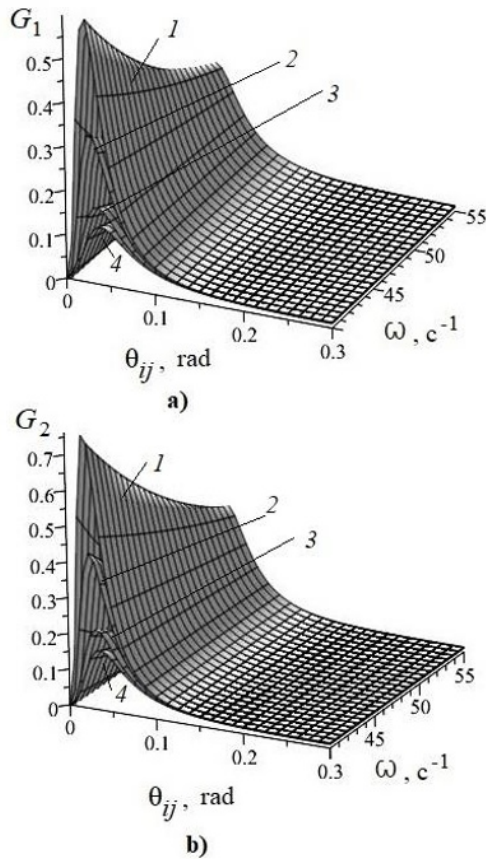
As an example, Table 1 shows the calculated values for the main energy characteristic of the studied mixing process  $E_{0ij}$ , corresponding to the values of the angular velocity  $\omega = 52.34 \text{ s}^{-1}$  and complex parameter  $d = 0.67$ .



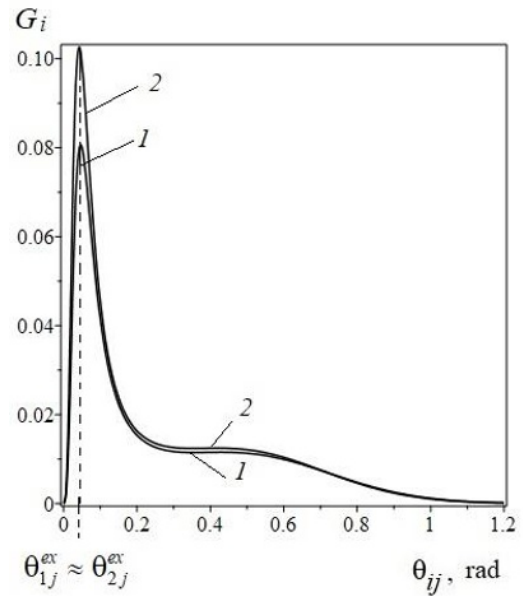
**Table 1.** The results of calculating the energy characteristic of the mixing process in a rotary apparatus.

$j$	$E_{01j}, 10^{-4}, J$	$E_{02j}, 10^{-4}, J$
1	1.45	1.49
2	3.23	3.36
3	5.01	5.15

As follows from Fig. 3, the results of the studies carried out have shown compliance with the main criterion for the effective mixing of bulk materials, identified in the works [23-26].



**Fig. 2.** Relationship between the complete differential distribution functions  $G_i(\theta_{ij}, \omega)$  by the angle of their spreading  $\theta_{ij}$  and the rotation speed of the drum  $\omega$ : a) natural sand GOST 8736-93 ( $i=1$ ); b) slag sand GOST 3344-83 ( $i=2$ );  $k=3$ ; 1 -  $d=0.55$ ; 2 -  $d=0.60$ ; 3 -  $d=0.67$ ; 4 -  $d=0.70$ .



**Fig. 3.** Relationship between the complete differential distribution functions  $G_i(\theta_{ij})$  by the angle of their spreading  $\theta_{ij}$ :  $\omega=52.34 \text{ s}^{-1}$ ;  $d=0.67$ ;  $k=3$ ; 1 - natural sand GOST 8736-93 ( $i=1$ ); 2 - slag sand GOST 3344-83 ( $i=2$ )

Analysis of these graphs in Fig. 3 confirms the convergence of not only the extreme values of the spreading angles  $\theta_{1j}^{ex} \approx \theta_{2j}^{ex}$  at  $\theta_{ij} < 0.1$  rad for components  $i=1,2$  but also the curves for dependencies  $G_1(\theta_{ij}), G_2(\theta_{ij})$ . For example, according to graphs 1, 2 (Fig. 3) the ratio  $G_2(\theta_{2j}^{ex}) / G_1(\theta_{1j}^{ex}) \approx 0.8$ .

## 5 Conclusions

An attempt is made in this work to expand the field of application of the energy method of Klimontovich Yu.L. [9,10] in stochastic modeling of one of the main processes of processing bulk components in the production of dry building mixtures, including from industrial waste. The results contribute to the development of a theoretical basis for calculating rotary mixers and evaluating the rational ranges of their design and operating parameters, taking into account the physical and mechanical properties of the mixture components. The main results include compliance with the criterion of effective mixing by the convergence of the obtained curves for the complete differential distribution functions of the number of particles by the angle of spread for each component. This is accompanied by the characteristic first bursts of the obtained distribution curves ( $\theta_{ij} < 0.1$  rad) for the number of particles of the tested bulk materials at the given ranges of change in the angular velocity of rotation of the drum (41.5-55.0)  $\text{s}^{-1}$  and the degree of blade deformation (0.55-0.70) after leaving the gap with the moving belt. This fact creates the prerequisites for the effective mixing of components in a rotary way.

So, the conclusions and results of the work include the following:

- The analysis of the efficiency of the first stage (rotary mixing) was carried out based on the basis of the obtained distribution functions of the number of particles of bulk components over the scattering angle, taking into account their physical and mechanical properties and a variety of design and operating parameters of the apparatus.

- A set of the most significant parameters of the rotary mixing process that affect the quality of the resulting mixture at the first stage of the apparatus operation (angular speed of rotation of the drum and a complex characteristic of deformation of elastic blades) has been identified.

- The general nature of the behavior of the particles of the tested bulk materials at the initial stage of their spreading after interaction with elastic blades installed perpendicular to the radius of the rotating drum was theoretically established.

- Theoretical compliance with the criterion of effective mixing of bulk components in obtaining dry building mixtures was obtained.

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