

# Evaluation of Manufacturing Process based on the Geometric Variation Model in Multi-station Machining Processes

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**Abstract.** The objective of this paper is to explore the evaluation method of manufacturing process to verify its effectiveness based on the limitation of the variations which occur in multi-station machining processes. Firstly, the manufacturing process of a mechanical part is considered as a mechanism mainly consisted of machine-tool, part-holders, machined part, and cutting tools; And small displacement torsors (SDTs) are applied to describe all deviations in the manufacturing process, including the variation deviations of the machined surfaces of a part with regards to their nominal positions, the gap deviations associated to each joint between two contact surfaces, etc; Then, the 3D manufacturing variation model is established based on the relations between the machining feature variations and the functional tolerance requirements to realize the evaluation of manufacturing process. Finally, an application example is given to illustrate the proposed method.

## 1 Introduction

Multi-station machining process is the most common, most important and most difficult to control in the manufacturing industry. The research on evaluation method of manufacturing process based on 3D manufacturing variation model for multi-station machining processes plays an important role in estimating the geometrical and dimensional quality of manufactured parts, optimizing the process route of products, generating robust process plans, and eliminating downstream manufacturing problems. In order to consider the influence of geometric process variations, Bourdet and Ballot [1] proposed a three-dimensional variations model by using the small displacement torsor (SDT) to model geometric deviations in manufacturing process. Based on the concept of SDT, Legoff et al. [2] put forward a method for performing tridimensional analysis and synthesis of machining tolerances. Villeneuve et al. [3] have proposed a three-dimensional model on manufacturing tolerancing for mechanical parts in which the SDT concept is used to model the machined parts, part-holders, and machining operations. Louati et al. [4] proposed a machining tolerancing method using SDT theory to optimize a manufactured part setting. Abellán-Nebot et al. [5] analyzed two 3D manufacturing variation models, the stream of variation model (SoV) and model of the manufactured part (MMP), in multi-station machining systems and compared their main characteristics and applications. Furthermore, Laifa et al. [6] presented a 3D formalization of manufacturing tolerancing which associates the concept of SDTs, the functional

constraints, and manufacturing process capability. This approach enables the evaluation of manufacturing process by limiting the variations which occur at the various production setups. Despite some research achievements have been made in this related field, there are still many open issues to be explored. This paper focuses on 3D modeling of the geometrical variations in the processes of part machining and exploring the evaluation of manufacturing process by using the SDT and its transfer formula.

## 2 Small displacement torsor and its transfer formula

It is generally known that any variation of a geometrical feature from its nominal position can be characterized by a SDT with three rotation components ( $\alpha, \beta, \gamma$ ) around  $x, y, z$  axes and three translation components ( $u, v, w$ ) along  $x, y, z$  axes, respectively [7]. The SDT  $\{\tau_{P_i/P}\}_{(O_i, R_i)} = \{\Phi \ \epsilon_{O_i}\}$ , which synthesizes the position and orientation of an associated feature  $P_i$  relative to its nominal feature at a given point  $O_i$  in a local reference frame  $R_i$ , can be expressed as:

$$\{\tau_{P_i/P}\}_{(O_i, R_i)} = \{\Phi \ \epsilon_{O_i}\} = \left\{ \begin{array}{cc} \alpha & u \\ \beta & v \\ \gamma & w \end{array} \right\}_{(O_i, R_i)} \quad (1)$$

For a special case, the variation torsor of a plane feature  $P_i$  with regard to its nominal plane can be expressed as:

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$$\{\tau_{P_i/P}\}_{(O_i, R_i)} = \{\Phi \ \varepsilon_{O_i}\} = \begin{Bmatrix} \alpha & U \\ \beta & U \\ U & w \end{Bmatrix}_{(O_i, R_i)} \quad (2)$$

where point  $O_i$  belongs to the plane and  $R_i$  is a local reference frame whose axis  $z_i$  is the normal of the plane, capital  $U$  represents the undetermined component in the expression of a torsor.

To calculate operations on these torsors, the following two properties are defined [3]:

$$\forall a \in R, \quad a + U = U \quad (3)$$

$$\forall a, b \in R^2, \quad a \cdot U + b \cdot U = U \quad (4)$$

Considering  $\{\tau_{P_i/P}\}_{(O_i, R_i)} = \{\Phi \ \varepsilon_{O_i}\}$  a SDT at point  $O_i$  in a local reference frame  $R_i$ , this SDT at a given point  $O$  expressed in the global reference frame  $R_0$ , will become:

$$\{\tau_{P_i/P}\}_{(O, R_0)} = \left\{ \mathbf{R}_{0,i} \cdot \Phi \quad \mathbf{R}_{0,i} \cdot (\varepsilon_{O_i} + (\mathbf{R}_{0,i}^T \cdot \overline{OO_i}) \times \Phi) \right\} \quad (5)$$

where  $\mathbf{R}_{0,i}$  is the rotation matrix from  $R_0$  to  $R_i$ ,  $\mathbf{R}_{0,i}^T$  is the transposed matrix of  $\mathbf{R}_{0,i}$ , and  $\overline{OO_i}$  is the translation vector from  $R_0$  to  $R_i$  expressed in  $R_0$ .

If there is only translation transformation between  $R_0$  and  $R_i$ , the above transfer formula will be simplified as:

$$\{\tau_{P_i/P}\}_{(O, R_0)} = \left\{ \Phi \quad \varepsilon_{O_i} + \overline{OO_i} \times \Phi \right\} \quad (6)$$

### 3 Modelling of manufacturing variations

The manufacturing process of a part is generally composed of different set-ups, each set-up is regarded as a mechanism mainly consisted of machine-tool, part-holders, machined part, and cutting tools. We suppose that the geometrical variations in the machining process are small enough to be modeled with SDT.

#### 3.1 The SDT chain in the machining process

In order to model the whole manufacturing process, three types of torsors need to be defined: the global variation SDT; the deviation SDT; the gap SDT.

For set-up  $S_k$ , the global variation SDTs associated with the machined part  $P$ , the part-holder  $H$ , and the machining operation  $M$  are defined respectively:  $\tau_{P/R}^{S_k}$ ,  $\tau_{H/R}^{S_k}$ , and  $\tau_{M/R}^{S_k}$ . For set-up  $S_k$ , the deviation SDTs associated with the machined surface  $P_i$ , the part-holder surface  $H_i$ , and the machining operation surface  $M_i$  are defined respectively:  $\tau_{P_i/P}^{S_k}$ ,  $\tau_{H_i/H}^{S_k}$ ,  $\tau_{M_i/M}^{S_k}$ . For set-up  $S_k$ , the gap SDT  $\tau_{P_i/H_i}^{S_k}$  expresses the variations of the interface between surface  $P_i$  of the machined part and the corresponding surface  $H_i$  of the part-holder. Given that the parts do not interpenetrate at the contacts, each fixed component of the torsors is regarded as nil. So, for any

set of two interacting surfaces ( $P_i, H_i$ ), we can obtain the SDT chain as follows:

$$\tau_{P_i/H_i} = \tau_{P_i/P} + \tau_{P/R} - \tau_{H/R} - \tau_{H_i/H} \quad (7)$$

#### 3.2 Modelling of the geometrical variations

The manufacturing process of a part generally consists of several different set-ups, and we will discuss how these geometrical variations are transferred between set-ups.

It is assumed that torsor  $\tau_{P_b/P_a}$  can represent the functional tolerance between two machined surfaces  $P_a$  and  $P_b$  of part  $P$ , and its expression is as follows:

$$\tau_{P_b/P_a} = \tau_{P_b/P} + \tau_{P/P_a} = \tau_{P_b/P} - \tau_{P_a/P} \quad (8)$$

Suppose surfaces  $P_a$  and  $P_b$  are machined in set-ups  $S_1$  and  $S_2$  respectively, Eq. (8) becomes:

$$\tau_{P_b/P_a} = (\tau_{P_b/R}^{S_2} + \tau_{P_i/P}^{S_2} - \tau_{P_i/H_i}^{S_2} - \tau_{H_i/H}^{S_2} - \tau_{H/R}^{S_2}) - (\tau_{P_a/R}^{S_1} + \tau_{P_i/P}^{S_1} - \tau_{P_i/H_i}^{S_1} - \tau_{H_i/H}^{S_1} - \tau_{H/R}^{S_1}) \quad (9)$$

Thus, the relationship between functional tolerances and various variations in each set-up can be established to reveal how geometric variations are transferred between set-ups. This relationship can further guide the process engineers to evaluate the manufacturing process.

### 4 Evaluation of the manufacturing processes

In order to verify the effectiveness of the manufacturing process, we will discuss the manufacturing process evaluation method that takes into account the limitations on the manufacturing variations by the tolerances related to the functional requirements.

#### 4.1 Parallelism tolerance requirement

As shown in Fig. 1, the parallelism tolerance of surface  $P_7$  related to datum  $A$  ( $P_1$ ) indicates that the actual plane  $P_7$  shall be contained between two parallel planes  $T_{pa}$  apart which are parallel to datum plane  $A$ .

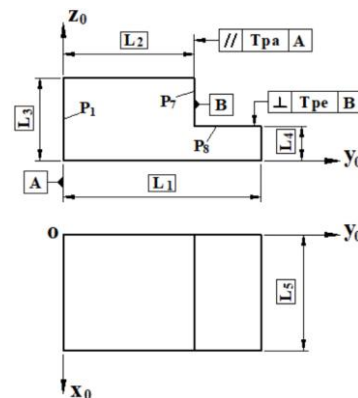


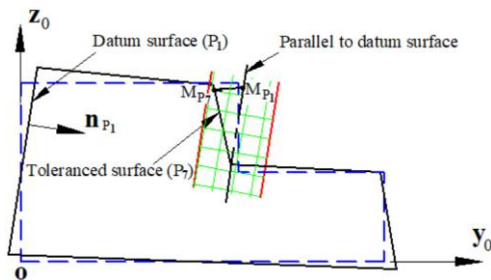
Fig. 1. Tolerance requirements

One uses the torsors to model the variations of machined plane  $P_7$  and datum surface  $A$  ( $P_1$ ) with regard to their nominal surface in the global reference frame  $R_0$ . And according to Eq. (8), one gets:

$$\begin{aligned} \{\tau_{P_7/P_1}\}_{(O,R_0)} &= \{\tau_{P_7/P}\}_{(O,R_0)} - \{\tau_{P_1/P}\}_{(O,R_0)} \\ &= \begin{Bmatrix} \alpha_{P_7/P} - \alpha_{P_1/P} & U \\ U & v_{P_7/P} - v_{P_1/P} \\ \gamma_{P_7/P} - \gamma_{P_1/P} & U \end{Bmatrix} \end{aligned} \quad (10)$$

As shown in Fig.2, the variations of tolerated surface  $P_7$  relative to datum surface  $P_1$  is defined by the displacement of any point of the tolerance surface  $M_{P_7}$  compared to the point corresponding  $M_{P_1}$ . Since the displacement depends only on the rotation variations, it can be calculated as:

$$\begin{aligned} \overrightarrow{M_{P_1}M_{P_7}} &= \begin{pmatrix} \alpha_{P_7/P} - \alpha_{P_1/P} \\ U \\ \gamma_{P_7/P} - \gamma_{P_1/P} \end{pmatrix} \times \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} \\ &= \begin{pmatrix} -y_i(\gamma_{P_7/P} - \gamma_{P_1/P}) \\ x_i(\gamma_{P_7/P} - \gamma_{P_1/P}) - z_i(\alpha_{P_7/P} - \alpha_{P_1/P}) \\ y_i(\alpha_{P_7/P} - \alpha_{P_1/P}) \end{pmatrix} \end{aligned} \quad (11)$$



**Fig. 2.** Variations between tolerated surface and datum surface

To satisfy the parallelism tolerance requirement, one gets:

$$\overrightarrow{M_{P_1}M_{P_7}} \cdot \mathbf{n}_{P_1} \leq T_{pa} \quad (12)$$

where  $\mathbf{n}_{P_1} = \begin{pmatrix} \gamma_{P_1/P} \\ 1 \\ -\alpha_{P_1/P} \end{pmatrix}$  is the normal vector to the datum

plane  $A$ . By neglecting the higher-order terms beyond the first order, inequality (12) becomes:

$$x_i(\gamma_{P_7/P} - \gamma_{P_1/P}) - z_i(\alpha_{P_7/P} - \alpha_{P_1/P}) \leq T_{pa} \quad (13)$$

#### 4.2 Perpendicularity tolerance requirement

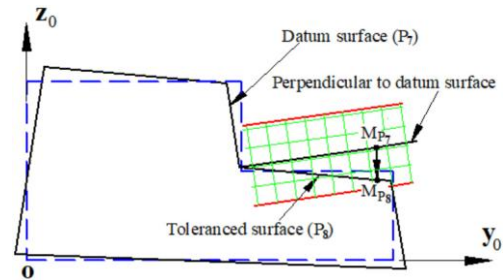
As shown in Fig.1, the perpendicularity tolerance of surface  $P_8$  related to datum surface  $B$  ( $P_7$ ) indicates that the actual surface  $P_8$  shall be contained between two parallel planes  $T_{pe}$  apart that are perpendicular to datum plane  $B$ .

One uses the torsors to model the variations of machined surface  $P_8$  related to datum surface  $B$  ( $P_7$ ) in the global reference frame  $R_0$ :

$$\begin{aligned} \{\tau_{P_8/P_7}\}_{(O,R_0)} &= \{\tau_{P_8/P}\}_{(O,R_0)} - \{\tau_{P_7/P}\}_{(O,R_0)} \\ &= \begin{Bmatrix} \alpha_{P_8/P} - \alpha_{P_7/P} & U \\ U & U \\ U & U \end{Bmatrix} \end{aligned} \quad (14)$$

As shown in Fig.3, the variations of tolerated surface  $P_8$  relative to datum surface  $P_7$  is defined by the displacement of any point of the tolerance surface  $M_{P_8}$  compared to the corresponding point  $M_{P_7}$  of the situation surface perpendicular to the datum surface  $P_7$ . Since the displacement also depends only on the rotation variations, it can be calculated as:

$$\overrightarrow{M_{P_7}M_{P_8}} = \begin{pmatrix} \alpha_{P_8/P} - \alpha_{P_7/P} \\ U \\ U \end{pmatrix} \times \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} = \begin{pmatrix} U \\ -z_i(\alpha_{P_8/P} - \alpha_{P_7/P}) \\ y_i(\alpha_{P_8/P} - \alpha_{P_7/P}) \end{pmatrix} \quad (15)$$



**Fig. 3.** Variations between tolerated surface and situation surface

Similarly, to satisfy the perpendicularity tolerance requirement, one has:

$$\overrightarrow{M_{P_7}M_{P_8}} \cdot \mathbf{n}_{P_7} \leq T_{pe} \quad (16)$$

where  $\mathbf{n}_{P_7} = \begin{pmatrix} \alpha_{P_7/P} \\ -\beta_{P_7/P} \\ 1 \end{pmatrix}$  is the normal vector to the

situation plane perpendicular to datum surface  $P_7$ . By neglecting the higher-order terms beyond the first order, inequality (16) becomes:

$$y_i(\alpha_{P_8/P} - \alpha_{P_7/P}) \leq T_{pe} \quad (17)$$

## 5 Application Example

This section presents a mechanical part (see Fig. 4) to demonstrate how the proposed method can be used for evaluating the manufacturing process. The functional requirements which are transferred are the parallelism of plane  $P_7$  with respect to datum  $A$  on plane  $P_1$ , the perpendicularity of plane  $P_8$  with respect to datum  $B$  on plane  $P_7$  as described in Fig. 4. And Figs. 5-7 show the machining process of this part, which consists of three set-ups performed on a numerical control (NC) machine-tool.

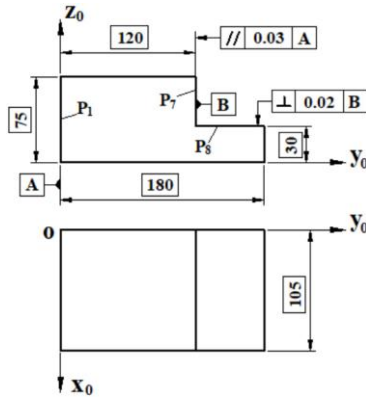


Fig. 4. Machined part geometry

### 5.1 Set-up 10

In set-up 10, there are no positioning surfaces. As shown in Fig. 5, the machined surfaces are marked as  $P_i$ , the local reference frame  $R_i (O_i, x_i, y_i, z_i)$  for each machined surface is defined as: Axis  $z_i$  is normal to surface  $P_i$  pointing towards the outside of the entity; Origin  $O_i$ , axes  $x_i$  and  $y_i$  of the reference frame belong to surface  $P_i$ . In the global reference frame  $R_0 (O, x_0, y_0, z_0)$  of Fig. 5, we have:

$$\{\tau_{P_i/P}\}_{(O_i, R_i)} = \begin{Bmatrix} \alpha_{P_i} & U_P \\ \beta_{P_i} & U_P \\ U_P & w_{P_i} \end{Bmatrix}, \quad i \in \{1, 2, \dots, 6\}$$

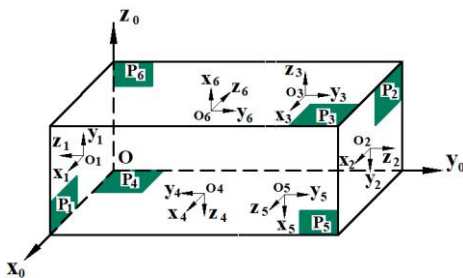


Fig. 5. The primarily machined part and reference frames

### 5.2 Set-up 20

In this set-up, surface 1 of the part is machined. As shown in Fig. 6 (a), the machined part is positioned on a plane (main positioning surface  $H_4/P_4$ ), a cylindrical surface with radius  $r_c$  (2<sup>nd</sup> positioning surface  $H_2/P_2$ ) and a spherical surface whose radius is  $r_s$  (3<sup>rd</sup> positioning surface  $H_6/P_6$ ) in an isostatic set-up. The global SDT  $\tau_{P/R}^{20}$  of the machined part can be obtained by combining the torsors associated with joints between the part and the part-holder. Here, the part-holder support points are also marked as  $O_i$  in a local frame  $R_i (O_i, x_i, y_i, z_i)$ ,  $i \in \{2, 4, 6\}$ .

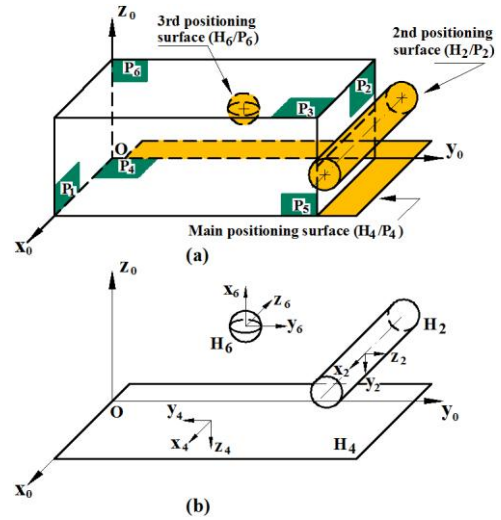


Fig. 6. Set-up 20: Milling of surface  $P_1$

As shown in Fig. 6(b), in a local frame  $R_i (O_i, x_i, y_i, z_i)$ ,  $i \in \{2, 4, 6\}$ , suppose the global variations of part-holder  $H$  are integrated within the deviation torsor of surface  $H_4$  relative to its nominal position in this set up for the main positioning surface  $H_4$ , and combine with the expression of the torsor matrices in Table 1, we have:

$$\{\tau_{H_4/H}^{20}\}_{(H_4, R_4)} = \begin{Bmatrix} \alpha_{H_4}^{20} & U_H \\ \beta_{H_4}^{20} & U_H \\ U_H & w_{H_4}^{20} \end{Bmatrix}$$

$$\{\tau_{H_2/H}^{20}\}_{(H_2, R_2)} = \begin{Bmatrix} U_H & U_H \\ \beta_{H_2}^{20} & v_{H_2}^{20} \\ \gamma_{H_2}^{20} & w_{H_2}^{20} \end{Bmatrix}$$

$$\{\tau_{H_6/H}^{20}\}_{(H_6, R_6)} = \begin{Bmatrix} U_H & u_{H_6}^{20} \\ U_H & v_{H_6}^{20} \\ U_H & w_{H_6}^{20} \end{Bmatrix}$$

We use Eq. (6) to calculate torsors  $\{\tau_{H_i/H}^{20}\}_{(O_i, R_i)}$ ,  $i \in \{2, 4, 6\}$ , in which the translation vectors from  $\{R_{H_i}\}$  to  $\{R_{O_i}\}$  are  $\overline{H_2 O_2} = (0, 0, -r_c)$ ,  $\overline{H_4 O_4} = (0, 0, 0)$  and  $\overline{H_6 O_6} = (0, 0, -r_s)$ , respectively. So for these three positioning surfaces, one will have:

$$\{\tau_{H_4/H}^{20}\}_{(O_4, R_4)} = \{\tau_{H_4/H}^{20}\}_{(H_4, R_4)} = \begin{Bmatrix} \alpha_{H_4}^{20} & U_H \\ \beta_{H_4}^{20} & U_H \\ U_H & w_{H_4}^{20} \end{Bmatrix}$$

$$\{\tau_{H_2/H}^{20}\}_{(O_2, R_2)} = \begin{Bmatrix} U_H & U_H - r_c \beta_{H_2}^{20} \\ \beta_{H_2}^{20} & v_{H_2}^{20} + r_c U_H \\ \gamma_{H_2}^{20} & w_{H_2}^{20} \end{Bmatrix}$$

$$\{\tau_{H_6/H}^{20}\}_{(O_6, R_6)} = \begin{Bmatrix} U_H & u_{H_6}^{20} - r_s U_H \\ U_H & v_{H_6}^{20} + r_s U_H \\ U_H & w_{H_6}^{20} \end{Bmatrix}$$

Then we can calculate the torsors  $\{\tau_{P_i/P}^{20}\}_{(O_i, R_0)}$  and

$\{\tau_{H_i/H}^{20}\}_{(O,R_0)}$   $i \in \{2, 4, 6\}$  at point  $O$  expressed in the global reference frame  $R_0$  ( $O, x_0, y_0, z_0$ ) by using Eq. (5), suppose the origin coordinate of the local reference frame  $R_i$  is  $O_i$  ( $a_i, b_i, c_i$ ),  $i \in \{2, 4, 6\}$  (see Fig.5), the rotation matrices  $\mathbf{R}_{0,i}$  from  $R_0$  to  $R_i$  respectively are:

$$\mathbf{R}_{0,2} = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{Bmatrix}, \mathbf{R}_{0,4} = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{Bmatrix}, \mathbf{R}_{0,6} = \begin{Bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{Bmatrix}$$

We get the expressions of  $\{\tau_{P_i/P}\}_{(O,R_0)}$ ,  $\{\tau_{P_2/P}\}_{(O,R_0)}$ ,  $\{\tau_{P_6/P}\}_{(O,R_0)}$ ,  $\{\tau_{H_4/H}^{20}\}_{(O,R_0)}$ ,  $\{\tau_{H_2/H}^{20}\}_{(O,R_0)}$ ,  $\{\tau_{H_6/H}^{20}\}_{(O,R_0)}$ .

According to Eq. (7), we can get the gap SDT  $\{\tau_{P_i/H_i}^{20}\}_{(O,R_0)}$  at point  $O$  expressed in global reference frame  $R_0$  ( $O, x_0, y_0, z_0$ ). Suppose  $\{\tau_{P_i/H_i}^{20}\}_{(O,R_0)} = \{\varphi \quad \varepsilon_O\}$  then,

$$\{\tau_{P_i/H_i}^{20}\}_{(O_i,R_i)} = \left\{ \mathbf{R}_{0,i}^T \cdot \varphi \quad \mathbf{R}_{0,i}^T \cdot (\varepsilon_O + \varphi \times \overline{OO_i}) \right\} \quad (18)$$

Using Eq. (18) to calculate the gap SDT  $\{\tau_{P_i/H_i}^{20}\}_{(O_i,R_i)}$  at point  $O_i$  expressed in local reference frame  $R_i$  ( $O_i, x_i, y_i, z_i$ ). And considering the properties (3) and (4), one gets:

$$\{\tau_{P_6/H_4}^{20}\}_{(O_4,R_4)} = \begin{Bmatrix} \alpha_{P_4} + \alpha_P^{20} - \alpha_H^{20} - \alpha_{H_4}^{20} & U \\ \beta_{P_4} - \beta_P^{20} + \beta_H^{20} - \beta_{H_4}^{20} & U \\ U & w_{P_4} - w_P^{20} + w_H^{20} - w_{H_4}^{20} - b_4 \alpha_P^{20} \\ & + b_4 \alpha_H^{20} + a_4 \beta_P^{20} - a_4 \beta_H^{20} \end{Bmatrix}$$

$$\{\tau_{P_2/H_2}^{20}\}_{(O_2,R_2)} = \begin{Bmatrix} U & U \\ \beta_{P_2} - \gamma_P^{20} + \gamma_H^{20} - \beta_{H_2}^{20} & U \\ U & w_{P_2} + v_P^{20} - v_H^{20} - w_{H_2}^{20} - c_2 \alpha_P^{20} \\ & + c_2 \alpha_H^{20} + a_2 \gamma_P^{20} - a_2 \gamma_H^{20} \end{Bmatrix}$$

$$\{\tau_{P_6/H_6}^{20}\}_{(O_6,R_6)} = \begin{Bmatrix} U & U \\ U & U \\ U & w_{P_6} - u_P^{20} + u_H^{20} - w_{H_6}^{20} - c_6 \beta_P^{20} \\ & + c_6 \beta_H^{20} + b_6 \gamma_P^{20} - b_6 \gamma_H^{20} \end{Bmatrix}$$

Considering the hierarchy of the part/part-holder positioning in an isostatic set-up, the components of  $\{\tau_{P_i/H_i}^{20}\}_{(O_i,R_i)}$  are nil because the contact between the two main positioning surfaces ( $H_4/P_4$ ) has no interpenetrating parts. One gets the following equations:

$$\begin{cases} \alpha_{P_4} + \alpha_P^{20} - \alpha_H^{20} - \alpha_{H_4}^{20} = 0 \\ \beta_{P_4} - \beta_P^{20} + \beta_H^{20} - \beta_{H_4}^{20} = 0 \\ w_{P_4} - w_P^{20} + w_H^{20} - w_{H_4}^{20} - b_4 \alpha_P^{20} + b_4 \alpha_H^{20} + a_4 \beta_P^{20} - a_4 \beta_H^{20} = 0 \end{cases}$$

Similarly, for the second and third positioning surfaces, we have:

$$\begin{cases} \beta_{P_2} - \gamma_P^{20} + \gamma_H^{20} - \beta_{H_2}^{20} = 0 \\ w_{P_2} + v_P^{20} - v_H^{20} - w_{H_2}^{20} - c_2 \alpha_P^{20} + c_2 \alpha_H^{20} + a_2 \gamma_P^{20} - a_2 \gamma_H^{20} = 0 \end{cases}$$

$$w_{P_6} - u_P^{20} + u_H^{20} - w_{H_6}^{20} - c_6 \beta_P^{20} + c_6 \beta_H^{20} + b_6 \gamma_P^{20} - b_6 \gamma_H^{20} = 0$$

Then,  $\{\tau_{P/R}^{20}\}_{(O,R_0)}$  can be derived as:

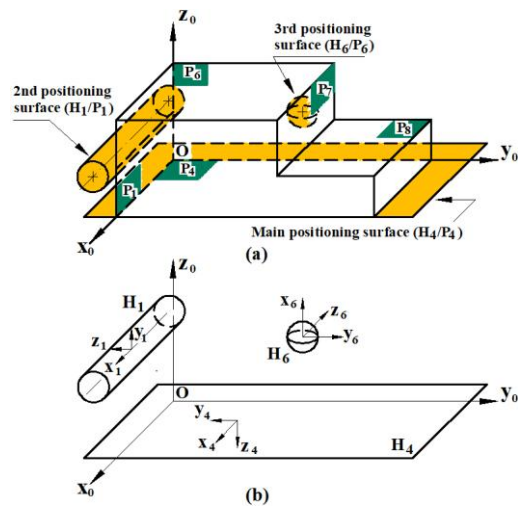
$$\{\tau_{P/R}^{20}\}_{(O,R_0)} = \begin{Bmatrix} -\alpha_{P_4} + \alpha_H^{20} + \alpha_{H_4}^{20} & w_{P_6} + u_H^{20} - w_{H_6}^{20} - c_6 \beta_{P_4} \\ & + c_6 \beta_{H_4}^{20} + b_6 \gamma_{P_4}^{20} - b_6 \gamma_{H_4}^{20} \\ \beta_{P_4} + \beta_H^{20} - \beta_{H_4}^{20} & -w_{P_2} + v_H^{20} + w_{H_2}^{20} - c_2 \alpha_{P_4} \\ & + c_2 \alpha_{H_4}^{20} - a_2 \beta_{P_2} + a_2 \beta_{H_2}^{20} \\ \beta_{P_2} + \gamma_H^{20} - \beta_{H_2}^{20} & w_{P_6} + w_H^{20} - w_{H_6}^{20} + b_4 \alpha_{P_4} \\ & - b_4 \alpha_{H_4}^{20} + a_4 \beta_{P_4} - a_4 \beta_{H_4}^{20} \end{Bmatrix}$$

Using Eq. (18), we get the expressions of  $\{\tau_{P/R}^{20}\}_{(O_i,R_i)}$  and  $\{\tau_{P_i/P}\}_{(O_i,R_i)}$ . According to Eq. (5), for the machined surface  $P_1$  in this set-up we have:

$$\{\tau_{P_1/P}^{20}\}_{(O,R_0)} = \begin{Bmatrix} \alpha_{P_1} + \alpha_{P_4} - \alpha_H^{20} - \alpha_{H_4}^{20} & U \\ U & -w_{P_1} + w_{P_2} - v_H^{20} - w_{H_2}^{20} + c_2 \alpha_{P_1} - c_2 \alpha_{H_4}^{20} \\ & + a_2 \beta_{P_2} - a_2 \beta_{H_2}^{20} - a_1 \beta_{P_1} + c_1 \alpha_{P_1} \\ \beta_{P_1} - \beta_{P_2} - \gamma_H^{20} + \beta_{H_2}^{20} & U \end{Bmatrix}$$

### 5.3 Set-up 30

In set-up 30, surfaces 7 and 8 are machined. As shown in Fig.7 (a), the part is positioned on a plane (main positioning surface  $H_4/P_4$ ), a cylindrical surface with radius  $r_c$  ( $2^{nd}$  positioning surface  $H_1/P_1$ ) and a spherical surface with radius  $r_s$  ( $3^{rd}$  positioning surface  $H_6/P_6$ ) in an isostatic set-up.



**Fig.7.** Set-up30: Milling of surface  $P_7$

Fig. 7 (b) shows the positioning device  $H_i$  of the part-holder in a local frame  $R_i$  ( $O_i, x_i, y_i, z_i$ ),  $i \in \{1, 4, 6\}$ . Consider the main positioning surface  $H_4$ , and suppose the global variations of part-holder  $H$  are integrated within the deviation torsor of surface  $H_4$  relative to its nominal position in set up 30. Similarly, according to Eq. (6) we have the expressions of  $\{\tau_{H_4/H}^{30}\}_{(O_4,R_4)}$ ,  $\{\tau_{H_1/H}^{30}\}_{(O_1,R_1)}$  and  $\{\tau_{H_6/H}^{30}\}_{(O_6,R_6)}$ .

Suppose the origin coordinate of the local reference

frame  $R_i$  is  $O_i (a_i, b_i, c_i)$ ,  $i \in \{1, 4, 6\}$  (see Fig. 5), we can calculate the torsors  $\{\tau_{P_i/P}\}_{(O,R_0)}$  and  $\{\tau_{H_i/H}\}_{(O,R_0)}$  at point  $O$  expressed in the global reference frame  $R_0 (O, x_0, y_0, z_0)$  by using Eq. (5), one gets the expressions of  $\{\tau_{P_6/P}\}_{(O,R_0)}$ ,  $\{\tau_{P_7/P}\}_{(O,R_0)}$ ,  $\{\tau_{P_8/P}\}_{(O,R_0)}$ ,  $\{\tau_{H_4/H}\}_{(O,R_0)}$ ,  $\{\tau_{H_6/H}\}_{(O,R_0)}$  and  $\{\tau_{H_8/H}\}_{(O,R_0)}$ . According to Eq. (7), we can get the gap SDT  $\{\tau_{P_i/H_i}^{30}\}_{(O,R_0)}$  at point  $O$  expressed in global reference frame  $R_0 (O, x_0, y_0, z_0)$ . Using Eq. (18)

$$\begin{aligned} \{\tau_{P/R}^{30}\}_{(O,R_0)} &= \left\{ \begin{array}{l} -\alpha_{P_6} + \alpha_H^{30} + \alpha_{H_4}^{20} \quad w_{P_6} + u_H^{30} - w_{H_6}^{30} - c_6\beta_{P_6} + c_6\beta_{H_4}^{20} - b_6\beta_{P_6} + b_6\beta_{P_2} + b_6\gamma_H^{20} - b_6\beta_{H_2}^{20} + b_6\beta_{H_1}^{30} \\ \beta_{P_6} + \beta_H^{30} - \beta_{H_4}^{20} \quad w_{P_6} - w_{P_2} + v_H^{20} + w_{H_2}^{20} - c_2\alpha_{P_6} + c_2\alpha_{H_4}^{20} - a_2\beta_{P_2} + a_2\beta_{H_2}^{20} \\ \quad + v_H^{30} - w_{H_1}^{30} - c_1\alpha_H^{20} + a_1\beta_{P_6} - a_1\beta_{H_1}^{30} \\ -\beta_{P_6} + \beta_{P_2} + \gamma_H^{20} - \beta_{H_2}^{20} + \gamma_H^{30} + \beta_{H_1}^{30} \quad w_{P_6} + w_H^{30} - w_{H_4}^{30} + b_4\alpha_{P_6} - b_4\alpha_{H_4}^{20} + a_4\beta_{P_6} - a_4\beta_{H_4}^{20} \end{array} \right\} \\ \{\tau_{P_7/P}\}_{(O,R_0)} &= \left\{ \begin{array}{l} \alpha_{P_7} + \alpha_{P_4} - \alpha_H^{30} - \alpha_{H_4}^{20} \quad U \\ U \quad w_{P_7}^{30} + a_7\beta_{P_7}^{30} + c_7\alpha_{P_7}^{30} - w_{P_7} + w_{P_2} - v_H^{20} - w_{H_2}^{20} + c_2\alpha_{P_4} - c_2\alpha_{H_4}^{20} \\ \quad + a_2\beta_{P_2} - a_2\beta_{H_2}^{20} - v_H^{30} + w_{H_1}^{30} + c_1\alpha_H^{20} - a_1\beta_{P_7} + a_1\beta_{H_1}^{30} \\ -\beta_{P_7}^{30} + \beta_{P_1} - \beta_{P_2} - \gamma_H^{20} + \beta_{H_2}^{20} - \gamma_H^{30} - \beta_{H_1}^{30} \quad U \end{array} \right\} \\ \{\tau_{P_8/P}\}_{(O,R_0)} &= \left\{ \begin{array}{l} \alpha_{P_8} + \alpha_{P_4} - \alpha_H^{30} - \alpha_{H_4}^{20} \quad U \\ \beta_{P_8}^{30} - \beta_{P_4} - \beta_H^{30} + \beta_{H_4}^{20} \quad U \\ U \quad w_{P_8}^{30} + a_8\beta_{P_8}^{30} - b_8\alpha_{P_8}^{30} - w_{P_8} - w_H^{30} + w_{H_4}^{30} - b_4\alpha_{P_8} + b_4\alpha_{H_4}^{20} - a_4\beta_{P_8} + a_4\beta_{H_4}^{20} \end{array} \right\} \end{aligned}$$

### 5.4 Respect of the functional tolerances

According to Section 3, we establish the relationship between the functional tolerances and the geometrical variations in the manufacturing process to evaluate the validity of the manufacturing process.

- Consider the parallelism tolerance requirement between surface  $P_1$  and  $P_7$ . The variation torsor of surface  $P_7$  relative to surface  $P_1$  (datum surface) is:

$$\{\tau_{P_7/P_1}\}_{(O,R_0)} = \left\{ \begin{array}{l} \alpha_{P_7} - \alpha_H^{30} - \alpha_{P_1} + \alpha_H^{20} \quad U \\ U \quad w_{P_7}^{30} + a_7\beta_{P_7}^{30} + c_7\alpha_{P_7}^{30} - v_H^{30} \\ \quad + w_{H_1}^{30} + c_1\alpha_H^{20} + a_1\beta_{H_1}^{30} - c_1\alpha_{P_1} \\ -\beta_{P_7}^{30} - \gamma_H^{30} - \beta_{H_1}^{30} \quad U \end{array} \right\}$$

The geometrical variation between tolerance surface  $P_7$  and its nominal position related to the datum plane  $P_1$  is defined by the displacement of any point of  $P_7$  compared to the corresponding point of  $P_1$ . And this variation only depends on rotation variations, which can be calculated as:

$$45(-\alpha_{P_7}^{30} + \alpha_H^{30} + \alpha_{P_1} - \alpha_H^{20}) - 105(\beta_{P_7}^{30} + \gamma_H^{30} + \beta_{H_1}^{30}) \leq 0.03$$

- Consider the perpendicularity tolerance requirement between surface  $P_7$  and  $P_8$ . And this variation also depends only on rotation variations. It needs to check all the points of the toleranced surface are located in the spatial region between two parallel planes spaced with a distance of the perpendicularity tolerance (theoretically perpendicular to the specified datum plane). The variation of toleranced surface  $P_8$  relative to datum surface  $P_7$  can be written as:

to calculate the gap SDT  $\{\tau_{P_i/H_i}^{30}\}_{(O_i,R_i)}$  at point  $O_i$  expressed in reference frame  $R_i (O_i, x_i, y_i, z_i)$ . And considering the properties (3) and (4), one gets the expressions of  $\{\tau_{P_6/H_4}^{30}\}_{(O_4,R_4)}$ ,  $\{\tau_{P_7/H_1}^{30}\}_{(O_1,R_1)}$  and  $\{\tau_{P_8/H_6}^{30}\}_{(O_6,R_6)}$ .

So  $\{\tau_{P/R}^{30}\}_{(O,R_0)}$ ,  $\{\tau_{P_7/P}\}_{(O,R_0)}$  and  $\{\tau_{P_8/P}\}_{(O,R_0)}$  can be derived as:

$$\begin{aligned} \{\tau_{P_8/P_7}\}_{(O,R_0)} &= \{\tau_{P_8/P}\}_{(O,R_0)} - \{\tau_{P_7/P}\}_{(O,R_0)} \\ &= \left\{ \begin{array}{l} \alpha_{P_8}^{30} - \alpha_{P_7}^{30} \quad b_8U_P - c_8\beta_{P_8}^{30} + c_7U_P + b_7\beta_{P_7}^{30} \\ \beta_{P_8}^{30} - U_P \quad U_P - a_8U_P + c_8\alpha_{P_8}^{30} - w_{P_7}^{30} - a_7\beta_{P_7}^{30} - c_7\alpha_{P_7}^{30} \\ \beta_{P_7}^{30} + U_P \quad w_{P_8}^{30} + a_8\beta_{P_8}^{30} - b_8\alpha_{P_8}^{30} + U_P - a_7U_P + b_7\alpha_{P_7}^{30} \end{array} \right\} \end{aligned}$$

In order to respect the parallelism tolerance requirement, the following inequality should be satisfied:

$$|60(\alpha_{P_8}^{30} - \alpha_{P_7}^{30})| \leq 0.02$$

## 6 Conclusions

Our research focuses on the development of 3D manufacturing variation model and verifying the validity of manufacturing process in multi-station machining processes. In multi-station machining process, each machining set-up is regarded as a single mechanism, and SDTs are employed to express the geometrical variations of parts caused by the machining operations and positioning dispersions during the successive machining set-ups. The SDT chain can be established based on the link between the process planning and functional tolerances, which makes it possible to evaluate the manufacturing process. The proposed method is illustrated by an example.

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