The effect of slenderness ratio on water entry

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Abstract. In the paper, the air cavity created by vertical water entry of cylinders with different slenderness ratio (defined as “ $s=L/D$ ”) is investigated experimentally, theoretically, and computationally. The study is focused on the range of Froude numbers, $Fr = V/(gr)^{0.5}$, ($12 < Fr < 30$). Particular attention is given to the effect of slenderness ratios on the surface seal time, the evolution process of the cavities. To understand the water entry physical processes, we conduct several experiments of water entry of different length cylinders. A theory model considering the added mass (induced by the water motion), buoyancy force, gravity, and water-resisting force is developed to predict the relationship between the falling distance and evolution time. Studies show that for the same size bullet the surface seal time of the cavity decreases with the increase of impact speed. Under the identical impact speed, the cavity seal time decreases with the increase of the slenderness ratio. For the consistent impact speed, with the value of “ s ” increase, the falling distance increase (the same moment). The falling distance captured by the developed theoretical model match with the experimental and numerical data.

1 Introduction

Water entry is a very complex physical phenomenon, but it is common in nature and engineering. The water entry of torpedoes, skipping of boats, dip-coating procedures and even the motion of some animals are both associated with water entry [1]. Physical phenomenon can also be applied to sports performance research, relating to the water entry of athletes, the drag reduction of swimmers near the free surface, the decrease in cavity formation for divers, and the entry and exit of oars in rowing [2]. When a solid object strikes a liquid surface, it may create an air cavity and splash curtain. The existing of the air cavity and splash crown have grate influence of the object's subsequent trajectory [3]. Due to the wide presence, water entry has been of interest for the scientists for over a century [4]. Many researchers have done huge works on water entry problems both in experimental and numerical and theoretical methods. Worthington & Cole (1897) firstly use single-spark illumination photography to investigate cavity formation and splash characteristics produced by spheres impact onto liquid surfaces [5]. Following their work, Mallock studied the sound produced by drops falling on water [6]. Bell investigated the impact of solid sphere with a liquid surface [7]. May investigated the effect of sphere's surface condition on the water-entry cavity [8] and the cavity-running behavior of water entry missiles [9]. They found that the cleaner of sphere surface the easier to induce a cavity under the same low impact speed, and they also show that the increase of the fluid viscosity produce a bigger cavity under the same speed. May & Hoover collectively investigated water-entry phenomenon [10]. Abelson measured the pressure fluctuation of water-entry process [11] and made a

prediction of the water-entry cavity [12]. Glasheen J W et al. studied the vertical water entry of disks at low Froude numbers [13]. Yan H et al. investigated the cavity dynamics in water entry at low Froude numbers [14]. They developed a theory method to predict the cavity profile. Fan C et al. have done many experiments to study the different behaviors of spheres entering water and PEO solution [15]. Duclaux et al. investigated the transient performances of cavities [16]. Yao E et al. did some investigation of vertical water-entry of bullet-shaped projectiles [17]. Guo Z et al. have done an investigation into horizontal water entry behaviors of projectiles with different nose shapes [18].

For the current research, the main research target of water entry is a sphere, and the studies are mostly concentrating on the cavity evolution process, the impact of physical characters (projective geometry, material properties, fluid properties et al.). But the effect of cylinder shape projectile with different slenderness ratio on the water entry, especially on the surface seal time and the different evolution process of the cavity, are seldom investigated. Those are what we will explore in the paper.

In the paper the air cavity created by vertical water entry of cylinders with different slenderness ratio (defined as “ s ”) is investigated experimentally, theoretically, and computationally. The study is focused on the range of Froude numbers, $Fr = V/(gr)^{0.5}$, ($12 < Fr < 30$). Particular attention is given to the effect of slenderness ratios on the surface seal time, the evolution process of the cavities. To understand the water entry physical processes, we conduct several experiments of water entry of different length cylinders. A theory model considering the additional mass (induced by the water motion), buoyancy force, gravity, and water-resisting force is developed to predict the relationship between the falling distance and evolution

time.

The paper is structured as follows: section 2 provides the experimental set-up and the experimental process. In section 3 the experimental results of the water entry are introduced. Subsequently, in section 4 the theoretical model is introduced, and the comparisons with the experimental data are analyzed. Finally, a summary and conclusion of the study is provided in section 5.

2 Experimental set up

A schematic representation of the experimental set-up is shown in figure 1.

Fig 1. Schematic of the experimental set-up.

The experimental system mainly includes four parts: the high-speed camera and computer, the guided release device, the water tank (transparent acrylic), and the light source. The frame rates of the camera (NAC HX-7s) is up to 200000 frames per second, the 1500 frames s⁻¹ is adopted in the experiment. The minimum exposure time of the camera is 1.1μs but it can be adjusted according to the need, our exposure time is 5μs. The guided release device is placed above the center of the water tank (50cm × 50cm × 80cm), and the height of the device can change through the height adjuster unit. The depth of the liquid, however, was kept at 60 cm to minimize splashing out of the tank. Two DISON 100w LED lights were used as light sources.

The experimental process is shown in figure 2. The camera adopts the post-trigger mode. The bullet can release before the camera shoots. And the video sequences can download to a computer for analysis.

Fig 2. Experimental process.

In the study, the aluminum cylinder was chose to explore the water entry phenomenon. The dimeter of the bullet is $D=6\text{mm}$, the length of the bullet is $L=18\text{mm}$, 30mm , 42mm , and 54mm respectively. The slenderness ratio is defined as $s=L/D$. The density of the bullet is $\rho=2700\text{kg/m}^3$. The normal contact angle is $\theta=75^\circ$. The bullet with different “s” is showed in figure 3.

Fig 3. The bullet models. (a) $s=3$; (b) $s=5$; (c) $s=7$; (d) $s=9$.

The liquid pool temperature was measured to be approximately the same as the room temperature of 25°C . The key parameters of water are the density $\rho_w = 998\text{kg/m}^3$, the viscosity $\mu = 0.895 \times 10^{-3}\text{Pa}\cdot\text{s}$, and surface tension $\sigma_w = 0.072\text{kg}\cdot\text{m}^{-2}$.

3 Experiment results

3.1 Impact speed (Fr) effect

The water entry process of cylinder bullet with different slenderness ratio under different Froude number ($Fr = V/(gr)^{0.5}$, $12 < Fr < 30$) were studied in the paper.

Figure 4 shows the comparison of water entry process of “s=5” bullet under the $Fr = 12.47$ and 17.02 .

The water entry cavity pinch-off models are different in Fig.4 “A” and “B”. The cavity in Fig.4 “A” is pitched off under the water surface (deep seal), while in “B” is pitched off above the water surface (surface seal). And the dimension of the cavity, carried into the water by the bullet, is also different. The cavity diameter and length induced in “B” is larger than in “A”.

The results in Fig.4 indicates that as the increase of the Fr (impact speed) the cavity dimension increase, and the cavity seal models varies from deep seal to surface seal.

According to our study the result is also applicable to $s=3$, $s=7$, $s=9$ bullet models. That is to say when the $Fr > 17$, the cavity with seal above the liquid surface. And the with the further increase of the impact speed the cavity seal time will decrease. As shown in figure 5.

Figure 5 shows the relationship between the cavity surface seal time and the impact speed of “s=3” bullet. When the bullet strikes the water in a lower speed ($Fr=12.59$), the cavity (deep sealed at time =38ms) is never sealed above the water surface. With the increase of the Froude number the cavity turned from deep seal to surface seal. The seal time decreases as the Froude number further increase.

Fig 4. Water entry process of “s=5” bullet under different Fr numbers. A, $Fr = 12.47$; B, $Fr = 17.02$.

Fig 5. The cavity seal time under different Froude number. The cavity is induced by s=3 bullet. The time starts when the bullet touches the water surface. (a) $Fr=12.59$; (b) $Fr=17.08$; (c) $Fr=23.96$; (d) $Fr=29.55$.

3.2 slenderness ratio effect

Section 3.1 shows the influence of the impact speed (Froude number) on the cavity pinch off models and surface seal times. Compared with the influence of the Froude number on the cavity, the influence of the

slenderness ratio is different. The slenderness ratio has no effect on the cavity pinch off models, as shown in Fig.6. It can affect the cavity surface seal time (Fig.7) and the bullet falling distance (Fig.8 and 9).

The Fig.6 shows when the $12.2 < Fr < 12.8$, the cavity pinched off under the water, the change of the slenderness ratio do not change the cavity seal model. When $s=5$ the cavity pinched off above the bullet and the cavity can cover the whole bullet. When $s=7$ and $s=9$ the cavity pinched off at the body of the bullet, the cavity cannot cover the whole bullet. The cavity pinched off time of $s=7$ and $s=9$ is smaller than $s=5$.

For the surface seal cavity, the slenderness ratio can affect the seal time (Fig.7). As the increase of the slenderness ratio (the increase of the bullet length) the seal time of the cavity decrease. Because with the bullet length increase the cavity will close on the bullet, so the closing time reduced.

Fig 6. The slenderness ratio effect on the cavity ($12.2 < Fr < 12.8$). “A” $s=5$, “B” $s=7$, “C” $s=9$.

Fig 7. The cavity surface seal time($23.3 < Fr < 24$). (a) $s=3$, (b) $s=5$, (c) $s=7$. The cavity seal time is 6ms, 5.3ms, 4ms, respectively.

Fig 8. The bullet falling distance (from water surface). “A” $s=3$, “B” $s=5$, “C” $s=7$.

Fig 9. The relationship of falling distance and time under different slenderness ratios. ($17.02 < Fr < 17.5$).

The falling distance of the bullet under the same water entry speed is related to the slenderness ratio. As the slenderness ratio increase the falling distance increase (Fig.8 and Fig.9) but the increase speed of the falling distance of the bullet slows down (Fig.9).

In the study the Froude number is relatively small, and the influence of gravity cannot be ignored. The density of the bullet is heavier than water. According to Newton's second law, the acceleration of the bullet can be described as:

$$a = \frac{mg - f}{m} = g - \frac{f}{m}$$

Where a is the acceleration of the bullet, g is the gravitational acceleration, m represents the mass of the bullet, and f is the other external force. The force f mainly includes the pressure resisting force f_d and the buoyancy force f_b .

The increase of the slenderness ratio of the bullet induced the increase of the mass (the diameter and density of the bullets are the same and is constant).

The mass of different length bullets is shown in table 1.

Table 1. Mass of different “s” bullets.

s	3	5	7	9
Mass(g)	1.31	2.19	3.00	3.88

When the bullet is totally covered by the cavity and the bullet moves at a similar speed $V(0)$ (Fig.8), the pressure resisting force and the buoyancy force of the bullet is very close, so the acceleration of the bullet would increase with the increase of the mass. The falling distance (at the same moment) will increase with the mass increase. But as the slenderness ratio increase further, the cavity induced by the bullet probably cannot cover the whole bullet (Fig.6 B, C). When the water wetting the bullet, the resistance force of the bullet increase, so the acceleration of the bullet decrease, causing the falling distance changing speed slows down.

The slenderness ratio can also affect the cavity size (Fig.10). when the slenderness ratio is relatively small the cavity can cover the whole bullet, the cavity dimension (diameter and length) is nearly the same. As the length of the bullet increase, the bullet affects the cavity evolution process. In Fig.10(d) the cavity sticks to the bullet and falls off along the bullet causing the increase of the cavity length.

4 Theoretical analyze

In the study the Froude number is relatively small, so the gravity effect on the bullet motion cannot be ignored. Considering the water-entry process, the bullet sinks under the combined influence of gravity and its own inertia and is resisted by buoyancy and hydrodynamic forces, respectively. A vertical force balance on the bullet may be expressed as

$$(m + \delta)Z''(t) = mg - f_d - f_b \quad (1)$$

Fig 10. The cavity profile of different slenderness ratio bullet, $17.02 < Fr < 17.5$. (a) $s=3$, (b) $s=5$, (c) $s=7$, (d) $s=9$.

Where $Z(t)$ is the depth of the center of the bullet, $Z''(t)$ is the acceleration of the bullet at t moment, $m = \rho\pi R^2 L$ is the bullet mass. The force required to accelerate the surrounding fluid is expressed in terms of an added mass $\delta = C_m \rho_L V$, where C_m is the added mass coefficient, for the sake of simplicity assume a constant value, V is the bullet volume, and ρ_L is the liquid density. And the $f_d = \frac{1}{2} C_d \rho_L A_0 Z' Z'$ is the pressure resisting force.

The drag coefficient, C_d is typically taken to be constant with respect to the impact parameters. The $f_b = \rho_L A_0 g L$ is the upward buoyancy force. L is the length of the bullet A_0 is the area of the bullet. Substituting the f_d, f_b and δ into Eq. (1) yields

$$Z'' = a - b Z' Z' \quad (2)$$

$$\text{where } a = \left(\frac{m - \delta}{m + \delta} \right) g, \quad b = \frac{\delta C_d}{2(m + \delta) L}.$$

Integrating Eq. (2) once in Z gives an expression for the bullet speed,

$$U(t) = U_0 - \frac{\sqrt{a} \tanh(\sqrt{ab} \cdot t)}{\sqrt{b}} \quad (3)$$

where U_0 is the initial speed (the speed arriving the water surface) of the bullet.

Integrating Eq. (3) once in Z gives the expression of the bullet fall distance,

$$Z(t) = U_0 t - \frac{\ln(\cosh(\sqrt{ab} \cdot t))}{b} \quad (4)$$

Figure 11 shows the experimental, theoretical and numerical comparison of the falling distance of the bullet over time. The theoretical results of the bullet falling distance match the experimental data well. So the theoretical model developed in the paper is feasible. To a certain extent, it can reflect the relationship between the falling distance of the bullet and time.

Fig 11. The comparison of the falling distance of the bullet over time. $s=7$, $Fr=17.05$.

5 Conclusions

The hydrodynamic problem of air cavity development and evolution in vertical water entry of an axisymmetric body is studied. The study is focused on the range of Froude

numbers, $Fr = V/(gr)^{0.5}$, ($12 < Fr < 30$). Particular attention is given to the effect of slenderness ratios on the surface seal time, the evolution process of the cavities. A theory model considering the added mass (induced by the water motion), buoyancy force, gravity, and water-resisting force is developed to predict the relationship between the falling distance and evolution time. Studies show that for the same size bullet the surface seal time of the cavity decreases with the increase of impact speed (Fig.5). Under the identical impact speed, the cavity seal time decreases with the increase of the slenderness ratio (Fig.7). For the consistent impact speed, with the value of “s” increase, the falling distance increase (Fig.8), but the falling distance changing speed slows down (Fig.9). The theoretical results of the bullet falling distance match the experimental data well. So the theoretical model developed in the paper is feasible. To a certain extent, it can reflect the relationship between the falling distance of the bullet and time.

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