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Design and evaluation of a quasi-zero-stiffness isolator using flexibly supported negative stiffness mechanism

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Abstract: In traditional quasi-zero-stiffness (QZS) isolation system, the negative stiffness part is usually fixed rigidly, lacking of effective amplifying mechanism. For reaching a quasi-zero state, the value of negative stiffness need to be very large to offset the positive stiffness of the structure. This paper proposes a novel isolator incorporating a flexible support to magnify negative stiffness part for effective realization of quasi-zero state. First, the concept and formulation of the innovative quasi-zero isolator are presented. Equivalent model for the flexibly supported negative stiffness part is established, followed by a parametric analysis to reveal the amplification effect. Thereafter, a design method is developed and numerical simulation is performed to verify the isolating performance. The results show that a flexible support enlarges the negative stiffness remarkably, thereby resulting in a cost-effective design. The optimized QZS isolator is capable of isolating external disturbance significantly.

1 Introduction

In recent years, the quasi-zero-stiffness (QZS) isolation system has attracted increasing attention owing to its excellent capacity of isolating external disturbance. To realize the characteristics of high static and low dynamic stiffness, a QZS isolator usually incorporates a negative stiffness mechanism to offset the positive stiffness of the structure self. Therefore, how to produce the negative stiffness behaviour turns out to be of great importance for a QZS isolator.

Many researchers have studied the realization of negative stiffness mechanism. To form the isolation device, Zhou et al [1] proposed an adjustable negative stiffness configuration based on the electromagnetic interaction. Benjamin et al [2] used the buckling beam fixed at both ends to achieve the passive negative stiffness, thereby isolating the external vibration or impact. Xu et al [3] obtained the unique relationship between geometry configuration and stiffness of spring elements, aiming to design the property of high static low dynamic stiffness. Combining cam, roller, and spring, Zhou et al [4] developed a kind of nonlinear vibration isolator, in which the over stiffness approaches nearly zero.

For the aforementioned QZS isolators, the negative stiffness mechanism is usually in parallel with the positive stiffness in mechanical sense. Hence, the value of the negative stiffness part needs to be as large as the positive stiffness part. Thereafter, the entire device exhibits the quasi-zero behaviour. In general, the positive stiffness produced by the structure is very large. With the positive stiffness increasing, the negative stiffness should also increase accordingly. As a result, the whole system will become complicated and inconvenient, which surely blocks further application.

As discussed above, an amplification mechanism is needed for the negative stiffness part to make the design cost-effective. In the field of utilizing negative stiffness dampers for structural vibration control [5], Zhou et al [6] found that a serial flexible support, rather than a rigid support, has the ability to amplify negative stiffness to improve the control performance. Inspired by this, the current study will develop a QZS isolator using flexibly supported negative stiffness mechanism, so as to realize cost-effective design.

2 Novel QZS isolator using flexibly supported NS mechanism

2.1 Conceptual structure

As shown in Fig. 1, the novel QZS isolator contains a flexible support for the negative stiffness part. In a traditional QZS isolator, the negative stiffness part k_n is in parallel with the positive stiffness part k to lift the mass together. Being different from this, in the novel QZS isolator, a flexible support is introduced. Therefore, the negative stiffness part is first in series with the flexible support, then in parallel with the positive stiffness.

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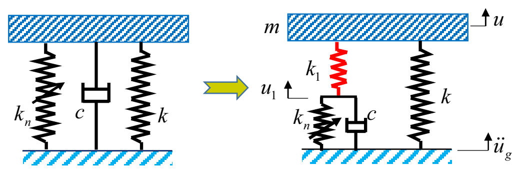

Fig. 1. Conceptual structure of the proposed QZS isolator.

2.2 System formulation

According to the structural configuration, the equation of motion for the isolating system is written as,

$$m\ddot{u} + ku + k_1\left(u - u_1\right) = -m\ddot{u}_g \tag{1}$$

where m is the mass, k is the positive stiffness element, k_1 is the flexible support, u and $\ddot{u}$ are the relative displacement and acceleration of the mass, respectively, u_1 is the deformation across the flexible support, $\ddot{u}_g$ is the base excitation.

For the flexible support, there exists the force relationship,

$$c\dot{u}_1 + \left(k_1 + k_n\right)u_1 - k_1 u = 0 \tag{2}$$

where c is the viscous damping, k_n is the negative stiffness.

Assumes the base excitation is a sinusoidal load, having the form,

$$\ddot{u}_g = A_0 e^{i\omega t} \tag{3}$$

where A_0 is the amplitude of the base excitation, $i = \sqrt{-1}$ is the imaginary unit, ω is the angular frequency.

Under such an external load, the relative displacement of the mass is also sinusoidal,

$$u = Ae^{i\omega t} \tag{4}$$

where A is the amplitude.

To keep the stability of the system, the lower limit of the flexible support satisfies,

$$k_1 + k_n > 0 \tag{5}$$

Therefore, defines the following dimensionless indices,

$$\gamma_k = k_1/k_n < -1 \tag{6}$$

$$\gamma_c = \omega c/k_n < 0 \tag{7}$$

Substituting eq. (4) into eq. (3), after simplification, it can be obtained,

$$\frac{k_e}{k_n} = \gamma_k \frac{1 + \gamma_k + \gamma_c^2}{\left(1+\gamma_k\right)^2 + \gamma_c^2} \tag{8}$$

$$\frac{c_e}{c} = \frac{\gamma_k^2}{\left(1+\gamma_k\right)^2 + \gamma_c^2} \tag{9}$$

where k_e is the equivalent negative stiffness after introducing the flexible support, c_e is the equivalent viscous damping after introducing the flexible support.

Hence, the equation of motion for the isolating system becomes,

$$m\ddot{u} + c_e\dot{u} + \left(k + k_e\right)u = -m\ddot{u}_g \tag{10}$$

Then, the amplitude of the relative displacement is obtained as

$$A = -\frac{A_0}{\omega_n^2 - \omega^2 + k_e/m + i\omega c_e/m} \tag{11}$$

where $\omega_n = \sqrt{k/m}$ is the natural frequency of the system.

Lastly, the acceleration transmissibility is derived as,

$$TR = \frac{\left|\ddot{u} + \ddot{u}_g\right|}{\left|\ddot{u}_g\right|} = \left[\frac{\left(1 + k_e/k\right)^2 + \left(\theta c_e/m\omega_n\right)^2}{\left(1 - \theta^2 + k_e/k\right)^2 + \left(\theta c_e/m\omega_n\right)^2}\right]^{\frac{1}{2}} \tag{12}$$

where TR is the acceleration transmissibility, $\theta = \omega/\omega_n$ represents the ratio of excitation frequency to the natural frequency.

3 Optimization of the flexible support

3.1 Parameter ranges

As shown in eq. (8), k_e/k_n represents the magnification of negative stiffness due to the introduction of flexible support. Letting $k_e/k_n > 1$, it is obtained that,

$$\gamma_c + \sqrt{-\left(1+\gamma_k\right)/\left(1-\gamma_k\right)} > 0 \tag{13}$$

Similarly, letting $c_e/c > 1$, it will lead to,

$$\gamma_c + \sqrt{-1 - 2\gamma_k} > 0 \tag{14}$$

Considering the range of $\gamma_k < -1$, eq. (14) will always hold when the dimensionless index γ_c satisfies eq. (13). Under this condition, the flexible support is able to amplify the negative stiffness and viscous damping at the same time.

3.2 Design procedure

In the above parameter range, using a flexible support can simplify the design procedure of a QZS isolator. If the negative stiffness k_n is much less than the positive stiffness k, a flexible support can be incorporated, of which the stiffness is,

$$k_1 = -kk_n/\left(k + k_n\right) \tag{15}$$

Thereafter, the whole design procedure is illustrated as in Fig. 2.

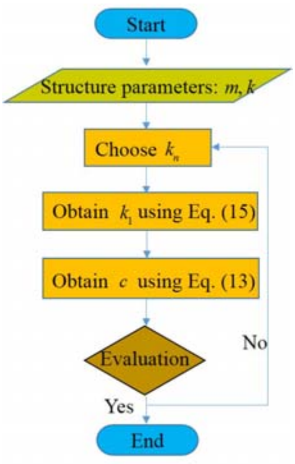

Fig. 2 Optimized design procedure for the flexible support.

As a start, the first step is the collection of structure parameters, including the mass and positive stiffness. Then, a negative stiffness, much less the positive stiffness, is chosen. Thereafter, according to eq. (15), the viscous damping is calculated. Determining all the parameters, numerical simulation is conducted to evaluate the isolating performance. If not satisfied, a new value of negative stiffness is selected for another try, until the performance meets the given requirement.

4 Results and discussions

4.1 Effect of flexible support

Figure 3 shows the amplification factor of negative stiffness. The horizontal axis is exciting frequency and the vertical axis is the ratio of k_e/k_n . In theory, when the support is an idealized rigid one, there is no amplification effect on the negative stiffness. For $\gamma_k = -10$, the support is still relatively rigid and the amplification factor is slightly greater than unity. When $\gamma_k = -1.2$, the support becomes a flexible one. When the frequency is zero, k_e/k_n is equal to 6, which means the original negative stiffness value is magnified by 6 times due to the connection of a flexible support. With the frequency increasing, the amplification factor exhibits a slight decrease trend, but is always larger than 1.

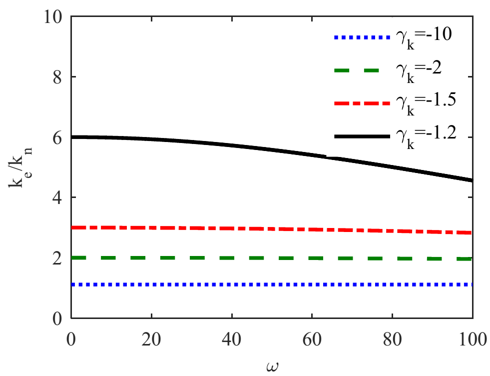

Fig. 3. Amplification factor **of** the negative stiffness**.**

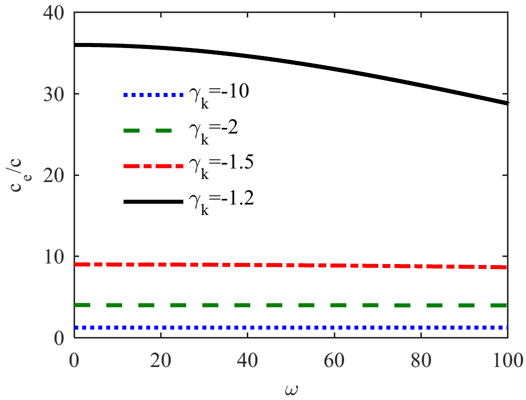

Fig. 4. Amplification factor of the viscous damping**.**

Similarly, the changing trend for the amplification factor of viscous damping is portrayed in Fig. 4. When $\gamma_k = -1.2$, the support has large flexibility. Under this case, the magnification of viscous damping is about more than 30, meaning a remarkable magnification effect. With γ_k decreasing to -10, the support is becoming more rigid. However, the ratio c_e/c approaches unity. Hence, a rigid support has no amplification effect for the viscous damping.

4.2 Design prototype

As analysed above, a flexible support has the ability to amplify the negative stiffness and viscous damping at the same time. This section will investigate how to utilize this beneficial effect by design examples.

Assume the mass $m = 100\text{kg}$ and the positive stiffness is $k = 3.95\text{kN/m}$. According to the design procedure in section 3.2, there exists a series of combination of support stiffness and negative stiffness to realize the quasi-zero state in the isolation system, as shown in Fig. 5. When the support stiffness k_1 is about 0.1 kN/m, the needed negative sitffness k_n keeps at a lower level. When the support stiffness k_1 increases to 10^4 kN/m, which approaches a rigid state, the needed negative stiffness closes to -3.95 kN/m, whose value is the same to the positive sitffness. Therefore, introducing a flexible supprot can achieve a cost-effective design of the isolator.

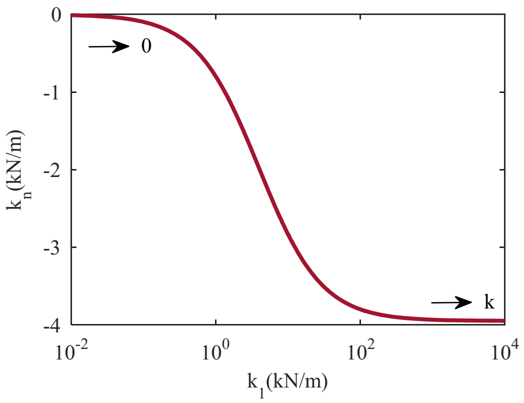

Fig. 5. Combination of the support stiffness and negative stiffness to realize quasi-zero state for the isolation.

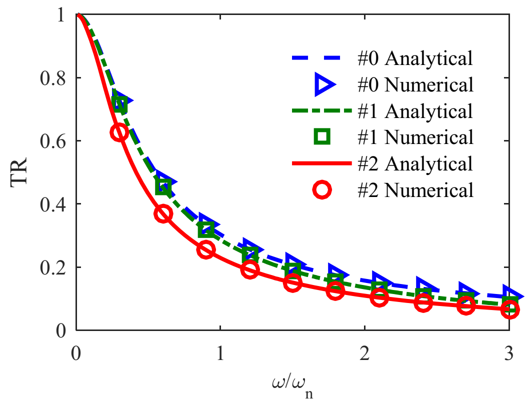

Fig.6 Isolation performance of three design cases under harmonic loads.

To be more exactly, three examples are selected for further comparison in Tab. 1. Among them, case #0 is the traditional method and the support is completely rigid. Therefore, the required negative stiffness is -3.95 kN/m to offset the positive stiffness. The required damping is 200 Ns/m. For case #1, a flexible support with stiffness 0.79 kN/m is incorporated. Owing to the advantageous effect of support flexibility, the required negative stiffness becomes -0.66 kN/m and viscous damping changes to 5 Ns/m. For case #2, when the support stiffness decreases to 0.79 kN/m, the negative stiffness alters to -0.33 kN/m, while the required viscous damping reduces to 1 Ns/m.

Tab. 1. Three design cases for the quasi-zero stiffness isolation system using different support stiffness.

	k_1 (kN/m)	k_n (kN/m)	c (N·s/m)
Case #0	rigid	-3.95	200
Case #1	0.79	-0.66	5
Case #2	0.36	-0.33	1

According to the above parameters, numerical models for the quasi-zero isolating system can be established using Matlab\Simulink. The adopted solver is ode 5, of which the time step is fixed with a value of 1×10^{-3} s.

4.3 Performance evaluation

First, the sinusoidal load is considered. Figure 6 shows the comparison of vibration isolation performance under case #0, case #1, and case #2. The numerical results are in good agreement with the analytical results. Compared with case #0, the acceleration transmissibility under case #1 is slightly smaller. Under case #2, the acceleration transmissibility even decreases further. Therefore, utilizing a flexible support can lead to a cost-effective design. Moreover, the flexible support performs out the rigid support condition from the respective of isolating performance.

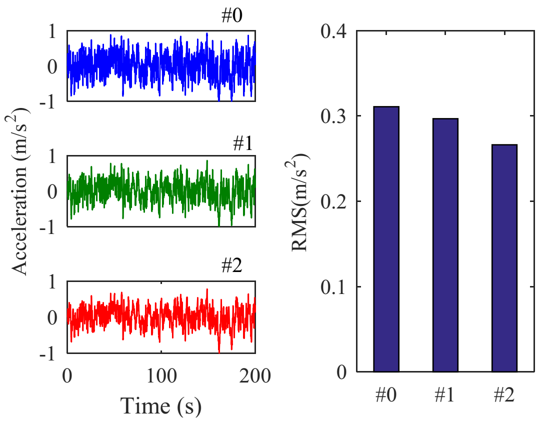

Fig. 7. Acceleration responses of the mass under white noise.

Then, the white noise is considered as external disturbance for further assessment. As shown in Fig. 7, the root mean square values of acceleration of the mass in three cases are 0.31 m/s2, 0.30 m/s2, and 0.27 m/s2, respectively. Compared with a rigid support, a flexible support can achieve better vibration isolation effect using smaller size of negative stiffness and viscous damping.

5 Conclusions

This paper proposed a quasi-zero stiffness vibration isolation system based on the flexibly supported negative stiffness mechanism. Equivalent model for the negative stiffness part is established. The effect of flexible support on the negative stiffness is analysed. Correspondingly, the optimized design procedure is presented. By numerical simulations, vibration isolation performance of the system is verified. The results show that the introduction of flexible support can effectively enlarge the original negative stiffness. Thus, the vibration isolation performance can be improved by optimizing the design in a cost-effective way.

Acknowledgments

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