About the geographic grid of the three-dimensional hypersphere

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Abstract. According to the Poincaré conjecture (1904) proved by Grigory Perelman (2002-2003) that any simply connected compact three-dimensional manifold without edges is homeomorphic to a three-dimensional hypersphere [1], to solve the problems of visualizing four-dimensional objects in three-dimensional space [2], it is proposed to choose a suitable manifold, in in this case, a ball, establishing a homeomorphism between objects located in different spaces by technological means of cartography. As a result of this work, it seems possible to build a dynamic video of the population distribution process on a map of the globe, which provides informational four-dimensional data flow, following the ideas embodied in 4D Anatomy [3]. The proposed technology opens up new ways of visualizing four-dimensional space This work was performed within the framework of the state assignment of the ICM MG SB RAS (project 0315-2019-0003).

1 Hypersphere formula

The formula for a hypersphere can look like this, where *R* is its radius:

$$R^2= X^2+Y^2+Z^2+H^2 \tag{1}$$

then, after a small transformation, which consists in transferring the parameter *H* along the fourth dimension to the left side of the equation, we can see on the right side of it a parametrically defined ball.

1.1 Ball formula

The formula for a parametrically defined ball can look like this:

$$R^2-H^2 = X^2+Y^2+Z^2 \tag{2}$$

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Thus, each point on the hypersphere corresponds to an interior point on one of two balls glued along the outer surfaces, depending on whether the parameter $-R \leq H \leq +R$ is positive or negative.

1.1.1 Interaction of homeomorphic varieties

The diagram of such correspondence, which explains the interaction of homeomorphic manifolds, is shown in Fig. 1, for two-dimensional spaces of smaller dimension.

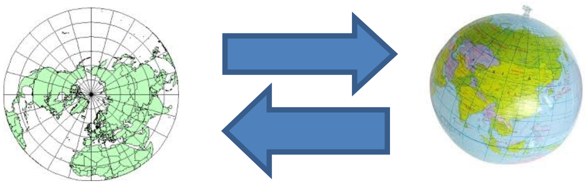

Fig. 1. The geographic map and the globe are informationally the same system, but visualized in different spaces

On the left side of the figure, the maps of the northern and southern hemispheres are glued together along the equator to form a flat, two-sided map. Returning to the hypersphere, we note that the centers of two generated balls are the poles of the hypersphere and, accordingly, the radii will form the guiding "meridians" of the hypersphere, like a dandelion inflorescence in Fig. 2. The section of the meridian with a parallels of subsphere selected inside an arbitrary ball uniquely determines the location of the point both inside this ball and on the surface of the hypersphere.

Fig. 2. The layout of the meridians in the form of the structure of a dandelion inflorescence

1.1.2 Cartographic grid

So, the coordinate of a point will be determined by a set of geographic coordinates (*hyperparallel H, natural parallel, natural median*) - that is, a point on the hypersurface of the hypersphere will be determined by three coordinates, while a continuous line drawn in the ball is also continuously displayed on the surface of the hypersphere.

1.1.3 Four-dimensional shapes

The issues of the geometric representation of multidimensionality began to be given scientific attention from the middle of the 19th century, which was soon reflected in Einstein's four-dimensional space-time concept, linking the relationship between space and time. So, according to the theory of relativity, with the simultaneous beginning of motion any two arbitrary objects from starting point to one target point with different speeds and trajectories in space, the durations of the times spent will differ when they meet. Modern theoretical physicists are currently developing an unusual physical theory of strings that works only in multidimensional space, dimensions from 10 or 26 dimensions.

The simplest geometric body for an n-dimensional space is a simplex, which is a convex polyhedron with the number of vertices which are per one unit more than the dimension of the space itself. Opposite any vertex of the simplex lies a face containing all the other vertices. So for a zero-dimensional space, a simplex is represented by a point. For a one-dimensional space, a simplex is already a line segment. In two-dimensional space, a simplex looks like a regular triangle, and in three-dimensional space, a simplex becomes a tetrahedron. Let's represent the described structures in a geometric manner. For example, a point represents a simplex of zero-dimensional space. By "pulling out" another point from a first point and connecting them with a line segment of length "one", we obtain a simplex of one-dimensional space. Building a regular triangle on a line segment, we obtain a simplex of two-dimensional space. Transforming a regular triangle into a regular tetrahedron, we represent a simplex of three-dimensional space. From geometric considerations, it is possible to calculate the radii of the spheres circumscribed around the "unit" simplexes:

$$r0 = 0$$

for a sphere in zero-dimensional space,

$$r1 = 1/2$$

for a sphere in one-dimensional space,

$$r2 = 2 * sqrt(1-r1^2) / 3$$

for a sphere in two-dimensional space and finally

$$r3 = 3 * sqrt(1-r2^2) / 4$$

for a sphere in three-dimensional space. The algorithm for constructing the subsequent simplex is as follows. In the presence of an n-dimensional simplex, with n vertices, the addition of the next vertex is postponed orthogonally to the face of the n-dimensional simplex at a distance calculated by the Pythagorean theorem with the hypotenuse “unit” side length and the known mathematical cathetus of the circumscribed sphere radius calculated in the previous step. The center of mass of the system is calculated on the constructed cathetus from the ratio *1:n*, which allows you to calculate the length of the next radius using the formula

$$r_n = n * sqrt(1-r_{n-1}^2) / (n + 1).$$

Calculating the passage to the infinity limit, we obtain the key constant:

$$r = 1 / sqrt(2)$$

In the work [4], infinite-dimensional spaces are considered, thus in this connection, it is possible to derive an iterative formula for calculating geographic coordinates when passing to spaces of higher dimension - *(H…, (H3, (H2, (H1, parallel, meridian))))*. Let's see what the simplex framework looks like on a hypersphere. Any five points inside a 3-D ball sets wireframe points on the hypersphere, forming a simplex-five-cell.

1.1.4 Matryoshka structures

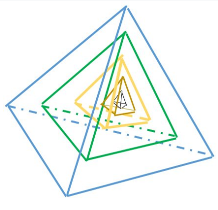

Fig. 3. Nested tetrahedron

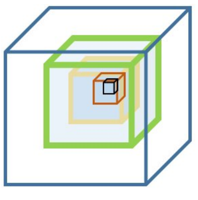

Fig. 4. Nested Cubes

"Matryoshka structures" [5], allow a cartographic way to "see" the hyper-relief in four-dimensional space. The approach is based on the expansion of the cartographic representation of the relief in the form of level contours in the direction of structures nested in three-dimensional space that correspond to one "height" along the fourth dimension (Figs. 3-4). Linear marking of the volume elements of a regular Platonic polyhedron with zero value on its boundary surface and values equal to the minimum distance to the boundary, which specify the height in the fourth dimension, creates nested structures in the form of regular Platonic solids, forming five four-dimensional pyramids, with their three-dimensional bases. The nested sphere markup generates a *4D* cone

$$R^2 = X^2 + Y^2 + Z^2 = (R-h)^2.$$

These primitives will form the basis for constructing a prototype design hyper-relief on a hyper-sphere, creating the prerequisites for creating a GIS for a four-dimensional space, which allows solving typical geographic information problems like of determining "visibility zones". The applicability of the "natural clustering of multidimensional space" developed in the laboratory was tested for economic problems with multifactor data, and was reported at a number of conferences.

2 Bulbous layers of the interior of the ball

A cartographic grid is an image on a map of geographic meridians and parallels in a particular cartographic projection. Serves for building a cartographic image and allows you to determine the coordinates of points on the map. When using a map of such a grid, it is possible to determine the coordinates of any point (geographic or rectangular, depending on the type of grid) and azimuths of the lines, as well as to judge the magnitude of distortions of the cartographic projection in different parts of the map.

2.1 Cartographic film

Setting the adjacent cartographic surfaces of the spheres inside the ball with maps of cha nges in the geography of the Earth in time, we get layer-by-layer parallels in four-dimensional space, which can be tracked in the form of a motion picture showing the process of changing the world (Fig. 3), sequentially moving to the next geographical map in the order of layers. At the same time, on the surface of the hypersphere, the graphic image will not be directly related to the existing film display formats, and it can be considered that we are talking about a different reading or another way of visualizing four-dimensional information.

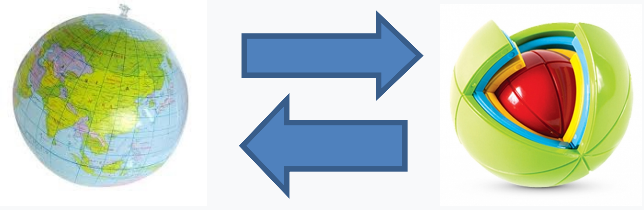

Fig. 5. The dynamic development in time of the cartographic image of the Earth can be represented by multilayer schemes of its changes.

2.2 Obstacles

According to the expertise of the Federal Service for Intellectual Property of the FEDERAL INSTITUTE OF INDUSTRIAL PROPERTY of Russia (No. 2016138924/12 (062081) dated 01/30/2018), the concept of a four-dimensional body (object) cannot be recognized as relating to the object of the invention, since it characterizes only a scientific theory not confirmed by an official source of the RAS, in while an important global indicator of a newly forming technology trend, the US patent should be highlighted. Yao, 01/31/2017, United States Patent 9554776 "Method for adjusting ROI and 3D/4D imaging apparatus using the same", which presents a device that creates a virtual 4D object. Another example of the development of the latest information technologies is the 4D-Anatomy virtual textbook - a cloud-based, interactive, resource for modeling the dissection of a physical body, as well as a modern educational platform. The creators of the program have implemented a powerful interactive resource on human anatomy. You get not just an image, but you can rotate it and even see from different angles how the human heart beats [3]. It is easy to imagine how useful such opportunities are in cardiology, in the face of increasing global attention to the ever-growing role of the digital economy.

3 Conclusion

If we recall that photography, appeared more than a century and a half ago, which was based on a camera obscura, and was always perceived as an exclusively two-dimensional object, was developed into the modern film industry, which gave rise to a much more complex object than simple “two-dimensionality”. The desire to go beyond the plane of the screen, prompted the inventors to stereocinematography - a variety of cinematic systems that imitate the presence of a third dimension, or cause the viewer to have the illusion of the depth of space. It is based on the phenomenon of human binocular vision and the optical parallax effect. Modern computer technologies make it possible to create pseudo-stereo images using computer graphics, without using stereo cameras. Similarly, it is possible to convert an existing "flat" image into three-dimensional by synthesizing the second part of a stereo pair. The terms “three-dimensional graphics” (3D-graphics) and “3D-cinema” used on the Internet describe fundamentally different phenomena and technologies. With the help of modern computer technology, there is no need to be fixed on a flat screen, while the dynamics of the process can be produced in volume space. And here the "impossibility to look behind the screen" is no longer an obstacle. Thus, holographic objects under development will certainly lead us to the four-dimensional world of 4D cinematography, which, due to human inertia, is still referred to as a marketing term.

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