Mathematical modeling of lithodynamic processes

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Abstract. A mathematical model for beach reshaping in the vicinity of transverse hydraulic structures has been developed on the basis of the developed unified approach for determining the alongshore transport of heterogeneous sediments as applied to the conditions of pebble and sandy beaches.

1 Introduction

Existing studies of sediment movement in the coastal zone of the sea and the models and calculation methods developed on their basis make it possible to solve many engineering problems associated with sediment transport and their interaction with hydraulic structures. At the same time, there are questions that require additional research.

The processes of sediment transport of a pebble beach have serious differences from the processes of sediment transport of a sandy beach [1]. The reason is the specific properties of the pebble beach, which is characterized by the presence of steeper coastal slopes, in contrast to the sandy beaches. This entails a number of characteristic processes for pebble beaches.

1) Waves crash relatively close to the water's edge, and the energy dissipation during collapse is concentrated in a narrower area than on sandy beaches. That is, pebble beaches have a narrower surf zone. An important consequence of this is that the wave run-up zone can have a width similar to that of the surf zone, and, accordingly, sediment transport in such a zone can be greater than on sandy beaches.

2) The processes of wave refraction are also limited to a narrower zone than that of sandy beaches. As a result, the refraction is incomplete, and the waves approach the shore at a greater angle than is the case of sandy beaches.

3) The pebble beach has a fairly high hydraulic permeability. This increases the ability of the pebbles to penetrate into the stream in the wave run-up zone. But, for the same reason, pebbles do not linger in the wave run-up zone, and the ability to be carried out into the sea during a wave rollback is higher than on a sandy beach. All this is the reason for the pressure pulsation in the body of the beach and leads to the fact that the particles on the surface of the beach slope are more mobile. Pebble deposits are highly mobile despite large grain sizes: the average grain size of sediments on a typical pebble beach ranges from 10 to 40 mm [2].

From the noted features of the processes on pebble beaches, it follows that the total alongshore sediment transport on pebble beaches can be relatively greater than on sandy

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beaches with all other things being equal. And this is despite the large size of the pebble material.

It should be noted that, due to the difficulties of in situ measurements on pebble beaches, the processes of its interaction with waves have not yet been sufficiently studied, and there is no complete understanding of the processes of sediment movement on them, in contrast to sandy beaches, where these processes have been studied in more detail, including in natural conditions.

Most studies on the sediment transport and the development of both numerical and analytical models focus on the effect of sediment size on sediment transport. However, it is the specifics of the pebble beach processes that also affect the sediment transfer process.

Moreover, in engineering calculations, to estimate the sediment rates of sandy and pebble beaches, various dependencies are used, including the average or other representative diameter of the sediments. However, geotechnical surveys, as well as experimental studies [3, 4] show that real beaches have sandy and pebble sections of the profile, i.e. they are heterogeneous in composition.

And if, solving engineering problems of coastal hydraulics related to sediment transport, one use the existing formulas for calculating the flow rate of only sandy or only pebble sediments for a coastal section composed of their mixture, this can lead to a decrease in the reliability of the calculation results, which in turn can lead to inaccurate prediction of coastal deformations and cause the destruction of hydraulic structures.

Therefore, an important task is to develop a universal approach to determining the value of alongshore sediment rate for the conditions of inhomogeneous pebble and sandy beaches.

2 Methods and approaches

A unified approach to determining sediment rate on pebble and sandy beaches is based on the modification of energy formulas for alongshore sediment rate. At the same time, in addition to the features of beaches, this approach also takes into account the distribution of sediments along its profile [5, 6].

It is assumed that sediments in the coastal zone of the sea are transported under the combined action of waves and currents, which is observed in real conditions in the surf zone. Therefore, when modifying the energy formulas for alongshore sediment rate, the Ackers-White (AW) hydraulic technique [7] is used, modified by I. G. Kantarzhi and S. M. Antsyferov for the conditions of waves propagating in the current [8], and allowing the determination of the total sediment transport, including bottom and suspended sediment.

Modification of the formulas consists in obtaining an empirical coefficient for them, which makes it possible to take into account the differences in hydraulic processes occurring on pebble and sandy beaches. This is due to the fact that the empirical coefficient largely reflects the features of the beaches (lateral slope, the size of the sediments that make up the beach), which form characteristic processes that have a dominant effect on the flow rate.

The empirical coefficient of the energy formula for alongshore sediment transport is determined by comparing the value of alongshore sediment rate determined by the AW method with the energy dependence. The determination of the coefficients in the energy dependences completes their modification.

Modification of the energy formulas makes it possible to obtain design dependencies for assessing the alongshore sediment flow rate with the aim of their further use on the same coastal area, for example, when predicting the reformation of a beach that is heterogeneous in composition in the vicinity of various transverse hydraulic structures.

For short-term and long-term forecasts of the interaction of sediment movements in the coastal zone of the sea with port barriers and coastal protection structures, a one-dimensional diffusion model is proposed that calculates the evolution of the beach in the zone of influence of the transverse structure. In the model, beach changes are studied by analyzing the retention of the beach material flow and the corresponding material accumulation and loss in the structure area.

In the proposed model, the solution to the problem consists of two stages. At the first stage, the distribution of alongshore sediment rate is determined from the diffusion equation (4). At the second stage, according to the calculated sediment rate, the position of the coastline is determined from the balance equation (1).

The wave climate is set by the direction of the wave approach α0, the significant height of the waves in deep water H0 and the wave period T. The outer boundaries on both sides of the structure are set at distances from the structure, where its influence is negligible. At these boundaries, the sediment rate is determined by a modified energy formula for alongshore sediment transport, which takes into account wave refraction. Other limit values are set directly on both sides of the structure. For an impermeable structure and in the absence of bypassing, the sediment rate at the structure is zero. Otherwise, this flow rate must be determined. The initial value of the amount of sediment rate before the construction of the structure is also determined by the modified formula for alongshore sediment transport, taking into account the refraction of waves. The initial position of the beach is set according to field measurements.

The behavior of the coastline is determined from the continuity equation, which is written for the beach in the following form:

$$\frac{\partial y}{\partial t} = -\frac{1}{h}\frac{\partial Q}{\partial x}, \tag{1}$$

where: y (x,t) – determines the position of the coastline; x – the distance along the coast; t – time; h – the limiting water depth at which the beach material moves.

The position of the coastline is determined from (1) on the basis of the sediment rate Q calculated using the one-dimensional diffusion equation. The derivation of the diffusion equation for alongshore sediment rate is based on the fact that the orientation of the coastline with respect to the x axis can be expressed through the angle α between the x axis and the wave front, and the angle between the wave front and the coastline α0:

$$\frac{\partial y}{\partial x} = tg(\alpha - \alpha_0) \tag{2}$$

The combination of equations (1) and (2) makes it possible to obtain a one-dimensional diffusion equation for sediment rate:

$$\frac{\partial Q}{\partial t} = \frac{\frac{\partial Q}{\partial \alpha_0}}{h\left[1+\left(\frac{\partial y}{\partial x}\right)^2\right]}\frac{\partial^2 Q}{\partial x^2} \tag{3}$$

Equation (3) can be rewritten as:

$$\frac{\partial Q}{\partial t} = L(t,x)\frac{\partial^2 Q}{\partial x^2}, \quad L(t,x) = \frac{A(z\cos\alpha_0\cos z\alpha_0 - \sin\alpha_0 \sin z\alpha_0)}{h\left[1+\left(\frac{\partial y}{\partial x}\right)^2\right]}, \tag{4}$$

where L(t,x) is the diffusion coefficient, which takes into account the change in sediment rate with a change in the angle between the wave front and the coastline in deep water.

Equations (1) and (4) are solved numerically using the finite difference method for given initial and boundary conditions, which makes it possible to obtain the distribution of sediment flow and the position of the coastline at the nodes of a discrete grid along the coastline and with a discrete time step.

Equation (4) is linearized and integrated according to the absolutely stable Crank-Nicholson scheme:

$$\frac{Q_i^{n+1}-Q_i^n}{\Delta t} = \frac{1}{2}\left(L\frac{\partial^2 Q}{\partial x^2}\bigg|^{n+1} + L\frac{\partial^2 Q}{\partial x^2}\bigg|^{n}\right), \tag{5}$$

where the index i represents the nodes of the spatial grid, and the index n represents the points of time steps, Δt - the grid step in the time coordinate, and:

$$\frac{\partial^2 Q}{\partial x^2} = \frac{Q_{i+1}-2Q_i+Q_{i-1}}{\Delta x^2} \tag{6}$$

where Δx - spatial grid step.

Similarly, the initial and boundary conditions are replaced by their difference approximations.

Taking approximately $L_i^{n+1} = L_i^n$ and denoting $\lambda = \Delta t/\Delta x^2$, equation (5) can be represented as:

$$a_i Q_{i-1}^{n+1} + b_i Q_i^{n+1} + c_i Q_{i+1}^{n+1} = d_i, \tag{7}$$

where

$$a_i = c_i = -L_i^{n+1} \tag{8}$$

$$b_i = \frac{2}{\lambda} + 2L_i^{n+1} \tag{9}$$

$$d_i = \frac{2}{\lambda}Q_i^n + L_i^n\left(Q_{i+1}^n - 2Q_i^n + Q_{i-1}^n\right) \tag{10}$$

Using known values of sediment rate at the upper and lower boundaries, equations (7-10) are a tridiagonal system solved by standard methods. Sediment rates Q_i^{n+1} are determined starting from the last node: $i = N, N-1, \dots, 1$.

Equation (1) is then integrated to determine the new position of the coastline ($y_i, i = 1,2,\dots,N$):

$$y_i^{n+1} = y_i^n - \frac{\Delta t}{2h}\left(\frac{\partial Q}{\partial x}\bigg|^{n} + \frac{\partial Q}{\partial x}\bigg|^{n+1}\right) \tag{11}$$

Figure 1 shows a comparison of the developed mathematical model of beach reshaping in the vicinity of transverse hydraulic structures, which is based on a unified approach for determining the alongshore transport of heterogeneous sediments as applied to the conditions of pebble and sandy beaches with an analytical solution to the problem [9] at $\alpha_0 = 6^0$, Q=$10^4 \frac{m^3}{m\cdot year}$.

The maximum relative calculation error with respect to the analytical solution is 13%.

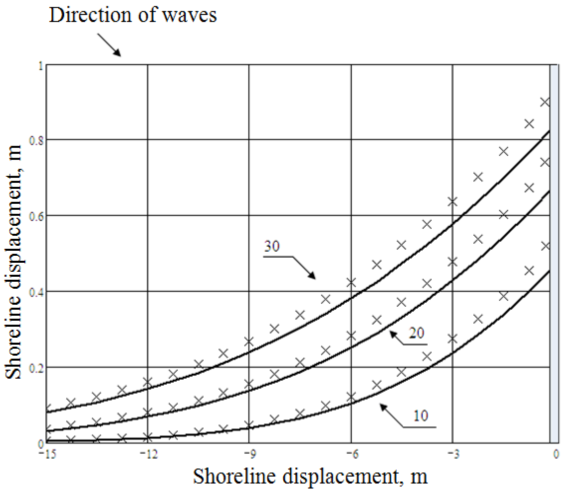

Fig. 1. Comparison of the 1-D model with the analytical solution of the problem: line with crosses - coastal contour according to the analytical solution; solid line - coastal contour according to 1-D model.

3 Calibration of the model of alongshore transport of sandy and pebble sediments based on the results of numerical modeling

Calibration of the developed model of alongshore transport of sandy and pebble sediments is based on a comparison of sediment rate calculated using the CERC energy formula [10], the modified CERC formula and the formula [11] obtained for the BOREZED numerical model [12, 13] and verified by field measurements.

Table 1. Alongshore sediment flow rates depending on wave height (T=4.85 s, slope=0.1, D=0.02 m, θ_b=10^0). Where H is a significant wave height along the collapse line; T is the wave period; D is the average diameter of the sediment; θ_b- angle at breaking; $tan\,\alpha$ - the coastal slope.

According to BOREZED, $\frac{m^3}{s}$	H, m	Calculation according to the proposed model				According to CERC with K=0.01, $\frac{m^3}{s}$
		wave speed at breaking, $\frac{m}{s}$	calculation error, %	according to CERC with new K, $\frac{m^3}{s}$		
0.00353	1.25	0.626	3	K=0.0346	0.00343	0.00099
0.00859	1.75	0.772	19	K=0.0435	0.01026	0.00236
0.01520	2.25	0.913	23	K=0.0258	0.01174	0.00454
0.02470	2.75	1.133	11	K=0.0353	0.02731	0.00774

The comparison was carried out for different wave parameters, as well as parameters that form characteristic processes on the beaches, provided that all parameters except one are constant. Tables 1-5 show the influence of wave parameters, slope, sediment size on sediment transport. The flow rate calculated by the formula obtained for the BOREZED numerical model [11] is compared with the results determined by the CERC formula [10] with the base coefficient and by the modified CERC formula with the coefficient determined by the proposed model of alongshore sediment transport. As a constant coefficient, the coefficient 0.01 was chosen, not 0.41, since it was obtained at average slopes and is intended to be used for diameters larger than 1mm. The choice of the CERC formula [10] was determined in order to show the possibility of estimating the rate on pebble beaches using the formula obtained in the conditions of sandy beaches.

The distribution of the alongshore sediment flow rates depending on the wave height is shown in Figure 2.

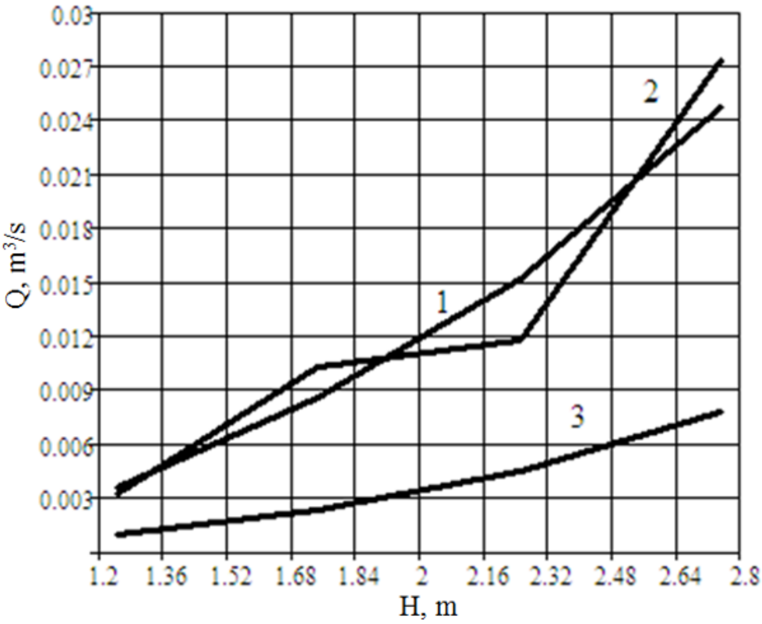

Fig. 2. Distribution of alongshore sediment flow rate depending on wave height: 1 - formula [10]; 2 - modified formula CERC; 3 - formula CERC [9] with the base coefficient.

Table 2. Alongshore sediment flow rates depending on the value of the period (H =1.25 s, slope=0.1, D=0.02 m, θ_b=10^0).

According to BOREZED, $\frac{m^3}{s}$	H, m	Calculation according to the proposed model				According to CERC with К=0.01,$\frac{m^3}{s}$
		wave speed at breaking,$\frac{m}{s}$	calculation error, %	according to CERC with new К, $\frac{m^3}{s}$		
0.00218	2.91	0.399	5	К=0.02	0.00231	0.00113
0.00289	3.88	0.479	12	К=0.032	0.00324	0.00103
0.00353	4.85	0.626	3	К=0.0346	0.00343	0.00099
0.00431	5.83	0.63	15	К=0.038	0.00367	0.00097

The distribution of alongshore sediment flow rates depending on the value of the period is shown in Figure 3.

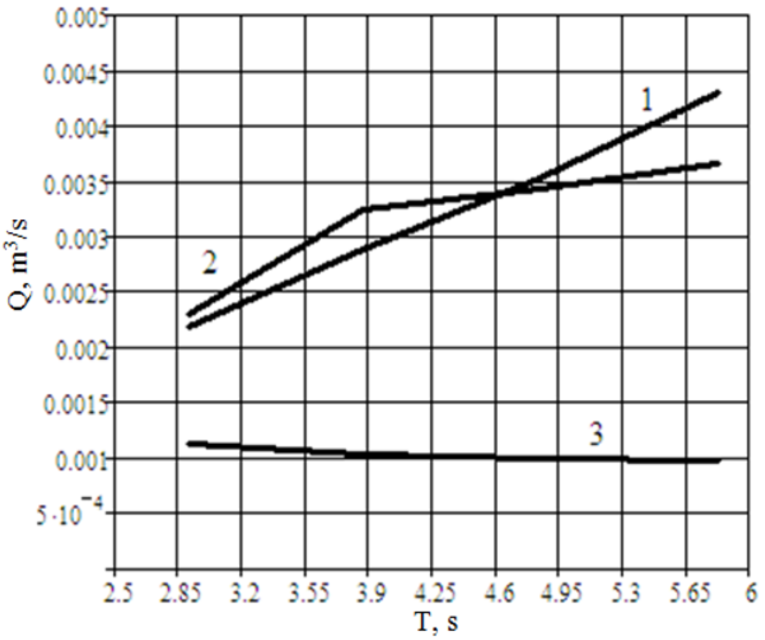

Fig. 3. Distribution of alongshore sediment flow rates depending on the value of the period: 1 - formula [10]; 2 - modified formula CERC; 3 - formula CERC [9] with the base coefficient.

Table 3. Alongshore sediment flow rates depending on the coastal slope (H =1.25 s, T=4.85 s, D=0.02 m, θ_b=10^0).

According to BOREZED, $\frac{m^3}{s}$	H, m	Calculation according to the proposed model				According to CERC with K=0.01, $\frac{m^3}{s}$
		wave speed at breaking, $\frac{m}{s}$	calculation error, %	according to CERC with new K, $\frac{m^3}{s}$		
0.00182	0.04	0.95	21	K=0.0122	0.00144	0.00118
0.00221	0.06	0.755	14	K=0.023	0.00252	0.00111
0.00278	0.08	0.646	17	K=0.031	0.00309	0.00105
0.00353	0.1	0.626	3	K=0.0346	0.00343	0.00099
0.00450	0.12	0.446	15	K=0.041	0.00384	0.00094

The distribution of alongshore sediment flow rates depending on the coastal slope is shown in Figure 4.

It can be seen from the table that the rate value in conditions of stationary waves increases with increasing slope, which confirms the property No. 1 indicated in the introduction.

Moreover, with an increase in the slopes, the speeds decrease due to the increase in friction, and the wave speeds will increase.

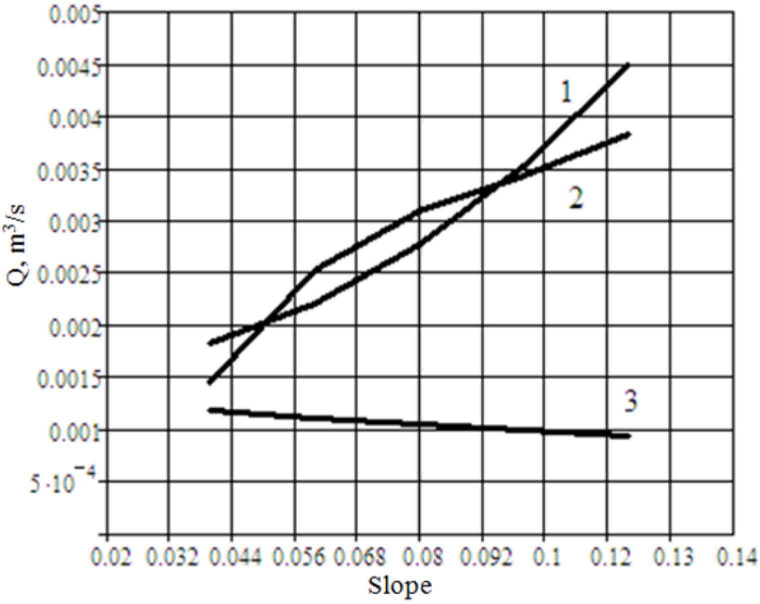

Fig. 4. Distribution of alongshore sediment flow rates depending on the coastal slope: 1 - formula [10]; 2 - modified formula CERC; 3 - formula CERC [9] with the base coefficient.

Table 4. Alongshore sediment flow rates depending on the size of the sediment diameter (H =1.25 s, T=4.85 s, slope=0.1 m, θ_b=10^0).

According to BOREZED, $\frac{m^3}{s}$	H, m	Calculation according to the proposed model				According to CERC with K=0.01$\frac{m^3}{s}$
		wave speed at breaking, $\frac{m}{s}$	calculation error, %	according to CERC with new K, $\frac{m^3}{s}$		
0.00420	0.01	0.704	10	K=0.047	0.00463	0.00099
0.00353	0.02	0.626	3	K=0.0346	0.00343	0.00099
0.00218	0.04	0.564	33	K=0.015	0.00146	0.00099

The distribution of alongshore sediment flow rates depending on the size of the sediment diameter is shown in Figure 5.

This wave will have more opportunity to move the material with a small diameter (0.01). In the case when the diameters grow (become more than 0.02), then there will already be overlapping sediments and waves will not be able to scatter them, which is the reason for an increase in friction and, accordingly, a decrease in speed.

With an increase in sediment size for a given wave, the calculation error increases. The error will be less with greater wave force.

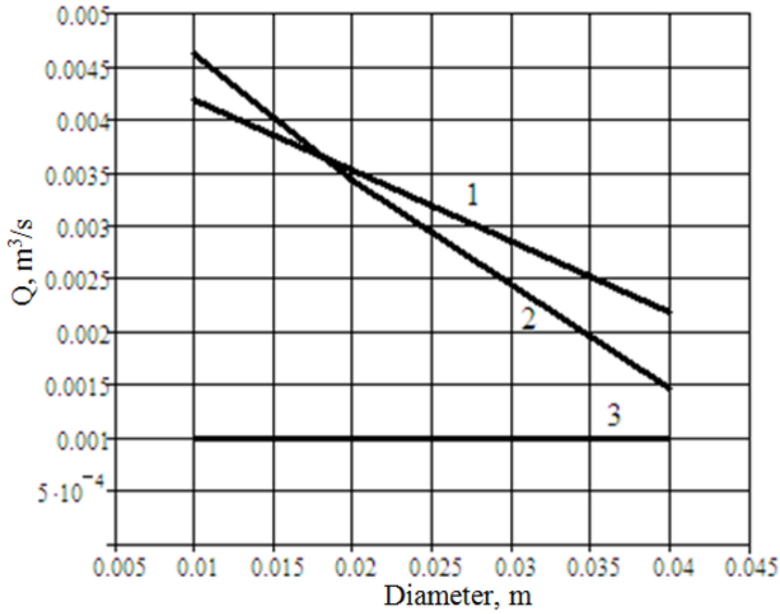

Fig. 5. Distribution of alongshore sediment flow rates depending on the size of the sediment diameter: 1 - formula [10]; 2 - modified formula CERC; 3 - formula CERC [9] with the base coefficient.

Table 5. Alongshore sediment flow rates depending on the angle of wave approach (H =1.25 s, T=4.85 s, slope=0.1 m, D=0.02).

According to BOREZED, $\frac{m^3}{s}$	H, m	Calculation according to the proposed model				According to CERC with K=0.01, $\frac{m^3}{s}$
		wave speed at breaking, $\frac{m}{s}$	calculation error, %	according to CERC with new K, $\frac{m^3}{s}$		
0.00244	7.5^0	0.437	17	K=0.0381	0.00286	0.00075
0.00353	10^0	0.626	3	K=0.0346	0.00343	0.00099
0.00488	12.5^0	0.639	12	K=0.0352	0.00432	0.00123
0.00656	15^0	0.8	31	K=0.0312	0.00453	0.00145
0.01120	20^0	1.144	37	K=0.0378	0.00705	0.00186

The distribution of alongshore sediment flow rates depending on the angle of wave approach to the coast is shown in Figure 6.

Thus, it is shown that using the CERC formula designed to determine the rate on sandy beaches, by determining the empirical coefficient in it for the conditions of a particular pebble beach, it is possible to estimate the sediment rate on pebble beaches.

If we use another formula for comparison, for example, the empirical dependence of V.A. Petrov and N.A. Yaroslavtsev, calculating the alongshore rate of pebble deposits [14] with a constant coefficient, then the rate value slightly differs from that calculated by the BOREZED model under conditions of a slope of 0.1. This is due to the fact that this formula was obtained with slopes of 0.1-0.12. Moreover, if new coefficients are used in the CERC formula [10] and the formula by V.A. Petrov and N.A. Yaroslavtsev [14], which are determined by the developed model of alongshore transport of heterogeneous sediments, then at different coefficients, the rates calculated from them will coincide.

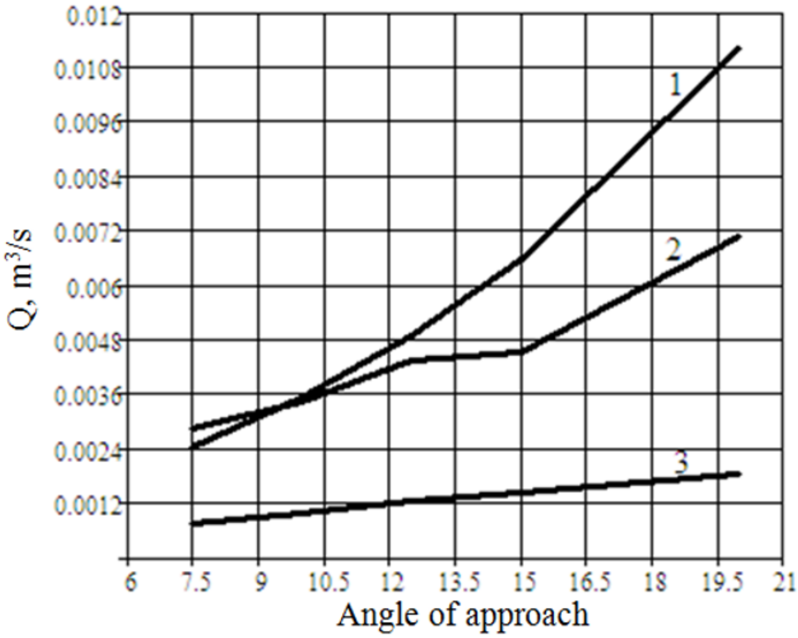

Fig. 6. Distribution of alongshore sediment flow rates depending on the angle of wave approach to the coast: 1 - formula [10]; 2 - modified formula CERC; 3 - formula CERC [9] with the base coefficient.

4 Conclusion

A refined method for calculating the alongshore sediment rate has been developed for the conditions of heterogeneous pebble and sandy coastal slopes. A calculation method is proposed based on energy dependences, including a correction coefficient determined for the conditions of the investigated coastal area. A method for obtaining the coefficient of energy dependence by using the method of transport of riverbed sediments has been developed.

On the basis of the developed model of the formation of an alongshore flow of inhomogeneous sediments, a mathematical model has been developed for the reformation of a beach inhomogeneous in composition in the vicinity of transverse hydraulic structures.

Based on the results of numerical modeling, the developed model of alongshore transport of heterogeneous sediments is calibrated, which is described by modified energy dependences. The maximum forecast error using the developed model is 37% in terms of material rate.

References

1. Quick M C, Dyksterhuis P 1994 *Cross-shore transport for beaches of mixed sand and gravel International Symposium: Waves-physical and numerical modeling* (Canadian Society of Civil Engineers) p 1443
2. Simm J D, Brampton A H, Beech N W, Brooke J S *Beach Management Manual Report 153* (Construction Industry Research and Information Association)
3. Kantarzhi I G, Rogachko S I, Vaytman V V 2007 *Hydraulic engineering* **2** 23
4. 2008 *Engineering and geological surveys as part of the project "Creation of a cargo*

area of the port of Sochi with the creation of coastal infrastructure at the mouth of the Psou River"

5. Bondareva E V, Kantarzhi I G 2009 *Hydraulic engineering* **12** 33
6. Bondareva E, Kantardgi I 2009 *Proc. of 9-th International Conference on the Mediterranean Coastal Environment (MEDCOAST 09)* (Sochi, Turkey) p 765
7. Ackers P, White W R 1973 *Proceedings of the ASCE* **99(HY11)** 2041
8. Kantarzhi I G, Antsyferov M S 2005 *Oceanology* **45(2)** 173
9. Pelnard-Considere R 1956 *Quatr. J. Hydraulique Paris* **3(N1)**
10. Leontiev I O 2001 *Coastal Dynamics: Waves, Currents, Sediment Flows* (M: GEOS) p 272
11. Van Wellen E, Chadwick A J, Mason T 2000 *Coastal Engineering* **40** 243
12. Chadwick A J 1991 *Proceedings of the Institution of Civil Engineers* **2(91a)** 719
13. Chadwick A J 1991 *Proceedings of the Institution of Civil Engineers* **2(91b)** 739
14. Petrov V A, Yaroslavtsev N A 1985 *Water resources* (M.)