Improving the efficiency of the hybrid algorithm for solving inhomogeneous minimax problem

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Abstract. The article deals with the problem of inhomogeneous minimax problem solution, what is typical of scheduling theory. This problem is NP-complete, and there is no exact algorithm for it, which has a polynomial time for large-scale problems. Therefore a quick algorithm that gets approximate tables is used. A possible method for solving this problem is considered a hybrid model, representing the synthesis of two genetic algorithms models, namely models Goldberg and CGS. Goldberg's model is viewed from multiple crossovers and most promising mutation. As it is difficult to make calculations analytically and often impossible to make it in practice, the computational experiment was carried out in this article. The results of the experiment are described in the tables, which graphically show a comparison of the hybrid model effectiveness. The comparison is based on the accuracy results obtained for two types of crossovers with the basic parameters of genetic algorithm. It is proved that the use of hybrid algorithm leads to the results which are more précised to the optimal ones, despite the deterioration in the temporary search characteristics solutions.

1 Introduction

Currently the development of methods for producing suboptimal approximate solutions to NP-complete problems in scheduling theory is relevant [1]. As the part of scheduling theory the solution of many problems are described, NP-complete problems are different from others that is why it is practically impossible to find a solution for polynomial time. For suboptimal solutions a variety of algorithms, including genetic ones are used. In terms of scheduling theory distribution problem can be formulated as follows. There is a service system consisting of N independent units $P = \{p_1, p_2, \dots, p_n\}$. Service end enters the flow of M - a plurality of independent parallel tasks (functional operators) $T = \{t_1, t_2, \dots, t_m\}$. $\tau(t_i p_j)$ - service duration assignment unit t_i and defined by a matrix T_τ . Devices are generally not identical, an assignment t_i can be performed on any of them. One device cannot

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simultaneously handle more than one assignment p_j . It is necessary to determine a distribution of tasks on devices without interruption to make the entire set of tasks with a minimum time spending. The criterion to minimize the task execution time is the minimax criterion, which is defined as follows: $f = \max_{l \leq j \leq n} f_j \rightarrow \min$, where $f_j = \sum_{\tau(t_i p_j) \in T} \tau(t_i p_j)$ processor time shutdown p_j [2,3].

2 Materials and methods

In this article, as a basic algorithm for solving a non-uniform minimax problem, the modified Goldberg model is taken, what is a type of genetic algorithms model (hereinafter GA). Goldberg modified model is different from a classical Holland one with the fact that the formation of a new generation uses a tournament selection, which allows to improve the algorithm results with various modification mutations by using various crossovers.

Goldberg modified model can be described as a sequence of the following steps:

Step 1. Initial generation, consisting of a predetermined number of individuals is being formed.

Step 2 Tournament selection of individuals and the use of crossover and mutation operators with a known probability of occurrence for a new generation.

Step 3. Check the conditions of the algorithm end, which usually lies in the immutability of the best solution for a given number of generations. If the test was not successful, go to the step 2.

Step 4. The best individual is chosen as the found solution [4, 5 and 6].

Graphically the functioning of Goldberg’s modified model can be represented in the Figure 1. The best individual will be selected and transferred to the next generation. The process is repeated as long as the best individual of a predetermined number of times will not be repeated in the generation.

Fig. 1. Goldberg’s modified model.

Fig. 2. One-point crossover.

Fig. 3. Two-point crossover.

Various modifications of mutations have been investigated from the whole spectrum which has been chosen as the most promising one, namely the simple mutation, is schematically illustrated in Figure 4:

Fig. 4. Simple mutation.

Goldberg modified model (with the earlier found parameters) should be strengthen with the adding suitable for CGS model, which can be described by the following sequence:

In the final generation, consisting of a predetermined number of individuals, we find a suitable and memorize individual [7, 8]. If we get the best individual from the first time, the final generation is exposed to strong mutations and rerun the modified model of Goldberg. If the newly received end generation of the best individual is repeated then the number of repetitions is increased. If the number of repetitions is equal to early given ones, the resulting specimen is selected as the best solution found. If during extra runs in the final generation specimen will turn out better than the previous ones, the counter will be reset, and additional launches relaunch a modified model of Goldberg.

3 The results of study

Due to the fact that it is very difficult to solve this problem analytically or maybe it is even not possible, computational experiments to gather statistics algorithms have been made [9, 10]. Software tool was written for the realization of computing experiment in modern programming language C# in a development environment of Microsoft Visual Studio 2017.

In the course of computing experiment realization, a series of calculations were done. The study evaluated parameters such as time of search solutions, mean and minimum values obtained in the experiment. Each experiment was repeated 50 times, with 100% probability of crossover and mutation.

The experimental results with a given amount of processing devices (5) and the number of tasks equal to 53 listed in Table No1. Where the column 1 - the estimated parameters; the column 2-data obtained for single-point crossover, without the use of strong mutations; the column 3-data obtained for the two-point crossover, without the use of strong mutations; the column 4 is the number of allowable retries the best individuals with strong mutations. The results were graduated according to the average obtainable meaning.

Table 1. The results for an example with 5 devices and 53 tasks.

1	2	3	Single-point crossover with a strong mutation							Two-point crossover strong mutation							
% of the individuals subjected to strong mutations	4		100	100	100	100	100	100	100	100	100	100	100	100	100	100	
% of tasks is subject to severe mutations			100	50	33	50	33	50	33	100	50	33	50	33	50	33	
% of bits subjected to strong mutation			12.5	12.5	12.5	50	50	75	75	12.5	12.5	12.5	50	50	75	75	
200 individuals; 200 repetitions																	
Min.	290	286	2	287	287	287	288	287	288	288	286	286	286	286	286	286	
Mid	293.9	289.08		290.12	289.34	289.64	290.36	290	290.08	289.78	287.74	287.94	287.98	287.8	287.64	287.8	287.68
Time	28	27		130	108	95	130	116	132	135	104	91	853	112	100	105	104
Min		3	287	287	288	287	288	287	287	286	286	286	286	286	286	286	
Mid			289.48	289.44	289.6	289.82	289.7	290.06	289.58	287.76	288.02	287.8	287.74	287.92	287.82	287.92	
Time			155	128	111	162	140	162	152	129	104	102	142	117	124	121	
Min		4	287	287	286	288	287	287	287	286	286	286	286	286	286	286	
Mid			289.28	289.14	289.24	289.5	289.28	289.5	289.3	287.5	287.74	287.9	287.54	287.5	287.76	287.7	
Time			179	140	130	181	173	191	169	155	123	120	154	152	152	151	
Min		5	287	287	286	287	287	287	287	286	286	286	286	286	286	286	
Mid			289.12	288.8	288.68	289.44	288.8	289.76	289.2	287.6	287.58	287.82	287.4	287.8	287.8	287.26	
Time			209	156	143	223	189	196	202	176	150	132	192	159	174	176	
Min		6	287	286	287	287	287	287	286	286	286	286	286	286	286	286	
Mid			288.92	288.38	289.1	289.24	288.86	289.24	288.88	287.58	287.62	287.72	287.3	287.58	287.36	287.48	
Time			233	185	146	252	215	250	231	189	158	156	209	179	213	190	

Min			7	287	286	286	288	287	287	286	286	286	286	286	286	286		
Mid				288.98	288.6	288.62	289.1	288.74	289.16	288.42	287.18	287.62	287.46	287.26	287.4	287.24	287.12	
Time				245	198	171	276	242	272	252	218	179	162	236	198	248	222	
400 individuals; 400 repetitions																		
Min	288	286	2	286	286	286	286	286	286	286	286	286	286	286	286	286		
Mid	290.7	287.6		288.38	288.54	288.78	288.42	288.48	288.72	288.68	287.02	287.22	287.42	287.24	287.3	286.82	287.3	
Time	107	101		354	286	279	342	326	352	316	321	275	263	321	283	349	285	
Min				3	287	286	287	286	286	286	286	286	286	286	286	286	286	
Mid					288.38	288.28	288.68	288.44	288.22	288.44	288.46	286.86	287.12	287.2	286.82	287.14	287.02	287.24
Time					413	334	324	404	389	453	380	391	346	341	413	380	406	357
Min				4	286	286	286	286	286	286	286	286	286	286	286	286	286	
Mid					287.8	288	288.3	288.4	288.24	288.32	288.2	286.86	287.12	287.22	287.02	287.3	286.76	287.02
Time					509	396	374	515	451	536	452	458	419	399	467	398	502	432
Min				5	286	286	286	286	286	286	286	286	286	286	286	286	286	
Mid					288.08	288.28	288.18	287.94	287.9	288.24	288.02	286.88	287	287.24	286.62	286.98	286.72	286.9
Time					541	416	430	585	484	560	533	530	460	433	574	504	597	503
Min				6	286	286	286	286	286	286	286	286	286	286	286	286	286	
Mid					287.68	288.1	288.28	288.12	287.98	288.02	287.64	286.8	287	286.98	286.68	287.04	286.6	286.84
Time					651	454	433	667	565	687	608	616	503	525	682	551	678	604
Min				7	286	286	286	286	286	286	286	286	286	286	286	286	286	
Mid					287.74	287.86	287.98	287.96	287.78	287.84	287.96	286.52	287.04	287.24	286.5	286.92	286.44	286.9
Time					730	523	500	746	639	805	625	686	563	547	712	621	743	642
	800 individuals;800 repetitions																	
Min	286	286	2	286	286	286	286	286	286	286	286	286	286	286	286	286		
Mid	288.88	287.2		287.94	287.8	287.72	287.78	287.58	287.72	287.72	286.8	286.72	286.8	286.76	286.5	286.68	286.56	
Time	300	298		974	847	840	1044	960	1060	960	1075	986	970	1067	1008	1018	1024	
Min				3	286	286	286	286	286	286	286	286	286	286	286	286	286	
Mid					287.52	287.56	287.9	287.4	287.96	287.68	287.84	286.56	286.82	286.9	286.62	286.8	286.56	286.7
Time					1217	1047	972	1280	1082	1281	1108	1295	1168	1138	1349	1209	1291	1231
Min				4	286	286	286	286	286	286	286	286	286	286	286	286	286	
Mid					287.56	287.62	287.84	287.5	287.56	287.46	287.52	286.6	286.78	286.78	286.64	286.86	286.42	286.78

Time		5	1411	1205	1153	1457	1264	1526	1394	1607	1364	1393	1535	1480	1562	1471
Min			286	286	286	286	286	286	286	286	286	286	286	286	286	286
Mid			287.48	287.36	287.6	287.38	287.36	287.52	287.56	286.48	286.88	286.6	286.58	286.74	286.44	286.78
Time			1505	1419	1313	1786	1553	1818	1527	1693	1531	1575	1770	1609	1853	1680
Min		6	286	286	286	286	286	286	286	286	286	286	286	286	286	286
Mid			287.32	287.48	287.86	287.1	287.22	287.34	287.52	286.44	286.62	286.8	286.4	286.78	286.46	286.62
Time			1621	1202	1085	1540	1290	1537	1262	1511	1389	1346	1531	1421	1608	1421

The experimental results for a task with a given number of processing devices equal to 4 and the number of tasks equal to 101 are given in the Table No2.

Table 2. The results for an example with 4 devices and 101 tasks.

1	2	3	4	Single-point crossover with a strong mutation							Two-point crossover strong mutation						
% of the individuals subjected to strong mutations				100	100	100	100	100	100	100	100	100	100	100	100	100	
% of tasks is subject to severe mutations				100	50	33	50	33	50	33	100	50	33	50	33	50	33
% of bits subjected to strong mutation				12.5	12.5	12.5	50	50	75	75	12.5	12.5	12.5	50	50	75	75
200 individuals; 200 repetitions																	
Min	686	680	2	679	680	680	680	679	681	681	679	678	678	678	677	679	678
Mid	696.62	686.88		684.56	683.94	683.76	685.74	685.04	687.06	685.28	681.7	680.56	681.16	681.74	681.48	681.96	682.02
Time	71	58		297	239	218	295	280	288	291	203	207	186	230	218	225	215
Min			3		680	679	679	681	680	682	681	678	678	678	678	678	679
Mid					684.26	682.5	682.58	685.74	684.88	685.76	684.74	680.08	680.72	680.94	681.12	680.8	682.02
Time					326	289	248	351	321	363	340	268	216	207	282	256	267
Min			4		680	678	678	680	681	681	679	677	678	677	678	678	678
Mid					683.56	682.7	682.72	685	683.58	685.78	684.28	680.4	680.64	680.52	681.02	680.34	681.04
Time					384	344	294	392	409	447	410	310	251	260	321	310	335
Min			5		679	679	679	681	680	679	679	678	678	679	679	677	678
Mid					683.18	682.16	682.08	684.58	683.58	685.42	684.1	679.92	680.26	680.48	681.02	680.56	681.26
Time					425	368	324	478	448	537	448	368	310	270	366	348	382
Min			6		679	678	678	680	678	681	679	678	677	678	678	677	679
Mid					682.38	681.72	681.88	684.76	682.74	684.5	683.48	679.84	679.82	680.32	680.5	680.36	681.08
Time					502	398	350	489	474	525	535	427	356	308	422	390	415
Min			7		678	678	679	680	679	680	680	677	678	678	678	677	678
Mid					682.2	681.48	681.58	683.78	682.12	684.6	682.9	679.72	679.78	679.92	680.36	679.8	680.62
Time					558	421	367	565	583	588	544	430	359	339	474	461	519
400 individuals; 400 repetitions																	
Min.	682	678	2	678	678	679	680	678	678	678	677	677	677	677	677	677	677
Mid	689.4	682.56		682.04	681.88	681.7	682.72	681.84	682.64	682.48	679.28	679.42	679.46	679.44	679.04	679.52	679.4

Time	197	165		745	615	615	796	723	840	784	669	582	559	717	625	627	680
Min			3		678	678	678	678	678	678	678	677	678	678	678	677	677
Mid					681.86	681.58	681.66	681.52	681.74	682.02	681.78	679.26	679.5	679.64	679.6	678.92	679.36
Time					857	740	701	1038	900	1012	997	7616	652	669	792	788	856
Min			4		678	678	677	679	677	678	678	677	678	677	677	677	678
Mid					680.74	680.74	680.88	681.84	681.14	681.94	681.36	678.8	679.1	678.94	678.84	678.92	679.26
Time					1116	862	824	1190	1044	1188	1082	919	811	796	1035	935	938
Min			5		678	678	678	678	678	678	679	677	677	677	678	677	678
Mid					680.18	680.4	680.5	681.38	680.68	681.5	680.92	678.96	678.84	678.88	679	678.56	678.84
Time					1279	988	887	1335	1260	1312	1346	1011	949	911	1149	1016	1192
Min			6		678	678	678	679	678	678	679	677	677	677	677	677	678
Mid					680.36	680.44	680.98	681.34	680.52	681.62	681.1	678.6	678.88	678.92	678.7	678.36	678.9
Time					1406	1068	966	1385	1346	1505	1386	1207	1023	985	1306	1268	1335
Min			7		678	678	678	677	678	678	678	677	677	677	677	677	677
Mid					680.44	680.42	680.2	681.14	680.3	680.92	680.56	678.44	678.88	678.86	678.44	678.44	678.84
Time					1528	1181	1089	1571	1476	1721	1527	1370	1096	1073	1427	1279	1416

The experimental results for a task with a given number of processing devices equal to 5 and the number of tasks equal to 101 are given in the Table No3.

Table 3. The results for an example with 5 devices and 101 tasks.

1	2	3	Single-point crossover with a strong mutation								Two-point crossover strong mutation						
% of the individuals subjected to strong mutations			4	100	100	100	100	100	100	100	100	100	100	100	100	100	
% of tasks is subject to severe mutations				100	50	33	50	33	50	33	100	50	33	50	33	50	33
% of bits subjected to strong mutation				12.5	12.5	12.5	50	50	75	75	12.5	12.5	12.5	50	50	75	75
200 individuals; 200 repetitions																	
Min	551	543	2	545	543	545	547	546	546	547	543	541	541	543	542	542	
Mid	566.68	553.36		552.1	550.38	550.1	553.8	552.2	554.8	553.02	546.84	546.24	545.46	546.88	546.02	547.12	546.96
Time	78	78		372	356	321	377	360	364	345	264	268	254	258	271	280	267
Min			3	545	543	542	546	545	546	545	543	541	541	542	542	543	541
Mid				552.12	549.28	548.4	552.32	551.62	552.46	551.14	546.36	544.36	545	545.8	545.86	545.66	545.82
Time				418	404	388	467	396	435	432	339	340	276	354	335	363	348
Min			4	546	543	543	548	545	546	546	542	540	541	542	541	542	543
Mid				550.44	548.7	547.2	551.76	550.52	551.8	551.24	545.38	544.54	544.32	545.6	545.26	546.4	545.56
Time				495	469	440	522	471	491	465	410	379	331	452	389	382	422
Min			5	544	544	543	546	546	546	545	542	541	540	542	542	543	542
Mid				550.12	547.9	546.98	551.26	550.22	551.54	550.64	544.48	544.1	543.86	545.76	544.64	545.8	545.46
Time				560	541	483	571	532	578	570	477	425	410	476	447	469	425
Min			6	546	542	543	546	544	546	544	541	540	540	542	542	543	542
Mid				550.56	547.78	545.92	550.92	549.68	551.36	549.56	545.04	543.62	543.98	545.26	544.98	545.88	544.82

Time			7	645	597	543	690	597	632	615	538	484	417	510	463	497	528	
Min				546	543	541	545	545	545	544	542	542	541	542	542	543	541	
Mid				549.98	547.56	545.8	550.26	549.84	550.64	549.76	544.72	543.76	543.42	544.9	544.54	545.52	544.58	
Time				733	657	575	740	668	668	646	564	554	495	593	554	585	549	
400 individuals; 400 repetitions																		
Min	547	543	2	542	543	541	543	542	544	542	539	540	540	540	541	541	540	
Mid	555.3	547.82		547.6	546.22	545.68	547.52	546.94	548.24	546.88	543.12	543.1	542.88	543.68	543.48	543.6	543.36	
Time	283	219		1060	960	869	1118	1077	1034	1099	812	792	766	793	863	857	765	
Min				3	542	541	541	543	543	544	542	541	540	540	541	540	541	541
Mid					546.28	545.66	544.8	546.98	546.64	547.52	546.06	543.26	542.78	542.08	543.54	542.7	543.24	543.12
Time					1311	1148	1099	1376	1210	1257	1332	944	902	961	994	993	1111	983
Min				4	542	542	541	544	541	543	542	540	540	540	541	540	541	540
Mid					546.22	544.9	544.98	547.14	546	546.94	546.2	542.48	542.46	542.18	542.96	542.5	543.12	542.38
Time					1518	1359	1149	1566	1440	1503	1395	1132	1067	1078	1195	1187	1221	1186
Min				5	542	541	541	542	540	542	542	540	539	540	540	540	541	540
Mid					546.42	544.62	544.74	546.02	545.38	546.3	545.52	542.42	542.12	542.22	542.76	542.2	542.94	542.34
Time					1557	1500	1302	1814	1653	1785	1684	1361	1215	1105	1368	1336	1407	1329
Min				6	543	541	541	542	542	542	542	540	540	539	540	540	541	540
Mid					545.64	544.44	544.32	545.8	545.16	546.28	545.46	542.44	541.6	541.92	542.44	542.26	542.74	542.02
Time					1926	1511	1437	1940	1858	1978	1862	1503	1439	1335	1642	1447	1517	1352
Min				7	542	541	540	543	542	543	542	541	540	540	540	540	541	540
Mid					545.1	544.1	544.4	545.88	544.82	546.36	544.98	542.12	541.58	541.84	542.72	542.02	542.46	542
Time					1704	1381	1150	1726	1485	1696	1553	1303	1214	1086	1383	1250	1358	1325

4 Discussion and conclusion

Thus, analyzing the results shown in the Tables 1-3, we can make the following conclusions:

Basic parameters of the genetic algorithm (number of individuals and the number of the best solutions repetition) affect the quality of the solutions by using a modified Goldberg model.

Using a two-point crossover gives the best results in the overwhelming number of experiments.

In the overwhelming number of experiments a hybrid algorithm gives better results.

Results of the hybrid algorithm depend on the number of repetitions. The more repetitions, the closer to the optimum results the average and the best solutions.

The number of repetitions degrades the temporal characteristics of the hybrid algorithm.

The number of repetitions over five rarely gives improvement with a serious deterioration of temporal characteristics.

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